Efficient Data-gathering using Graph-based Transform and Compressed Sensing for Irregularly Positioned Sensors

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***Abstract*—In this work, we propose a decentralized approach for energy efficient data-gathering in a realistic scenario. We address a major limitation of compressed sensing (CS) approaches proposed to data for wireless sensor network (WSN), namely, that they work only on a regular grid tightly coupled to the sparsity basis. Instead, we assume that sensors are irregularly positioned in the field and do not assume that sparsifying basis is known a priori. Under the assumption that the sensor data is smooth in space, we propose to use a graph-based transform (GBT) to sparsify the sensor data measured at randomly positioned sensors. We first represent the random topology as a graph then construct the GBT as a sparsifying basis. With the GBT, we propose a heuristic design of the data-gathering where aggregations happen at the sensors with fewer neighbors in the graph. In our simulations, our proposed approach shows better performance in terms of total power consumption for a given reconstruction MSE, as compared to other CS approaches proposed for WSN.**

I. Introduction

In wireless sensor networks (WSN), energy efficient data manipulation and transmission is very important for data gathering, due to significant power constraints on the sensors. This constraint has motivated the study of joint routing and compression for power-efficient data gathering of locally correlated sensor network data [1]. Recent work has shown how practical compression schemes such as distributed wavelets can be adapted to work efficiently with various routing strategies [2], [3]. The existing transform-based techniques can reduce the number of bits to be transmitted to the sink, thus achieving overall power savings. These transform techniques are essentially critically sampled approaches, so that their cost of gathering scales up with the number of sensors, which could be undesirable when large deployments are considered.

This motivated us to apply compressed sensing (CS) approaches to data-gathering in WSN. Traditional CS approaches have been focused on reducing the number of measurements, M, while achieving satisfactory reconstruction of the signal with the dimension of N ($M < N$). When the signal is K-sparse in a given sparsifying basis, $\mathbf{\Psi}$, and measurement (sensing) basis, $\mathbf{\Phi}$, and $\mathbf{\Psi}$ are incoherent, the signal can be successfully reconstructed with $M = O(K \log N)$ measurements [4], [5]. Thus, compared to the critically sampled approaches, the rate of increase of measurements is lower than that for the critically sampled approaches because it is a logarithmic function of N, $M = O(K \log N)$.

CS has been considered as a potential alternative in this context, as the number of measurements required depends on the characteristics (sparseness) and dimension of the signal [5]. Researchers have proposed various ways to apply CS to WSN in order to reduce the gathering costs. In [6], it was shown that CS could also operate using *sparse* random projections (SRP) but this work does not consider transport cost to collect measurements in a *multi-hop* network. In [7], the potential benefits of CS for sensor network applications have been recognized but significant obstacles remain for it to become competitive with more established (e.g., transform-based) data gathering and compression techniques. A primary reason is that CS theoretical developments have focused on *minimizing the number of measurements* (i.e., the number of samples captured), rather than on *minimizing the cost of each measurement*. To solve this problem, spatially-localized CS was proposed by optimizing the choice of measurement basis for a given sparsifying basis, taking into account both the distance between sensors and a new metric measuring the maximum energy overlap between measurement basis and sparsifying basis [8], [9].

CS-based approaches proposed to date have two major limitations in practice: (i) sensors are assumed to be uniformly placed on a 2D grid and (ii) performance is evaluated based on a discrete sparsifying basis, $\mathbf{\Psi}$, defined on the discrete sensor grid. The regular topology would be useful for monitoring buildings, bridges, or power plants but is not appropriate for many other applications such as monitoring of habitat, wild fire, or battle field. This motivates us to study how the CS-based approach can be extended to data-gathering with *irregularly* positioned sensors. Further, we consider the input data to be smooth or sparse in space (e.g., in a dense regular grid), *indenpendently* of the irregular position of the sensors. We investigate the use of graph-based transforms (GBT) to provide a sparse representation of realistic sensor data. In this paper, we propose a heuristic CS-based approach that exploits a characteristic of GBT in order to achieve energy efficient data-gathering among irregularly positioned sensors.

