Uplink Overloaded MU-MIMO OFDM Signal Detection Methods using Convex Optimization

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Abstract—This paper proposes a low latency and low complexity signal detection method for overloaded MU-MIMO OFDM (Multi-User Multi-Input Multi-Output) signals using a convex optimization approach for uplink IoT (Internet of Things) environments, in which there are a large number of IoT terminals and a base station having smaller number of antennas than that of the terminals. Simulation results demonstrate the validity of the proposed signal detection method.

I. INTRODUCTION

In the 5th generation mobile communications systems (5G), high speed broadband communications, reliable and low latency communications, and massive simultaneous access by IoT (internet of things) terminals are considered as important system requirements to be realized[1]. Massive MIMO (Multi-Input Multi-Output) is widely regarded as one of elemental technology of the 5G, where tens or hundreds of antennas are introduced at base stations (BSs)[2], while terminals in conventional mobile communications systems typically have 4 or 8 antennas at most. Massive MU (multi-user)-MIMO has been investigated in [3] , [4], where the the number of terminals is assumed to be less than that of antennas at the BS. The communications between terminals and BSs in the IoT environments can be considered as a special case of MU-MIMO communications, however, the number of IoT terminals will be typically greater than that of antennas at the BS even when the massive MIMO is introduced because massive deployment of IoT terminals is envisioned. One of naive approach to cope with such scenarios will be the utilization of multiple access technology, but it introduces additional latency of communications, which will not be appreciated in common IoT systems.

MIMO communications with the number of transmitted antennas (or rather say, transmitted streams) greater than that of receiving antennas is called overloaded MIMO. Although it is difficult to detect all transmitted signals in such environments because the estimation problem becomes underdetermined in general, maximum likelihood (ML) approach can obtain the estimate of the transmitted signals if a finite alphabet is used for the transmitted symbols as in digital communications systems[5]. Since the ML approach is not feasible in massive overloaded MIMO communications due to prohibitively high computational complexity, we have proposed a low complexity overloaded MIMO signal detection scheme[7] using SOAV (sum-of-absolute-values) optimization, which is based on the ideas of convex optimization and compressed sensing[6],[13]. It should be noted that the detection scheme is effective especially when the number of antennas is large and the alphabet size is small.

In this paper, taking advantage of the fact that each communication in typical IoT environments is low rate, we consider to increase the number of simultaneous accesses by IoT terminals to reduce the latency with the massive overloaded MIMO detection scheme in [7]. It is true that the uplink communications in the IoT environments can be naturally modeled by the massive overloaded MIMO, but it is not trivial whether the detection scheme is effective or not in the IoT environments because the performance of the detection scheme largely depends on the size and structure of the sensing matrix, while only rather ideal random sensing matrices are used in [7]. In this paper, we employ OFDM (Orthogonal Frequency Division Multiplexing) scheme, which results in structured sensing matrices, and consider transmitted signal reconstruction from its underdetermined linear measurements by the antennas at the BS via the SOAV optimization using frequency domain or time domain received signals, which correspond to different sensing (channel) matrices. Since theoretical performance guarantee can bee given when the sensing matrix is an i.i.d. random matrix[7], we also consider the impact of randomization of the channel matrix by using precoding with the Hadamard matrix. Moreover, we propose a signal detection scheme using not only the discreteness of the transmitted symbols but also the sparsity of them, because some of IoT terminals might be non-active in common IoT environments. Computer simulations using different reconstruction algorithms to solve the SOAV optimization problem demonstrate the validity of the proposed approach.
II. SYSTEM MODEL

In this paper, we consider uplink communications of IoT environments, which is modeled as an MU-MIMO OFDM system. Fig. 1 shows the system model, where $N$ is the number of IoT terminals having a single antenna, while $N_a$ IoT terminals out of $N$ terminals are active, and $M$ is the number of antennas at the base station. Note that we assume that only a single base station is in the system and that the number of antennas at each IoT terminal is one for the simplicity, but it is possible to extend the model to the scenario with multiple base stations and multiple antennas at each IoT terminal by appropriately modifying channel matrices to be defined later.

