A variable structure for linear phase maximally flat FIR digital filters

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***Abstract*—This paper proposes a novel variable structure for linear phase frequency-selective maximally flat FIR digital filters. The proposed method expresses the transfer function by cascading and paralleling the 2nd order variable subfilters. The multiplier of the subfilter requires very few multiplication in update compared with the conventional maximally flat variable FIR digital filters. Hence, the proposed structure can change the frequency response immediately.**

I. Introduction

Finite impulse response (FIR) digital filters are widely used because they are absolutely stable and can realize linear phase response [1]–[7]. Linear phase FIR digital filters don't cause any phase distortion, so that they are important in communication systems, image processing, and so on. Maximally flat (MF) FIR digital filters are well known by ripple-less frequency response which can realize high accuracy extraction of input signals in the passband [3]–[7]. These filters are designed to satisfy flatness constraints such that the frequency response and its first- to high-order derivatives at several frequencies must be exactly same as those of ideal frequency response. Here, we refer to the highest differential order in the flatness constraint for a frequency as the flatness degree at the frequency.

Recently, variable FIR digital filters have been investigated to adjust the frequency response without re-designing [8]–[10]. In connection with this, Wang *et al.* proposed a general structure for MF FIR digital filters [11]–[13]. Wang's structure connects subfilters in parallel as many as the flatness constraints, and has a variable multiplier to approximate various ideal frequency responses. Yoshida *et al.* modified the general structure for MF FIR digital filters by expressing the coefficients with convolution of subcoefficients [14]. Yoshida's structure can reduce the number of multiplier, adder, and delay even though it can approximate various ideal frequency responses as same as Wang's structure. Both structures can realize various frequency responses without re-design by changing the ideal frequency response and the frequency where flatness constraints are imposed. Hence, they can be regarded as structures for variable MF FIR digital filters. However, it is difficult to update the filter coefficients of these structures immediately because the filter coefficients are given by many multiplication. Furthermore, these structures premise that the flatness degree at each frequency is fixed. When the structures realize band-pass characteristics and the passband center frequency is changed, their frequency response has a peak in transition band caused by unsuitable flatness degree as shown in Fig. 1 [5]. Such peak is undesired because they would amplify noise. To avoid it, it is important that a variable structure for frequency selective MF FIR digital filters can adjust the flatness degree at the stopband candidate frequencies, $\omega = 0$ and $\omega = \pi$.

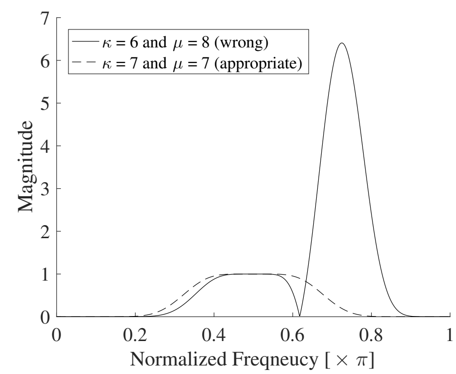


Fig. 1. The frequency response of band-pass MF FIR digital filters with wrong parameters and appropriate parameters

Hence, this paper proposes a novel variable structure for linear-phase frequency selective MF FIR digital filters. By limiting the filter characteristics among low-pass, band-pass and high-pass ones, the proposed structure can reduce the number of multiplication in coefficients update. To realize such structure, we introduce a variable subfilter whose coefficients are given by simple formula and zeros in z domain can be adjusted. Then, the transfer function for linear-phase band-pass MF FIR digital filters are rewritten in cascading and paralleling such variable subfilters. Since controlling the number of zeros at $\omega = 0$ and $\omega = \pi$ is exactly same as controlling the flatness degree at them [3], the proposed structure can avoid a peak in transition band when the passband center frequency is updated. Furthermore, it is also shown that the proposed transfer function can realize not only the band-pass characteristics but low-pass and high-pass ones by adjusting subfilters.

II. Proposed Structure

In general, the transfer function of linear phase FIR digital filter is given by

$$\begin{aligned} H(z) &= \sum_{n=0}^{N} h(n) z^{-n} \\ &= z^{-N/2} H_0(\omega) \end{aligned} \quad (1)$$

where N, $h(n)$ and $H_0(\omega)$ are filter order, filter coefficients and zero phase transfer function, respectively. Linear phase band-pass MF FIR digital filters are designed to satisfy the following flatness constraints

$$\left.\frac{d^l H_0(\omega)}{d\omega^l}\right|_{\omega=\omega_0} = \left.\frac{d^l 1}{d\omega^l}\right|_{\omega=\omega_0}, l = 0, 1, \cdots, L \quad (2)$$

$$\left.\frac{d^l H_0(\omega)}{d\omega^l}\right|_{\omega=\pi} = 0, l = 0, 1, \cdots, 2\kappa - 1 \quad (3)$$

$$\left.\frac{d^l H_0(\omega)}{d\omega^l}\right|_{\omega=0} = 0, l = 0, 1, \cdots, 2\mu - 1, \quad (4)$$

where ω_0 is the passband center frequency. L, κ and μ are the flatness degree parameters at each frequency and are an integer, 1/2 an integer and 1/2 an integer, respectively. The relation among N, L, κ and μ are

$$N = 2(L + \kappa + \mu). \quad (5)$$

In this paper, we assume that N and L are fixed, so that $\kappa + \mu$ is not changed.