The rest of this paper is organized as follows. We briefly introduce CS and formulate a practical data-gathering problem in CS framework in Section II. Then we propose our approach with graph-based transform (GBT) in Section III, and provide simulation results to evaluate its performance in Section IV.

II. Problem Formulation

A. Compressed Sensing (CS)

In CS the N-sample signal ($\boldsymbol{x}$) can be recovered from M measurements or projections ($M < N$) onto a sensing (measurement) basis, $\boldsymbol{\Phi}$, if $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are incoherent [4], [5]. More formally, if a signal, $\boldsymbol{x} \in \Re^N$, is K-sparse in a given basis, $\boldsymbol{\Psi}$ (i.e., the sparsity inducing basis), $\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{a}, |\boldsymbol{a}|_0 = K$, where $K \ll N$, then theoretically we can reconstruct the original signal with $M = O(K \log N)$ measurements by finding the sparsest solution to an under-determined, or ill-conditioned, linear system of equations, $\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{a} = \boldsymbol{U}\boldsymbol{a}$, where $\boldsymbol{U}$ is known as the holographic basis. Reconstruction is possible by solving the convex unconstrained optimization problem: $\min_{\boldsymbol{a}} \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{U}\boldsymbol{a}\|_2^2 + \gamma\|\boldsymbol{a}\|_1$.

B. Signal Model and Random Deployment of Sensors

In this work, we consider a realistic data model with irregularly positioned sensors. We assume that $\boldsymbol{x} \in \Re^N$ is a vector containing measurements obtained by N sensors in a 2D region at a given time. We assume that data is spatially smooth in a regular fine grid but is sampled at N random irregular locations to form $\boldsymbol{x}$. This is illustrated by the example in Fig. 1, where smooth data is generated in a 2D region using an AR model operating in a 600×600 grid, then 256 sensors are randomly deployed, and $\boldsymbol{x}$ is the vector of measured values at those irregularly placed sensors. Thus, unlike in prior CS work in this context, it is possible to decouple smooth data generation (on a regular grid) from (irregular) sensor placement.

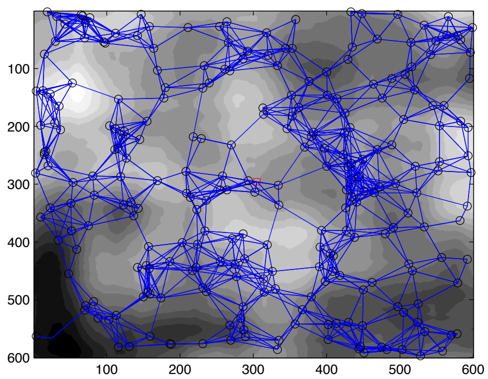


Fig. 1. Smooth data is generated using an AR model on a fine grid 600×600. Then, 256 sensors are assumed located in irregular positions. In order to represent WSN as a graph, the communication range is set as the minimum distance that results in a connected graph.

Under these realistic assumptions, we use graph-based transforms (GBT) as a sparsifying basis because the transform can be applied to various deployments of sensors if the topology is represented by a graph. For the construction of GBT, we first represent the WSN as a graph, $G(V, E)$ with nodes (sensors) and links (connections) between sensors as illustrated in Fig. 1. Note that the links can exist only if the two sensors are within a specific range. In this work, we set the range as the minimum, R_{min} such that the resulting graph is connected (i.e., there are no disconnected subgraphs) as shown in Fig. 1. Since the sensor data is likely to be highly correlated between adjacent sensors, the links between distant sensors that are farther apart can be disconnected for a sparser representation. The construction and the performance of GBT for the irregular sensor data will be presented in Section III.

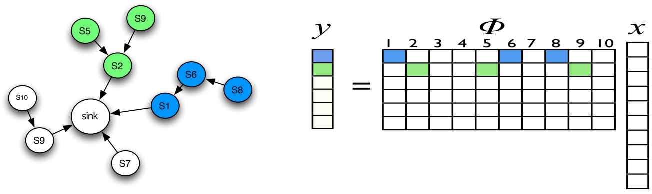


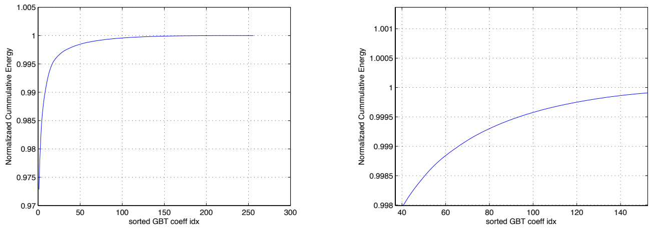
Fig. 2. Link between CS measurements and data aggregation in WSN.

