A RSS-Based passive locating algorithm for unknown signal sources

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***Abstract*— this paper studied a locating method to estimate a radio source's position in three-dimensional space based on wireless sensor networks (WSNs). We proposed a localization method that depends on the received signal strength (RSS) from sensing nodes and under the condition of an unknown transmission power of the radio source and uncertainty the path loss exponent from the source to nodes. A non-recursive method is adopted to derive the three-dimension coordinate analytic expression of the radio source rather than by setting initial position message and processing a complex iteration algorithm. The simulation results show that our proposal can achieve the locating estimate's Cramer-Rao Lower Bound when the mean square error of RSS is less than 3db.**

I. INTRODUCTION

WSN is a vital research direction in the field of information technology since the 21st century. Which is the most important basis of new type of networks. Such as Internet of things(IOT)[1],Cyber Physical System(CPS)[2],Internet of Vehicles(IOV)[3].Due to its special architecture and extensive application value. Many governments have been paying attention to WSN and a lots of scientists have been immersing in studying it for a long time. In the more than ten years since its appearance. A large number of research achievements and application systems have been produced. Which have promoted the development of network basic theory and network application technology[4].

Localization has always been a concern in WSN. At present, various locating approaches have been proposed for WSN. Such as Angle-of-Arrival (AOA), Received Signal Strength (RSS), Time-of-Arrival (TOA) and Time Difference of Arrival (TDOA). In these locating methods, RSS is widely used due to its low complexity and no extra hardware cost. For RSS the transmit power and the path loss exponent are two key parameters affect the locating accuracy. It is very difficult to use WSN to locate unknown sources, because the transmit power of the source is unknown, and the path loss exponent is a time-varying parameter which is influenced by the change of environments and time. In order to overcome this difficulty. Have proposed many two-dimensional locating algorithms based on RSS. Literature[5] proposed that weighted least square method(WLS) and unscented transformation (UT) were adopted to RSS to collectively estimate the location and transmit power of the sensor nodes. Aiming at such problems, the paper [6] used two-step weighted least square Method (2-WSL) to directly get the location closed solution without initial position and iteration. In [7], Generalized Total Least-Squares (GTLS) was used to solve the location parameters and transmit power of sensor nodes. In [8], A new Semidefinite Programming (SDP) method was proposed for non-convexity of Maximum Likelihood (ML).Which effectively solved the convex optimization problem of ML. And this paper also proposed the Weighted Total Least Squares (WTLS) algorithm. By simulation analysis, the locating accuracy of this paper was better than Least Square Method (LS) algorithm. In [9], the ratio method is used to eliminate the uncertainty of the transmission power, and then iteratively solves the path loss according to the range of path loss. The linear least squares is used to determine the position of the source node. Finally, the simulation results show that the algorithm Effectiveness Based on the above research achievement, we propose a source locating approach in 3D space and derive its analytic formula for the case where both source transmitting power and path loss exponent are unknown. it is shown that this method can reach the Cramer-Rao Lower Bound (CRLB) when the mean square error of RSS is small.

II. THE ESTABLISHMENT AND LINEARIZATION OF THE LOCATING EQUATIONS

By measuring the RSS parameters, the locating observation equation can be established based on the path loss model. However, in this equation, there is a non-linear relationship between the target location parameters and the locating observation parameters. Therefore, the first step is to make a certain approximation of the locating observation equation when we calculate the signal source location, so that the target position parameters are linear to the measurements, and then solve it. In the text, denote the measurement with noise of $\{*\}$ as $\{\hat{*}\}$ .The actual distance between the undetermined source and the ith sensor is defined as:

$$\begin{aligned} r_i^2 &= (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 \\ &= k_i - 2x_i x - 2y_i y - 2z_i z + k \end{aligned} \tag{1}$$

Where $k_i = x_i^2 + y_i^2 + z_i^2$, $k = x^2 + y^2 + z^2$.

In the above formulas. x_i, y_i, z_i is the coordinates of the ith sensor node. (x, y, z) is the coordinate of the signal source.