Zero phase transfer function $H_0(\omega)$ satisfying (2) is given by [5]

$$H_0(\omega) = (1 - x)^\kappa (1 + x)^\mu Q_L^{\kappa,\mu} \quad (6)$$

$$Q_L^{\kappa,\mu}(x) = \sum_{n=0}^{L} q_n^{\kappa,\mu}(x_0)(x - x_0)^n \quad (7)$$

$$q_n^{\kappa,\mu}(x) = (-2)^n (1 - x_0)^\kappa (1 + x_0)^\mu P_n^{-\kappa-n,-\mu-n}(x_0)$$

$$P_n^{\alpha,\beta}(x) = 2^{-n} \sum_{k=0}^{n} \binom{\alpha + n}{k} \binom{\beta + n}{n - k} \cdot (x - 1)^{n-k}(x + 1)^k, \quad (8)$$

where $P_n^{\alpha,\beta}(x)$ is called Jacobi polynomial and

$$\begin{aligned} x &= -\cos\omega \\ x_0 &= -\cos\omega_0 \\ \binom{\alpha}{n} &= \frac{\alpha(\alpha - 1) \cdots (\alpha - n + 1)}{n!}. \end{aligned}$$

Since $q_n^{\kappa,\mu}(x)$ is expressed in a closed-form function, $H_0(x)$ can change the passband center frequency ω_0. When ω_0 is changed, it is important to avoid a peak as shown in Fig. 1 by adjusting the flatness degree parameter κ and μ. However, updating of $q_n^{\kappa,\mu}(x_0)$ becomes time consuming according to the value of n. Furthermore, Wang's and Yoshida's general structures don't assume to change the value of κ and μ.

To solve this problem, the proposed method express the transfer function by cascading and paralleling the following variable subfilters. The proposed subfilter is the 2nd order FIR digital filter as shown in Fig. 2. The transfer function of the proposed subfilter is defined as

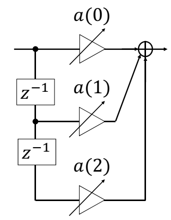


Fig. 2. The structure for the 2nd order variable FIR digital filters

$$G_n^\alpha(z, \omega_z, \omega_s) = a(0) - a(1)z^{-1} + a(2)z^{-2}. \quad (9)$$

The coefficients $a(\cdot)$ is defined with $n \geq 0$ as

$$\begin{aligned} a(0) &= g_n^\alpha(\omega_s) && (10) \\ a(1) &= -2\cos\omega_z a(1) && (11) \\ a(2) &= a(1), && (12) \end{aligned}$$

and with $n = -1$ as

$$\begin{aligned} a(0) &= \sqrt{g_0^\alpha(\omega_s)} && (13) \\ a(1) &= -\cos\omega_z a(1) && (14) \\ a(2) &= 0, && (15) \end{aligned}$$

where

$$g_n^\alpha(\omega_s) = \frac{-\alpha - n}{n + 1} \frac{1}{2(\cos\omega_0 - \cos\omega_s)}. \quad (16)$$

From above equations, it is clear that ω_z controls zero(s) of $G_n^\alpha(z, \omega_z, \omega_s)$. Multipliers are defined by α, n, ω_z and ω_s.

To express $H(z)$ by using $G_n^\alpha(z, \omega_z, \omega_s)$, the proposed method rewrite $H_0(\omega)$ as

$$H_0(\omega) = \left(\frac{-x + 1}{-x_0 + 1}\right)^\kappa \left(\frac{-x - 1}{-x_0 - 1}\right)^\mu \tilde{Q}_L^{\kappa,\mu} \quad (17)$$

$$\tilde{Q}_L^{\kappa,\mu}(x_0) = \sum_{n=0}^{L} \sum_{k=0}^{n} r_k^\kappa(\pi) r_{n-k}^\mu(0)(-x + x_0)^n, \quad (18)$$

where

$$r_k^\alpha(\omega_s) = \binom{-\alpha}{k} (-x_0 - \cos(\omega_s))^{-k}. \quad (19)$$

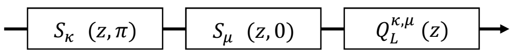
Furthermore, $\tilde{Q}_L^{\kappa,\mu}(x_0)$ can be transformed as same as the passband function in Yoshida's structure [14],

$$\tilde{Q}_L^{\kappa,\mu}(x_0) = \sum_{n=0}^{L} r_n^\kappa(\pi)(-x + x_0)^n \cdot \left\{ \sum_{k=0}^{L-n} r_k^\mu(0)(-x + x_0)^k \right\}. \quad (20)$$

