Trimmed-Mean FxLMS Algorithm for ANC in Impulsive Noise

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***Abstract*—The filtered-x least mean square (FxLMS) algorithm has been widely used for active noise control (ANC), which has a satisfying convergence performance in the Gaussian noise. Nevertheless, the ANC system can receive an unusual signal contaminated by the impulsive noise (IN) in practical applications, which results in serious convergence performance degradation. Some references can be found discussing this issue, but many of the proposed solutions highly rely on the setup of pre-determined parameters depending on special situations. In this paper, the IN problem is dealt with by some robust algorithms, which are adapted from order statistics. With the new FxLMS algorithms, there is no pre-requisite to determine the parameters as required in the past works. Through simulation results, we show that the proposed method has better average noise reduction convergence performance compared to other robust methods.**

***Index Terms*—Active noise control, Impulsive noise, FxLMS, Order statistics, Robust algorithm.**

I. Introduction

Noise cancellation methods can be generally divided into two different approaches: one is passive cancellation and the other is active cancellation. The passive cancellation approach basically relies on the material property to prevent from the noise. Due to practical limitations, the performance of the passive cancellation methods may be subject to the material engineering and physical design, usually also going with the cost of higher price. Thanks to the improvement of modern digital technology, the active cancellation approach has received a lot of attention, usually together with a satisfying performance and rich study in recent years.

In recent years, more and more audio applications have been introduced because of the advance in technology development. The requirement of active noise control (ANC) is widely found in cars, mobile phones, fans, etc. ANC goes to process the received sound waves in an earphone, for example, and the secondary path tries to generate the signal which is close to the noise appearing in the primary path. The noise is then canceled through a loudspeaker embedded in the earphone. ANC can improve the efficiency in noise control with lower volume and cost [1][2]. The most widely used method in the ANC approach is to apply the filtered-x least mean square (FxLMS) algorithm [2]. The algorithm has the advantages of robust performance [3], low computational complexity, and ease of implementation.

When the ANC system encounters the impulsive noise, the FxLMS algorithm will not reach a satisfying steady-state performance that can be achieved in the Gaussian noise condition. In [4], a new robust method called the filtered-x least mean p-power algorithm (FxLMP) was proposed, but its cost function $J(n) = E\{|e(n)|^p\} \approx |e(n)|^p$, where p is an integer number, indicates that the better results require knowing the prior parameter p, which is not an easy task. In [5], there is a simple variant of the FxLMS algorithms for ANC to deal with the impulsive noise. In [6] and [7], Akhtar's algorithm improved the performance better than the Sun's algorithm [5], where if the reference signal is over a pre-determined threshold. In above mentioned algorithms, the common problem using those methods is to find appropriate threshold parameters, which may be not easy to be well used with on-line operation in general ANC systems. In [8], some of the robust functions were introduced including classical functions such as Huber, Tukey bisquare, and Hampel. In [9], the Hampel function was brought into the LMS algorithm to resist impulsive noise in ANC.

The method proposed in this paper applies the order statistics. In [10]-[11], order statistics type LMS algorithms were developed. Here, the trimmed mean FxLMS is studied together with impulsive noise detection and preprocessing. From the analysis of averaged noise reduction performance, simulation results show that the trimmed mean FxLMS has good robustness compared to other robust method to deal with the impulsive noise.

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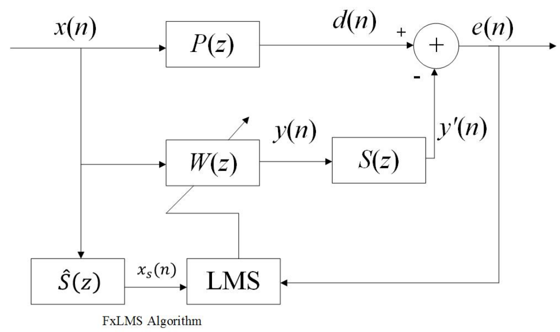

Fig. 1: Functional block of the conventional FxLMS algorithm.