The received power $\widehat{P}_i$ detected by the ith sensor node can be obtained from Shadowing model. r_i is the distance from the source to the ith sensor ,so the correlation of $\widehat{P}_i$ and r_i can be defined as:

$$\widehat{P}_i = P_0 - 10\beta \lg^{\left(\frac{r_i}{r_0}\right)} + n_i \quad (2)$$

Here P_0 is the reference power at the reference distance d_0, and it mainly depends on the transmit power. $\widehat{P}_i$ is the measurement of received signal power of the sensor node. β is the loss exponent. It mainly depends on the environment and ranges between 2 and 6, and this value can be obtained by experimental method. r_i is the distance from the target to the front of the sensor node. r_0 is reference distance. Generally, $r_0 = 1m$, n_i follows the Gaussian distribution where the mean is 0 and the variance is δ^2.

Considering the uncertainty of path loss exponent, β is modeled as:

$$\beta = \beta_0 + n_\beta \quad (3)$$

where β_0 can be measured in a specific environment; n_β is used to describe the uncertainty of path loss exponents caused by time and other exponents, which follows the Gaussian distribution where the mean is 0 and the variance is δ^2.

r_i can be obtained from (2), which is as follows:

$$r_i = 10^{(P_0 - P_i)/10\beta} \quad (4)$$

Substitute (4) into (1), gives:

$$2x_i x + 2y_i y + 2z_i z - k + 10^{P_0/5\beta} 10^{-P_i/5\beta} = k_i \quad (5)$$

With considering the RSS noise and PLE disturbance, (5) can be linearized into (6):

$$\mathbf{GZ} + \mathbf{e} = \mathbf{Y} \quad (6)$$

Where :

$$\mathbf{G} = \begin{bmatrix} 2x_1 & 2y_1 & 2z_1 & -1 & 10^{-\widehat{P}_1/5\beta} \\ 2x_2 & 2y_2 & 2z_2 & -1 & 10^{-\widehat{P}_2/5\beta} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2x_N & 2y_N & 2z_N & -1 & 10^{-\widehat{P}_N/5\beta} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_N \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} x \\ y \\ z \\ k \\ 10^{P_0/5\beta} \end{bmatrix}$$

In the above formulas. $(x_1, y_1, z_1), (x_2, y_2, z_2), \cdots, (x_N, y_N, z_N)$ is the coordinates of the sensor node. (x, y, z) is the coordinate of the signal source. k is the distance from signal source to the coordinate origin. k_i is the distance from sensor node to the coordinate origin. k is the distance from source to the coordinate origin. $\widehat{P}_1, \widehat{P}_2, \cdots, \widehat{P}_N$ is the received power of each sensor node. P_0 is the received power at reference distance d_0. $\mathbf{e}$ is the error.

III. SOLVING THE EQUATION BY WEIGHTED LEAST SQUARES (WLS)

Using WLS to solve $\mathbf{Z}$ from (6), gives (7):

$$\begin{aligned} \mathbf{Z} &= \arg\min\left\{ (\mathbf{Y} - \mathbf{GZ})^T \mathbf{\Psi}^{-1} (\mathbf{Y} - \mathbf{GZ}) \right\} \\ &= \left(\mathbf{G}^T \mathbf{\Psi}^{-1} \mathbf{G} \right)^{-1} \mathbf{G}^T \mathbf{\Psi}^{-1} \mathbf{Y} \end{aligned} \quad (7)$$

Where $\mathbf{\Psi}$ is the covariance matrix of $\mathbf{e}$, it is expressed as :

$$\mathbf{\Psi} = \mathrm{cov}(\mathbf{e}) = E(\mathbf{ee}^T) \quad (8)$$

If the squared error term is ignored in (6), e_i in $\mathbf{e}$ can be reformulated as:

$$e_i = \frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_i}{5\beta}} (\beta n_i + (P_0 - P_i) n_\beta) \quad (9)$$

The expectation of $e_{i,j}$ (10) and $e_{i,i}$ (11) can be computed by (9):

$$\begin{aligned} E(e_i e_j) &= E\left[\begin{array}{l} \left(\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_i}{5\beta}} \right) \left(\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_j}{5\beta}} \right) \\ \times \left(\beta n_i + (P_0 - P_i) n_\beta \right) \left(\beta n_j + (P_0 - P_j) n_\beta \right) \end{array} \right] \\ &= \left(\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_i}{5\beta}} \right) \left(\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_j}{5\beta}} \right) (P_0 - P_i)(P_0 - P_j) \sigma_\beta^2 \end{aligned} \quad (10)$$

$$\begin{aligned} E(e_i^2) &= E\left[\left(\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_i}{5\beta}} \right)^2 \left(\beta n_i + (P_0 - P_i) n_\beta \right)^2 \right] \\ &= \left(\left(\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_i}{5\beta}} \right)^2 \left(\beta^2 \delta^2 + (P_0 - P_i)^2 \delta_\beta^2 \right) \right) \end{aligned} \quad (11)$$