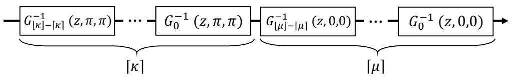
From above equations, this paper proposes $H(z)$ as

$$H(z) = S_\kappa(z, \pi) S_\mu(z, 0) \tilde{Q}_L^{\kappa,\mu}(z) \quad (21)$$

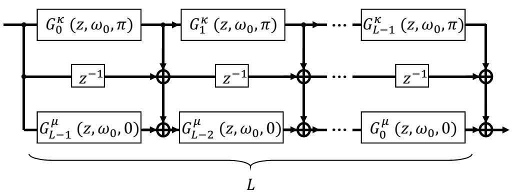
$$\tilde{Q}_L^{\kappa,\mu}(z) = \sum_{n=0}^{L} R_n^\kappa(z, \pi) \left\{ \sum_{k=0}^{L-n} R_k^\mu(z, 0) \right\}, \quad (22)$$



(a) Filter structure



(b) Structure of $S_\kappa(z,\pi)S_\mu(z,0)$



(c) Structure of $\tilde{Q}_L^{\kappa,\mu}(z)$

Fig. 3. The proposed structure for linear phase frequency-selective variable MF FIR digital filters

where

$$S_\alpha(z,\omega_s) = \begin{cases} 1, \alpha = 1 \\ G^{-1}_{\lfloor\alpha\rfloor-\lceil\alpha\rceil}(z,\omega_s,\omega_s) \\ \cdot \left(\prod_{i=1}^{\lceil\omega_s\rceil-1} G_0^{-1}(z,\omega_s,\omega_s)\right), \text{others} \end{cases} \quad (23)$$

$$R_n^\alpha(z,\omega_s) = \begin{cases} 1, n = 1 \\ \prod_{i=0}^{n-1} G_i^\alpha(z,\omega_0,\omega_s), \text{others.} \end{cases} \quad (24)$$

In (23), $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ are floor function and ceiling function, respectively.

Hence, we achieve a novel variable structure for linear phase band-pass MF FIR digital filters based on variable subfilter $G_n^\alpha(z,\omega_z,\omega_s)$ as shown in Fig. 3. Since $S_\kappa(z,\pi)$, $S_\mu(z,0)$ and $\tilde{Q}_L^{\kappa,\mu}(z)$ are composed by variable subfilters, each multiplier of the proposed structure requires 2 or 4 multiplication and 2 division regardless of the filter order. Hence, we can adjust κ, μ and ω_0 immediately. Moreover, even though (21) is a transfer function for band-pass filters, it can be applied to low-pass and high-pass filters. To design low-pass filters, we set $\mu = 0$ and $\omega_0 = 0$. On the other hand, To design high-pass filters, we set $\kappa = 0$ and $\omega_0 = \pi$.

III. Example

In this section, we show the frequency response of the 26th order variable MF FIR digital filters. The parameters of the proposed structure are passband center frequency ω_0 and the flatness parameter L, κ and μ. In this example, L is fixed as $L = 3$ and then we have $\kappa + \mu = 10$. Other parameters, ω_0, κ and μ are set as table I. Note that κ and μ are adjusted to avoid a peak in transition band.

TABLE I
Parameters in example

ω_0	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
κ	10	9	5	1	0
μ	0	1	5	9	10

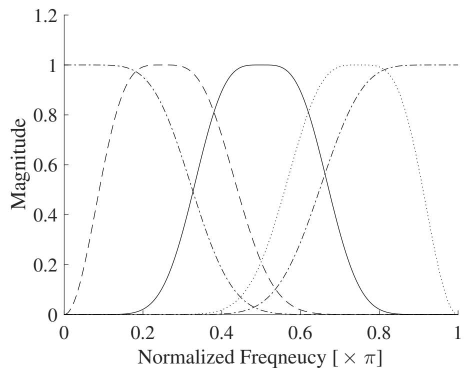


Fig. 4. The frequency response of the proposed structure

Figure 4 shows the frequency response of the proposed structure. From Fig.4, it is clear that the proposed structure can change the frequency response without a peak in transition band.

IV. Conclusion

In this paper, a novel variable structure for linear phase frequency-selective MF FIR digital filters was proposed. To achieve the structure, the 2nd order variable subfilter was introduced. The multiplier of the subfilter requires very few multiplication in update and control zero(s) in z domain. Then, we show that the transfer function of the linear phase band-pass MF FIR digital filters can be transformed by cascading and paralleling the subfilters. Hence, the proposed structure can adjust the center frequency and the flatness degree at the stopband candidate frequencies immediately. Moreover, the proposed structure can be applied to low-pass and high-pass filters by adjusting them. Through example, it was confirmed that the proposed structure can change the frequency response without a peak in transition band.

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