With a given GBT as sparsifying basis, $\boldsymbol{\Psi}$, any aggregation scheme can be represented in CS terms by generating the corresponding measurement matrix, $\boldsymbol{\Phi}$, and using it to reconstruct the original signal. As shown in Fig. 2, each row of $\boldsymbol{\Phi}$ represents the aggregation corresponding to one measurement: we place non-zero (e.g., random) coefficients in the positions corresponding to sensors that provide their data for a specific measurement, while the other positions are set to zero. Thus, the sparsity of a particular measurement in $\boldsymbol{\Phi}$ depends on the number of active nodes participating in this aggregation. In order to achieve low cost aggregation in WSN, we need to minimize the number of measurements as well as the cost of each measurement, which depends on both the sparsity of $\boldsymbol{\Phi}$ and the locations of non-zero entries in $\boldsymbol{\Phi}$. Thus, we propose to aggregate data from a few sensors along shortest-path tree (SPT), as will be discussed in the next section.

III. Proposed Approach

In order to achieve a sparse representation of sensor data, we construct GBT from the graph representation of a WSN, $G(V, E)$, with nodes (sensors) and links (connections). From the graph, the adjacency matrix $\boldsymbol{A}$ is formed. Here, $\boldsymbol{A}(i,j) = \boldsymbol{A}(j,i) = 1\ \forall i \neq j$ if the distance between sensor i and j is smaller than the minimum range, R_{min}. Otherwise $\boldsymbol{A}(i,j) = \boldsymbol{A}(j,i) = 0$. Then we define the degree matrix $\boldsymbol{D}$, where $\boldsymbol{D}(i,i)$ is the number of links connected to the i^{th} sensor and $\boldsymbol{D}(i,j) = 0, \quad \forall i \neq j$. Finally, the Laplacian matrix can be defined as $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$.

After the eigenvalue decomposition of the Laplacian matrix, $\boldsymbol{L}$, we use the eigenvector matrix as a sparsifying basis, $\boldsymbol{\Psi}$, whose columns are the eigenvectors of $\boldsymbol{L}$. Note that $\boldsymbol{\Psi}$ is orthogonal because $\boldsymbol{L}$ is symmetric, leading to real eigenvalues and a set of orthogonal eigenvectors (refer to [10] for more details). Fig. 3 shows the performance of the GBT as a sparsifying basis. Although the sensor data is not perfectly sparse, the GBT shows a good compressibility, i.e., more than 99% of energy is compacted in a few GBT coefficients. Note that for a given underlying smooth data the level of compressibility in the GBT will depend on the specific location of the sensors. But seeking solutions that are sparse on the GBT (instead of in the original regular grid) allows us to accommodate realistic irregular sensor deployments.



(a) (b)

Fig. 3. (a) Compressible WSN data in the GBT and (b) its zoomed-in version. The x-axis shows the indices of GBT coefficients and the y-axis shows the cumulated sum of normalized energy of GBT basis functions. The data is generated by a second order AR model described in Section IV.

For CS-based data-gathering, we consider two approaches: SPT aggregation and GBT-aware aggregation. For the SPT aggregation, we randomly choose a certain number of sensors, and aggregate data of all the sensors on the SPT as proposed in [7]. Then, the linear combinations of data with Gaussian random coefficients are transmitted along the SPT. Alternatively, we propose the GBT-aware aggregation that selectively chooses the sensors along SPT. The choice is made by considering the number of the links connected to the sensors in the graph that is used for GBT construction as in Fig. 1. From a certain number of randomly chosen sensors, the aggregation happens along the SPT as in the first approach. But, an aggregation takes place at a sensor if the number of neighbors connected to the sensor is less than a threshold. Otherwise, the sensor relays the received data to its parent sensor along SPT.