Substitute (10) (11) into (9) to get (12):

$$\boldsymbol{\psi} = \mathrm{cov}(\mathbf{e}) = E(\mathbf{ee}^T) = \mathbf{BQB} \quad (12)$$

Where: $\mathbf{B} = diag\left\{ \left[\frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_i}{5\beta}} \cdots \frac{\ln 10}{5\beta^2} 10^{\frac{P_0 - P_N}{5\beta}} \right] \right\}$

$$\mathbf{Q} = \begin{bmatrix} \beta^2\delta^2 + (P_0 - P_1)^2 \delta_{5\beta}^2 & \cdots & (P_0 - P_1)(P_0 - P_N)\delta_\beta^2 \\ (P_0 - P_2)(P_0 - P_1)\delta_\beta^2 & \cdots & (P_0 - P_2)(P_0 - P_N)\delta_\beta^2 \\ \vdots & \ddots & \vdots \\ (P_0 - P_N)(P_0 - P_1)\delta_\beta^2 & \cdots & \beta^2\delta^2 + (P_0 - P_N)^2 \delta_\beta^2 \end{bmatrix}$$

Solving the covariance matrix $\boldsymbol{\psi}$ depends on P_0 and P_i , so a closer approximation is necessary to achieve this problem. Firstly, we can use measurements of received power $\widehat{P}_i$ instead of actual value P_i .Secondly, P_0 can be substituted by an estimate of the LS method $\mathbf{Z} = \left(\mathbf{G}^T \mathbf{G} \right)^{-1} \mathbf{G}^T \mathbf{Y}$, this can be rewritten as:

$$\widehat{P}_0 = 5\beta \log_{10} \mathbf{Z}_5 \quad (13)$$

When ignore the influence of the uncertain path loss exponent δ_β^2 then solving (13) by LS method. Therefore, the disturbance can be used to calculating the covariance matrix $\mathbf{Z}$. Δ is denoted as error disturbance when there is noise interference:

$$\mathbf{G} = \mathbf{G}_0 + \Delta\mathbf{G}, \mathbf{Z} = \mathbf{Z}_0 + \Delta\mathbf{Z}, \mathbf{Y} = \mathbf{Y}_0 \quad (14)$$

So (7) can be written as:

$$\left(\mathbf{G}^T \boldsymbol{\psi}^{-1}\mathbf{G}\right)\mathbf{Z} = \mathbf{G}^T \boldsymbol{\psi}^{-1}\mathbf{Y} \quad (15)$$

Substituting (14) into (15) and ignoring its squared error term, gives:

$$\begin{aligned}&\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{G}_0\mathbf{Z}_0 + \mathbf{G}_0^T \boldsymbol{\psi}^{-1}\Delta\mathbf{G}\mathbf{Z}_0 + \Delta\mathbf{G}^T \boldsymbol{\psi}^{-1}\mathbf{G}_0\mathbf{Z}_0 + \mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{G}_0\Delta\mathbf{Z}\\ &= \mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{Y}_0 + \Delta\mathbf{G}^T \boldsymbol{\psi}^{-1}\mathbf{Y}_0\end{aligned} \quad (16)$$

Without noise of RSS and disturbance of PLE, $\mathbf{G}_0\mathbf{Z}_0 = \mathbf{Y}_0$, substitute it into (16) and we can obtain:

$$\begin{aligned}&\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\Delta\mathbf{G}\mathbf{Z}_0 = -\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{G}_0\Delta\mathbf{Z}\\ &\Delta\mathbf{Z} = -\left(\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{G}_0\right)^{-1}\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\Delta\mathbf{G}\mathbf{Z}_0\end{aligned} \quad (17)$$