II. FxLMS Algorithm

Fig. 1 depicts the functional block of the conventional single-channel feed-forward ANC structure using the FxLMS algorithm [2]. The noise source $x(n)$ is received from the receiver microphone, the system response $P(z)$ in the primary path is modeled for the physical channel between the receiver microphone and the error microphone, and the secondary-path response $S(z)$ models the characteristics of the secondary loudspeaker in the earphone. In addition, $\hat{S}(z)$ is the estimated response of $S(z)$ in the FxLMS algorithm, which can be obtained by on-line or off-line methods in practical applications. The objective of the adaptive filter $W(z)$ is to minimize the residual error signal $e(n)$, which essentially establishes the adaptation criterion in the FxLMS algorithm.

Assuming $W(z)$ is an finite impulse response (FIR) filter of the length of L taps, the corresponding output signal $y(n)$ is expressed as

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n), \tag{1}$$

where $\mathbf{w}(n) = [w_0(n), w_1(n), w_2(n), \cdots, w_{L-1}(n)]^T$ is the tap coefficient vector of $W(z)$ and $\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T$ is the $L \times 1$ input signal vector. The residual error signal $e(n)$ received by the error microphone is given by

$$e(n) = d(n) - y^{'}(n), \tag{2}$$

and $d(n) = p(n) * x(n)$ is the primary disturbance noise and $y^{'}(n) = s(n) * y(n)$ is the secondary antinoise signal, where $*$ denotes convolution, $p(n)$ is the impulse response of the primary path model $P(z)$, and $s(n)$ is the impulse response of the secondary path model $S(z)$.

It is known that the least mean square (LMS) algorithm minimizes the mean square error (MSE) of the error signal to adaptively find the optimum filter coefficients. The negative gradient direction with a step size μ is established for the LMS algorithm with the tap update equation, usually written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu}{2}\nabla J(n), \tag{3}$$

where ∇ denotes taking gradient, which is used to minimize the MSE of the cost function

$$J(n) = E[e^2(n)] \approx e^2(n), \tag{4}$$

where $E[\cdot]$ is the expectation operation. The FxLMS algorithm is modified by giving

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_1 e(n)\mathbf{x}_s(n), \tag{5}$$

where μ_1 is the step size for the FxLMS algorithm, $\mathbf{x}_s(n) = [x_s(n), x_s(n-1), \cdots, x_s(n-L+1)]^T$, and $x_s(n) = \hat{s}(n) * x(n)$, where $\hat{s}(n)$ is the impulse response of the estimated secondary path model $\hat{S}(z)$.

III. Robust FxLMS Algorithm

A. Impulsive Noise Preprocessing

In the FxLMS structure, the input signal $x(n)$ will be first passed through $\hat{S}(z)$. However, $\hat{S}(z)$ is an FIR filter that can span the influence of the impulsive noise in computing $W(z)$. Since the occurrence probability of the impulsive noise is small and the noise amplitude is very large in practical applications, we can preprocess the signal $x(n)$ to remove the large inputs before it passes through $\hat{S}(z)$.

Suppose σ_g is the standard deviation of $x(n)$ after removing the impulsive noise. Based on the $3\sigma_g$ property of the Gaussian signal, the probability of a Gaussian input signal which value does not exceed $3\sigma_g$ accounts for approximately 0.99. Hence, we may think that when the input signal exceeds $3\sigma_g$, it can be regarded as an impulsive noise. Then, a simple preprocessing rule is proposed as follows:

$$x'(n) = \begin{cases} x(n), & \text{if } x(n)\in[-3\sigma_g, +3\sigma_g] \\ x(n-1), & \text{otherwise} \end{cases} \tag{6}$$

That is, we simply replace the detected sample contaminated by the impulsive noise with the sample at its previous time.

B. Trimmed-Mean FxLMS Algorithm

The well-known mean filter is a linear method that is generally used to reduce noise and is suitable in a Gaussian noise environment, but it cannot provide good results if it encounters the impulsive noise. The principle of order statistics lies in robustness applications with sorting a range of values, which is nonlinear and can perform well under impulsive noise. The trimmed mean filter is based on the combination of linear and nonlinear methods. The first step is to select a range of samples and then to arrange the values from small to large. After removing the maximum and minimum outliers, the remainder is to calculate the result by averaging. When the adaptive filter $W(z)$ in an ANC system encounters the impulsive noise, the convergence of $W(z)$ will be destroyed, resulting in a performance decrease and even an entire convergence failure.