The proposed approach is based on our previous work, which showed that projections with less maximum energy overlap, β, with the sparsifying basis (i.e., more evenly distributed energy overlap between them) lead to better reconstruction [9]. Also, we observe that the energy of GBT functions (i.e., columns of $\mathbf{\Psi}$) is unevenly distributed over sensors, and higher energy is compacted in the sensors that are connected to more neighbors, as shown in Fig. 4. Therefore, the aggregation over the sensors with fewer neighbors has higher probability to achieve more evenly distributed energy overlapped between the aggregation and the GBT functions, which leads to higher reconstruction accuracy.

The threshold is empirically chosen in this work. Once the threshold is determined, the threshold does not need to be updated if the topology remains the same (i.e., if the graph does not change). Thus, in our proposed approach the complete GBT is only required at the sink to reconstruct the received signal, so that sensors in the field do not need to know the complete topology of the network in order to transmit data. This indicates that the aggregation decision can be based on local characteristics of the network, leading to a decentralized operation.

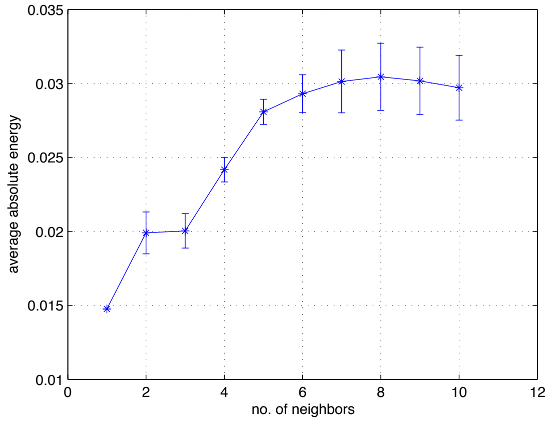


Fig. 4. The number of neighbors vs. average absolute energy with standard deviations. For a given number of neighbors, N_N, the energy metric is computed by averaging absolute energy of the GBT functions (i.e., column vectors of $\mathbf{\Psi}$) on the nodes connected to N_N neighbors in the graph used for the GBT construction as shown in Fig. 1.

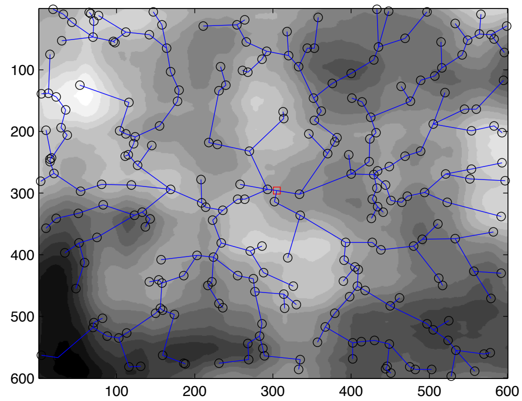


Fig. 5. 256 sensors in irregular positions and the corresponding SPT.

IV. Simulation Results

In this simulation, a second order AR model is used to generate 50 realizations with high spatial data correlation as shown in Fig. 5. More specifically, the AR filter $H(z) = \frac{1}{(1-\rho e^{jw_0 z^{-1}})(1-\rho e^{-jw_0 z^{-1}})}$, where $\rho = 0.99$ and $w_0 = 359$. For the simulation, 256 sensors are randomly positioned in the 600×600 grid and the data measured at each sensor is represented using 12 bits. Also, the measurements (or down-sampled data) are transmitted along the SPT as shown in Fig. 5. Note that the locations of the sensors do not change throughout our simulations. Also, for measuring energy consumption, we adopt a realistic cost model proposed in [11]. Energy in the sensors is dissipated when both transmitting, $E_T(k, D)$, and receiving data, $E_R(k)$. The energy consumption in k bit transmission over a distance D is $E_T(k, D) = E_{elec}k + \varepsilon_{amp}kD^2$ Joules and the consumption in k bit reception is $E_R(k) = E_{elec}k$.