Because of $\mathbf{G}_0\mathbf{Z}_0 = \mathbf{Y}_0$, eq.**(6)** can be formulated as $\mathbf{e} = -\Delta\mathbf{G}\mathbf{Z}_0$ and substitute it into (17), get $\Delta\mathbf{Z} = \left(\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{G}_0\right)^{-1}\mathbf{G}_0^T \boldsymbol{\psi}^{-1}\mathbf{e}$ .Substituting $\Delta\mathbf{Z}$ into cov($\mathbf{Z}$) ,t he covariance matrix of $\mathbf{Z}$ is defined as:

$$\mathrm{cov}(\mathbf{Z}) = E\left[\Delta\mathbf{Z}\Delta\mathbf{Z}^T\right] = (\mathbf{G}^T \boldsymbol{\psi}^{-1}\mathbf{G})^{-1} \quad (18)$$

In the above solution, we have been assuming that x, y, z and k are independent from each other. However, it can be seen from (1) that these parameters are related to each other. Therefore, the relationship between x, y, z and k can be used to further improve the estimation accuracy, the improved expression is as follows:

$$\mathbf{e}' = \mathbf{Y}' - \mathbf{G}'\mathbf{Z}' \quad (19)$$

Where: $\mathbf{Y}' = \begin{bmatrix}\mathbf{Z}_1^2\\ \mathbf{Z}_2^2\\ \mathbf{Z}_3^2\\ \mathbf{Z}_4\end{bmatrix}$ $\mathbf{G}' = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 1\end{bmatrix}$ $\mathbf{Z}' = \begin{bmatrix}x^2\\ y^2\\ z^2\end{bmatrix}$

Suppose the estimated errors of x, y, z and k are u_1, u_2, u_3 and u_4 ,the elements in $\mathbf{Z}$ can be written as:

$$\mathbf{Z}_1 = x + u_1, \mathbf{Z}_2 = y + u_2, \mathbf{Z}_3 = z + u_3, \mathbf{Z}_4 = k + u_4 \quad (20)$$

Substituting (20) into (19) and ignoring its squared error term, $\mathbf{e}'$ can be expressed as:

$$e'_1 = 2xu_1, e'_2 = 2yu_2, e'_3 = 2zu_3, e'_4 = u_4 \quad (21)$$

So the covariance matrix of $\boldsymbol{e}'$ can be reproduced as:

$$\boldsymbol{\psi}' = E(\mathbf{e}'\mathbf{e}'^T) = \mathbf{B}'\{\mathrm{cov}(\mathbf{Z})\}_{(1:4)\times(1:4)}\mathbf{B}' \quad (22)$$

Where, $\mathbf{B}' = diag\{[2x, 2y, 2z, 1]\}$ .Actually $\mathbf{B}'$ is unknown because it contains the true coordinate x, y, z of the signal source. Same as (12) to approximate the $\mathbf{B}'$ as $\mathbf{B}' = diag\{[2\mathbf{Z}_1, 2\mathbf{Z}_2, 2\mathbf{Z}_3, 1]\}$.

Therefore, the WLS method is used again to solve the coordinate parameters of the final location of the signal source, it is as follow (23):

$$\mathbf{Z}' = \left(\mathbf{G}'^T \boldsymbol{\psi}'^{-1}\mathbf{G}'\right)^{-1}\mathbf{G}'^T \boldsymbol{\psi}'^{-1}\mathbf{Y}' \quad (23)$$

The location coordinate of the signal source are finally solved by (24) :

$$\mathbf{Z} = \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}sign(\mathbf{Z}_1)\sqrt{\mathbf{Z}'_1}\\ sign(\mathbf{Z}_2)\sqrt{\mathbf{Z}'_2}\\ sign(\mathbf{Z}_3)\sqrt{\mathbf{Z}'_3}\end{bmatrix} \quad (24)$$

IV. SIMULATION AND EXPERIMENT RESULTS ANALYSIS

There are 6 sensor nodes which are randomly distributed within $50m*50m*50m$ sensor network monitoring range. The coordinate of the signal source is $(20,20,20)m$, and the signal received power is $P_0 = 40db$ at 1 meter away from the located source. Path loss exponent is β=3 . The RSS measurement of signal strength which is received by each sensor node involved in positioning, should satisfy the mathematical model of formula (2). The positioning accuracy is measured by (25)

$$RMSE = \sqrt{\frac{1}{\text{L-1}}\sum_{i=1}^{L}((\hat{\mathbf{Z}}_i - \mathbf{Z})^2)} \quad (25).$$

Where $L = 10000$ .There are two experiments. In the first experiment, we will simulate the impact of RSS measuring errors on positioning accuracy. In this experiment, the error variance of the uncertain PLE is δ_β^2=0.2 , and the measurement error noise of RSS is $\delta_R^2 \in [1,5]db$. In the second experiment, we will analyze the effect of the uncertain PLE on locating accuracy. In this experiment, measurement error noise of RSS is δ_R^2=3db and δ_R^2=3.5db respectively, the uncertain PLE is δ_β^2=[0.1,0.5] .