Let $s_n \in \mathbb{C}^C \ (n = 1, 2, \cdots, N)$ be the transmitted OFDM symbol by the $n$-th IoT terminal in the frequency domain, where $C$ is the number of subcarriers. For $N - N_a$ non-active terminals, we set $s_n = 0_C$, where $0_C$ denotes a vector having all zero elements with size $C \times 1$.

Assuming the sufficient length of cyclic prefix (CP), the frequency domain received OFDM symbol vector at the base station after the CP removal is written as

$$
\begin{bmatrix}
    r_1^f \\
    \vdots \\
    r_M^f
\end{bmatrix} =
\begin{bmatrix}
    A_{1,1} & \cdots & A_{1,N} \\
    \vdots & \ddots & \vdots \\
    A_{M,1} & \cdots & A_{M,N}
\end{bmatrix}
\begin{bmatrix}
    s_1 \\
    \vdots \\
    s_N
\end{bmatrix}
+ \begin{bmatrix}
    v_1^f \\
    \vdots \\
    v_M^f
\end{bmatrix},
$$
(1)

where $r_m^f \ (m = 1, 2, \cdots, M)$ is the frequency domain received OFDM symbol at the $m$-th antenna of the base station, and $v_m^f$ is the additive white noise vector in the frequency domain with the zero-mean and the covariance matrix of $\sigma_n^2 I_C$, where $I_N$ is the $N \times N$ identity matrix. $\Lambda_{m,n} \in \mathbb{C}^{C \times C}$ is a diagonal channel matrix having the channel frequency response between the $n$-th IoT terminal and the $m$-th antenna of the base station in the diagonal elements as

$$
\Lambda_{m,n} =
\begin{bmatrix}
    \lambda_1^{(m,n)} & 0 & \cdots & 0 \\
    0 & \lambda_2^{(m,n)} & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & \lambda_C^{(m,n)}
\end{bmatrix},
$$
(2)

where $\lambda_c^{(m,n)} \ (c = 1, 2, \cdots, C)$ is the frequency response between the $n$-th IoT terminal and the $m$-th antenna of the base station on the $c$-th subcarrier, which is defined as

$$
\begin{bmatrix}
    \lambda_1^{(m,n)} \\
    \lambda_2^{(m,n)} \\
    \vdots \\
    \lambda_C^{(m,n)}
\end{bmatrix} = \sqrt{\mathbf{D}} \begin{bmatrix}
    h_{1,m,n} \\
    h_{2,m,n} \\
    \vdots \\
    h_{C,m,n}
\end{bmatrix}^T,
$$
(3)

where $\mathbf{D}$ is the unitary $C$-point discrete Fourier transform (DFT) matrix, and $h_{m,n} = [h_{1,m,n}, h_{2,m,n}, \cdots, h_{L,m,n}]^T$ is the $L$ path frequency selective channel impulse response between the $n$-th IoT terminal and the $m$-th antenna of the base station.

Since it has been known that the performance of the signal reconstruction via SOAV optimization largely depends on the size as well as the structure of the sensing matrix (i.e., channel matrix in our problem), we also consider the time domain received signal vector given by

$$
\begin{bmatrix}
    r_1^t \\
    \vdots \\
    r_M^t
\end{bmatrix} =
\begin{bmatrix}
    D^H A_{1,1} & \cdots & D^H A_{1,N} \\
    \vdots & \ddots & \vdots \\
    D^H A_{M,1} & \cdots & D^H A_{M,N}
\end{bmatrix}
\begin{bmatrix}
    s_1 \\
    \vdots \\
    s_N
\end{bmatrix}
+ \begin{bmatrix}
    v_1^t \\
    \vdots \\
    v_M^t
\end{bmatrix},
$$
(4)

where $r_m^t \ (m = 1, 2, \cdots, M)$ is the time domain received OFDM symbol at the $m$-th antenna of the base station, $(\cdot)^H$ denotes Hermitian transpose, and $v_m^t$ is the additive white noise vector in the time domain with the zero-mean and the covariance matrix of $\sigma_n^2 I_C$.