To replace the original FxLMS with the proposed trimmed-mean FxLMS (short for Trm-FxLMS), we define an $L \times N$ matrix $\mathbf{Z}(n)$ as $\mathbf{Z}(n) = [e'(n-N+1)\mathbf{x}_s'^T(n-N+1) \cdots e'(n-1)\mathbf{x}_s'^T(n-1)\ e'(n)\mathbf{x}_s'^T(n)]$, where $\mathbf{x}_s'(n) = \hat{s}(n) * \mathbf{x}'(n)$, which can be represented as the matrix in the following:

$$\mathbf{Z}(n) = \begin{bmatrix} z_{11}(n) & z_{12}(n) & \cdots & z_{1N}(n) \\ z_{21}(n) & z_{22}(n) & \cdots & z_{2N}(n) \\ \vdots & \vdots & \ddots & \vdots \\ z_{L1}(n) & z_{L2}(n) & \cdots & z_{LN}(n) \end{bmatrix}. \tag{7}$$

After rearranging the elements from small to large for each row in the matrix, we have a new matrix

$$\widetilde{\mathbf{Z}}(n) = \begin{bmatrix} \widetilde{z}_{11}(n) & \widetilde{z}_{12}(n) & \cdots & \widetilde{z}_{1N}(n) \\ \widetilde{z}_{21}(n) & \widetilde{z}_{22}(n) & \cdots & \widetilde{z}_{2N}(n) \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{z}_{L1}(n) & \widetilde{z}_{L2}(n) & \cdots & \widetilde{z}_{LN}(n) \end{bmatrix} \tag{8}$$

where $\widetilde{z}_{i1}(n) \leq \widetilde{z}_{i2}(n) \leq \cdots \leq \widetilde{z}_{iN}(n)$, $i = 1, 2, \cdots, L$. Then, defining the trimmed mean operation of $\mathbf{Z}(n)$ as

$$\text{trm}\{\mathbf{Z}(n)\}_{N,M} = \frac{1}{N-2M} \begin{bmatrix} \sum_{i=M+1}^{N-M} \widetilde{z}_{1i}(n) \\ \sum_{i=M+1}^{N-M} \widetilde{z}_{2i}(n) \\ \vdots \\ \sum_{i=M+1}^{N-M} \widetilde{z}_{Li}(n) \end{bmatrix}, \tag{9}$$

the update equation of the trm-FxLMS algorithm is

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w \text{trm}\{\mathbf{Z}(n)\}_{N,M} \tag{10}$$

where N is assumed to be an odd number, for simplicity, and $2M$ samples are moved out from the average operation in the N-point trm-FxLMS algorithm. If M is too small, the FxLMS algorithm is easy to be affected by the impulsive noise.

There are two special cases that should be noticed when $M = (N-1)/2$ and $M = 0$.

a) median FxLMS: If $M = (N-1)/2$, the remained sample for the FxLMS method is the median of $[z_{i1}(n)\ z_{i2}(n)\ \cdots\ z_{iN}(n)]$, $i = 1, 2, \cdots, L$. The trm-FxLMS actually becomes the median FxLMS (or short for med-FxLMS).

b) mean FxLMS: If $M = 0$, no sample is discarded in the trm-FxLMS, that is, the trm-FxLMS becomes the mean FxLMS. In the Gaussian noise, the mean FxLMS approaches the conventional FxLMS and has better convergence performance than the trm-FxLMS with $M \neq 0$.

C. FxLMS/trm-FxLMS Switching

The trm-FxLMS has better performance than the conventional FxLMS in the impulsive noise while worse in the Gaussian noise. To profit from both merits, the robust algorithm is to switch the LMS update equation between the conventional FxLMS and trm-FxLMS based on the $3\sigma_g$ rule. Only if the impulsive noise is detected, the trm-FxLMS is employed, otherwise, the conventional FxLMS is performed.