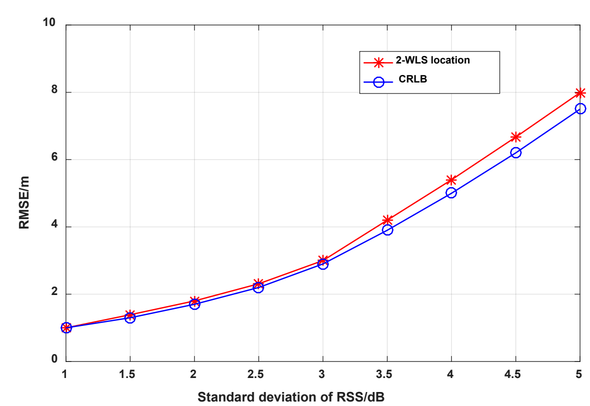

Fig. 1. Relationship between RSS noise and positioning accuracy

Figure1 shows when the RSS measurement error is less than 3db, the locating accuracy achieved by our method can reach CRLB. When the RSS measurement error is greater than 3db, the positioning error gradually deviates from the CRLB, but the deviation is only a little.

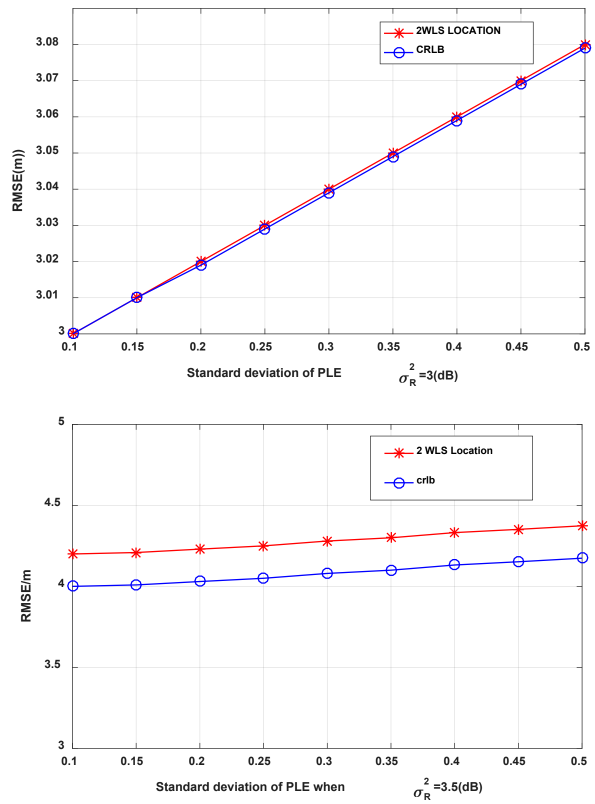

Fig. 2. correlation between PLE noise and locating accuracy

Figure2 shows the uncertainty of the path loss factor can affect the accuracy of the position and the positioning accuracy decreases as the error of the PLE increases. When the RSS noise variance is less than 3db, this algorithm can reach the CRLB, but it deviates from CRLB if it is greater than 3db. As can be seen from the above figure, when the variance of RSS noise is equal 3db, the positioning error curve deviates about only 0.2m.

V. CONCLUSION

In this paper, we propose a passive location algorithm of signal sources in a 3D space, which can directly give us the coordinates of the source without iteration. It is more suitable for practical application while reducing computation. The uncertainty of source transmit power in passive location is solved by two-step WLS, and it also overcomes the time varying of PLE caused by environment and time. Simulation results show that the proposed approach is effective.

VI. ACKNOWLEDGMENT

Fund Project: National Natural Science Foundation of China (61461056) and (61662089).

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