Moreover, we also consider the case with linear precoding in order to randomize the sensing matrix. Specifically, let $X_n \in \mathbb{C}^{C \times C}$ be the precoding matrix at the $n$-th IoT terminal and $X_n s_n$ be the frequency domain transmitted OFDM symbol by the $n$-th terminal, the corresponding frequency domain received OFDM symbol vector can be written as

$$
\begin{bmatrix}
    r_1^t \\
    \vdots \\
    r_M^t
\end{bmatrix} =
\begin{bmatrix}
    A_{1,1} & \cdots & A_{1,N} \\
    \vdots & \ddots & \vdots \\
    A_{M,1} & \cdots & A_{M,N}
\end{bmatrix}
\begin{bmatrix}
    X_1 s_1 \\
    \vdots \\
    X_N s_N
\end{bmatrix}
+ \begin{bmatrix}
    v_1^t \\
    \vdots \\
    v_M^t
\end{bmatrix},
$$
(5)

In the same manner, the time domain received OFDM symbol vector for the case with the precoding can be written as

$$
\begin{bmatrix}
    r_1^t \\
    \vdots \\
    r_M^t
\end{bmatrix} =
\begin{bmatrix}
    D^H A_{1,1} X_1 & \cdots & D^H A_{1,N} X_N \\
    \vdots & \ddots & \vdots \\
    D^H A_{M,1} X_1 & \cdots & D^H A_{M,N} X_N
\end{bmatrix}
\begin{bmatrix}
    s_1 \\
    \vdots \\
    s_N
\end{bmatrix}
+ \begin{bmatrix}
    v_1^t \\
    \vdots \\
    v_M^t
\end{bmatrix},
$$
(6)
III. PROPOSED SIGNAL DETECTION METHOD

Equations of input-output relations in (1), (4), (5) and (6) can be written in a unified form as
\[ \hat{r} = \hat{H} \hat{s} + \hat{v} \in \mathbb{C}^{CM}, \] (7)
where \( \hat{r} \) is a received signal vector, \( \hat{s} \) is a transmitted signal vector, \( \hat{H} \) is a channel matrix, and \( \hat{v} \) is an additive white noise vector.

In order to obtain the estimate of the transmitted signals, we solve an optimization problem, which is formulated by using real vectors and matrices. Thus, we transform the complex received signal model of (7) into a real received signal model as
\[ r = Hs + v \in \mathbb{R}^{2CM}, \] (8)
where
\[ r = \begin{bmatrix} \text{re}(r) \\ \text{im}(r) \end{bmatrix}, \] (9)
\[ s = \begin{bmatrix} \text{re}(s) \\ \text{im}(s) \end{bmatrix}, \] (10)
\[ H = \begin{bmatrix} \text{re}(H) & -\text{im}(H) \\ \text{im}(H) & \text{re}(H) \end{bmatrix}, \] (11)
\[ v = \begin{bmatrix} \text{re}(v) \\ \text{im}(v) \end{bmatrix}. \] (12)

A. Signal detection using discreteness of transmitted signals
(for the case that all terminals are active)

We firstly consider the case that all terminals are active, namely \( N_a = N \). For the simplicity, we assume all terminal employ QPSK (quadrature phase shift keying) for the base band modulation, although any QAM (quadrature amplitude modulation) can be applicable for the proposed detection schemes. In the proposed method, the estimate of transmitted signal \( \hat{s} \) is obtained by solving the SOAV optimization problem as
\[ \hat{s} = \arg \min_{s \in \mathbb{R}^{2CM}} \left( \frac{1}{2} \| s - 1 \|_1 + \frac{1}{2} \| s + 1 \|_1 + \frac{d}{2} \| r - Hs \|_2^2 \right), \] (13)
where \( d \) is a positive constant in order to reflect the fact that the prior distribution of the transmitted symbols is not uniform, and we set \( d = \frac{N - N_a}{N_a} \) using the information of the number of active terminals. Note that the SOAV optimization in (13) utilizes not only the discreteness but also the sparsity of the transmitted symbols. Again, the SOAV optimization problem can be solved by using either DFISTA or DAMP algorithms.