In our simulation, we compare five different approaches: (i) SPT aggregation (CS_{SPT}) [7], (ii) GBT

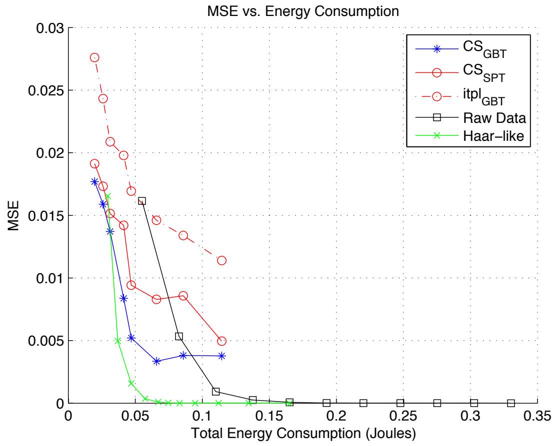


Fig. 6. Total energy consumption vs. MSE. The x-axis is the total energy consumption in Joules and the y-axis is MSE. The curves are generated by taking averages over 50 realizations of the sensor data.

interpolation($itpl_{GBT}$) [12], (iii) raw transmission without any compression, (iv) wavelet-based data gathering (Haar-like) [2], and (v) our proposed method (CS_{GBT}). For CS_{SPT}, we randomly choose M sensors, and aggregate data of all the sensors on the SPT from those sensors to the sink. For each aggregation, all the sensors on the SPT linearly combine the received aggregate with their readings using Gaussian random coefficients until the aggregates reach to the sink [7]. For $itpl_{GBT}$, we randomly choose M sensors and transmit the sampled data to the sink along the SPT. Then, data is reconstructed by the graph interpolation technique proposed in [12]. For CS_{GBT}, the aggregation happens along the SPT as in CS_{SPT} from M randomly chosen sensors. But, the aggregation takes place at a sensor if the number of neighbors connected to the sensor is less than a threshold. We empirically choose the threshold as 5, thus the aggregation happens at the nodes on SPT if the nodes have fewer than 5 neighbors in the graph in Fig. 1. For abovementioned approaches, we fix the quantization step size and change M to generate curves in Fig. 6. For the raw data transmission, every sensor transmits its reading (represented by 12 bits) to the sink along SPT. The wavelet-based approach performs the Haar-like transform in network and transmit the quantized transform coefficients to the sink [2]. The curves for the raw data transmission and wavelet approach are generated with different levels of quantization.

The result in Fig. 6 shows that our proposed approach outperforms the other methods in terms of the energy consumption and the reconstruction accuracy except for the wavelet-based approach. The GBT interpolation technique shows worse performance because it assumes a bandlimited graph signal supported only at frequencies $[0, w]$, but the cutoff frequency, w, in the graph of our simulation data is not small enough with respect to the downsampling rate (i.e., the ratio of the number of sensors providing samples with respect to the total number of sensors), so that the reconstruction quality is degraded.

The wavelet-based approach works better than CS_{SPT} and CS_{GBT} because the sensor signal is compressible but not exactly sparse as shown in Fig. 3 (b) and the CS-based approaches reconstruct signal from the measurements with quantization error caused by consecutive quantization processes (i.e., the quantization error propagates as more aggregations occur along SPT). Our proposed approach have some limitations that could be overcome with further work (e.g. quantization error and the transmission overhead to signal to the sink what random coefficients were used by the sensors). With respect to the wavelet-based approach, however, our proposed approach has the advantage that aggregation is simple and robust to the changes of sensor locations as long as the graph remains unchanged, which leads to the same GBT.

V. Conclusion

In this paper, we investigate how to overcome major shortcomings of CS approaches applied to WSN by taking into consideration practical scenario where sensors are irregularly positioned in 2D smooth field. The sensor data generated by the second order AR model is sparsely represented by graph-based transform (GBT) and successfully reconstructed by the proposed GBT-aware aggregation along SPT in energy efficient way. This approach can be decentralized since GBT is only required at the sink to reconstruct the received signal, so that sensors in the field do not need to know the complete topology of the network in order to transmit data. We plan to investigate the effect of different topology and density of sensors in our future work.

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