IV. Simulation Results

In this section, the performance of the trm-FxLMS algorithm is compared with those of the conventional FxLMS and other robust FxLMS algorithms such as the mean FxLMS, med-FxLMS, log-FxLMS, Huber-FxLMS, and Hampel-FxLMS algorithms. The trimmed mean [12] is defined as calculating the mean after discarding given parts of the samples of values at high and low ends. The tap lengths of the primary and secondary path models, $P(z)$ and $S(z)$, are 65 and 49, respectively. The frequency responses of the two models are plotted in Figs. 2 and 3. The effective tap length of the adaptive filter $W(z)$ is chosen as 50.

In this work, the impulsive noise is simulated with the symmetric alpha stable distribution which characteristic function is written by [13]

$$\varphi(t) = exp(jat - \gamma|t|^{\alpha}) \tag{11}$$

where α is a predetermined value in $[0, 2]$, a is the location parameter, γ is the scale parameter and $\gamma > 0$. The smaller the α, the longer the tail of this distribution becomes, and the proportion of impulsive noise will increase. If $\alpha = 2$, the distribution is Gaussian. If $\alpha = 1$, it becomes the Cauchy distribution. When $\gamma = 1$, it is called the standard SαS distribution. We consider two cases of $\alpha = 2$ and $\alpha = 1.8$ in our simulations. For Case 1, $\alpha = 2$, we choose $N = 11$ since the input signal is Gaussian. For Case 2, $\alpha = 1.8$, we choose $N = 61$ to deal with the impulsive noise. M is chosen to be 20% of N for the trm-FxLMS. Besides, the estimated secondary path model $\hat{S}(z)$ is obtained by Chang's online secondary path estimation FxLMS structure [2].

We compare the performance in terms of the metric of averaged noise reduction (ANR) [14]. Define ANR in decibel (dB) as

$$ANR(n)[\text{dB}] = 20\log_{10}\left(\frac{A_e(n)}{A_d(n)}\right) \tag{12}$$

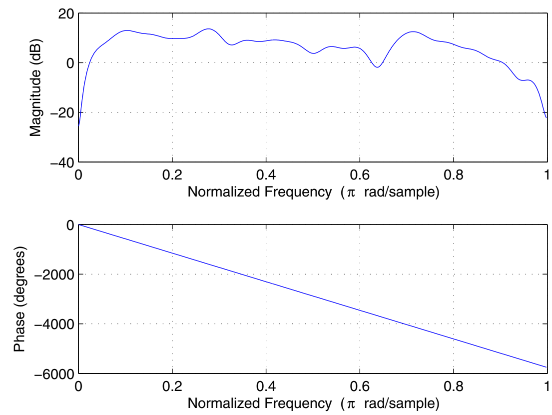

Fig. 2: Frequency response of the primary path model $P(z)$.

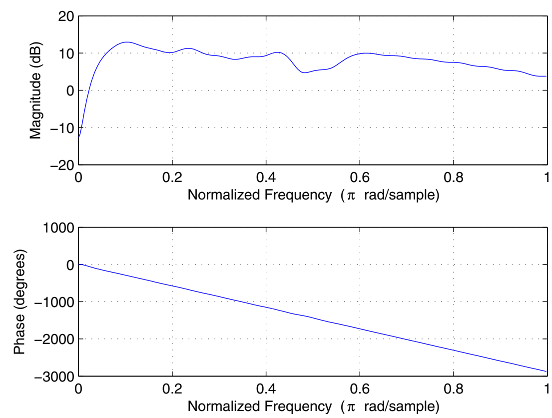

Fig. 3: Frequency response of the secondary path model $S(z)$.

where

$$A_e(n) = \lambda A_e(n-1) + (1-\lambda)|e(n)| \tag{13}$$

$$A_d(n) = \lambda A_d(n-1) + (1-\lambda)|d(n)| \tag{14}$$

where $|\cdot|$ represents absolute value, λ is a forgetting factor and is set as 0.999.

A. Experimental Case 1

The results of ANR are shown in Fig. 4. We turn off the online secondary path estimation at the 1.4×10^4 iteration to check the change of convergence performance. In the Gaussian noise, we can see that the FxLMS and mean FxLMS have better convergence rate while med-FxLMS and trm-FxLMS are worse and approach other robust FxLMS methods. In additions, adding the switching method is effective to let med-FxLMS and trm-FxLMS work perfectly in the Gaussian noise. As the on-line secondary path estimation function is turned off, the ANR performances of those robust FxLMS can be improved about 5dB except for the log-FxLMS.

B. Experimental Case 2

In this case, the input signal $x(n)$ is modeled by the SαS distribution with $\alpha = 1.8$ for simulating the impulsive noise as

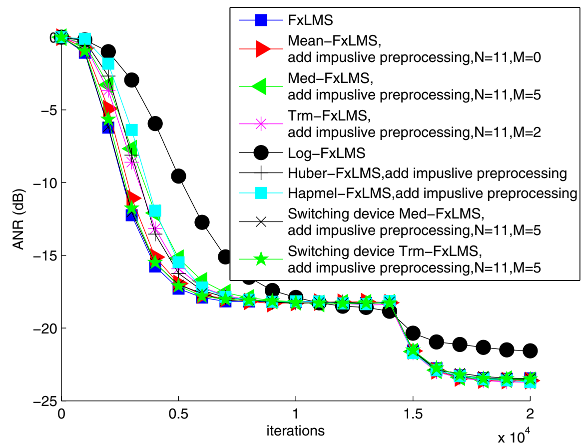

Fig. 4: ANR curves of the compared FxLMS algorithms in Experiment Case 1.

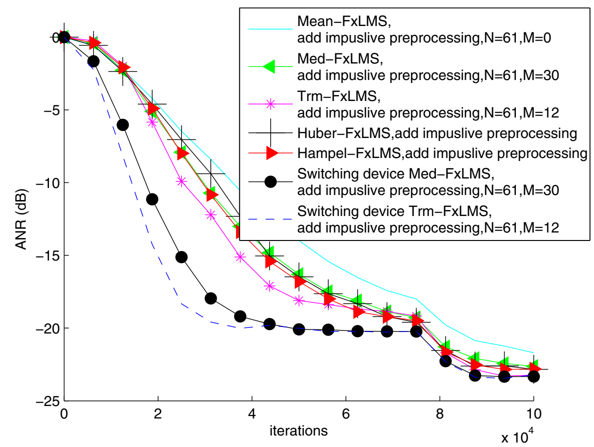

Fig. 6: ANR curves of the compared FxLMS algorithms in Experiment Case 2.

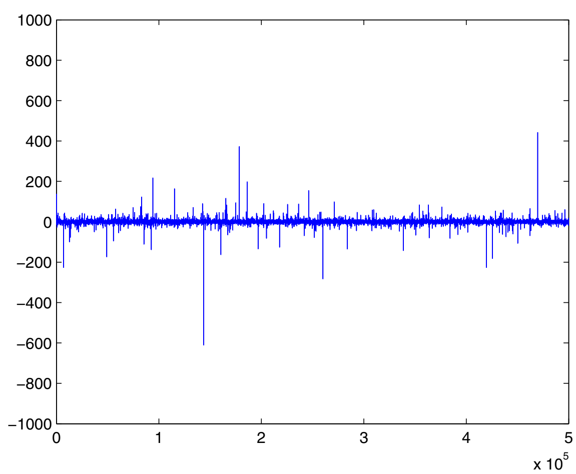

Fig. 5: Input signal $x(n)$ modeled by the SαS distribution with $\alpha = 1.8$ for simulating the impulsive noise in Experiment Case 2.

shown in Fig. 5. The online secondary path estimation mode is turned to offline at the 7.5×10^4 iteration. The results of ANR are shown in Fig. 6, where we can see that the convergence rate of the trm-FxLMS is better than other algorithms even when the switching method is not employed. The conventional FxLMS fails in convergence such that we do not plot its ANR result here and the mean FxLMS becomes the worst one in this comparison. As the switching method is applied, the performances of the med-FxLMS and the trm-FxLMS are greatly improved.

V. Conclusion

The proposed order statistics type FxLMS, specially the trm-FxLMS, has sufficient robustness to deal with the influence of the impulsive noise. According to the simulation results, even when the input signal has intensive impulsive noises, the proposed algorithm still can effectively suppress the impulsive noise with a good ANR performance. We can see that the proposed method has better convergence performance compared to other robust FxLMS methods.

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