B. Signal detection using discreteness and sparseness of transmitted signals (for the case that some terminals are not active)

Here, we consider a more general scenario, where there might be some non-active terminals. We assume that the basestation does not know which terminals are active or not, but it has the information of the number of active terminals \( N_a \). If we assume the QPSK modulation for the baseband modulation of active terminals as in the previous section, the effective alphabet of the real received signal model including both active and non-active terminals can be regarded as \( \{-1, 0, 1\} \), where the transmitted symbol by any non-active terminal is considered as 0. Note that the occurrence probability of the symbol 0 depends on the number of active terminals \( N_a \), and is different from that of the symbols 1 or -1 in general. Thus, the estimate of the transmitted symbol is given by the solution of the SOAV optimization problem as
\[ \hat{s} = \arg \min_{s \in \mathbb{R}^{2CM}} \left( \frac{1 - d}{2} \| s - 1 \|_1 + \frac{1 - d}{2} \| s + 1 \|_1 + d \| s \|_1 + \frac{\alpha}{2} \| r - Hs \|_2^2 \right), \] (14)
where \( d \) is a positive constant in order to reflect the fact that the prior distribution of the transmitted symbols is not uniform, and we set \( d = \frac{N - N_a}{N} \) using the information of the number of active terminals. Note that the SOAV optimization in (14) utilizes not only the discreteness but also the sparsity of the transmitted symbols. Again, the SOAV optimization problem can be solved by using either DFISTA or DAMP algorithms.

IV. NUMERICAL RESULTS

We have conducted computer simulations in order to demonstrate the performance of the proposed approach. Specifically, we evaluate the performance of the proposed approach for the cases using frequency and time domain received signals, with and without precoding, and with small and large scale systems. In the simulations, we use OFDM signaling with the subcarrier size of \( C = 64 \), and assume that all delayed signals are within the cyclic prefix (guard interval). The wireless channel between each IoT node and each receiving antenna of the BS is assumed to be 10 path Rayleigh fading channel. We use a common Hadamard matrix of order \( C \) for the precoding matrix at all IoT terminals when precoding is applied.

A. BER performance for the case that all terminals are active
\( (N_a = N, N > M) \)

We firstly evaluate the BER performance of the proposed approach using frequency domain received signals in a small system, namely, the number of receiving antennas at the BS of \( M = 6 \) and the number of IoT terminals of \( N = 8 \), for the cases with and without precoding. It should be noted that the received signal is a 133\% overloaded signal, meaning that the number of transmitted streams is 1.33 times greater than that of receiving antennas. We have set the regularization...
The performance for the case with the i.i.d Gaussian random channel matrix is much worse than that for the case with the i.i.d Gaussian random channel matrix, while, by introducing precoding with the Hadamard matrix, the performance of the proposed method with DFISTA is much improved and is almost comparable to the case with the i.i.d Gaussian random channel matrix. It should be emphasized that a common Hadamard matrix, rather than user dependent matrix, can improve the performance a lot, which is very attractive from a viewpoint of implementation. The reason for the poor performance of the proposed method with the DAMP will be the small system size, because AMP based algorithms are derived by using an approximation of a large system limit.

Figures 4 and 5 show the BER performance of the proposed approach using frequency domain received signals in a large system, namely, $M = 60$ and $N = 80$, for the cases with and without precoding. This case also corresponds to the 133% overloaded received signal. In the case of the large system, the proposed method with the DFISTA works well regardless of the precoding as shown in Fig. 4. Moreover, the proposed method with the DAMP and precoding considerably outperforms that with the DFISTA, and achieves almost comparable performance as the case with the i.i.d Gaussian random channel matrix.

Figures 6 - 9 show the BER performance of the proposed approach using time domain received signals both in the small system ($M = 6$, $N = 8$) and the large system ($M = 60$, $N = 80$) for the cases with and without precoding. From the figures, it can be concluded that there is not much performance difference between the cases using the frequency domain and time domain received signals, while the structures of the channel matrices are significantly different at a glance. Since we have observed that the difference of the performance using the frequency domain and the time domain signals is not significant in the rest of the scenarios as well, we’ll show the performance using the frequency domain signal only.
B. SER performance for the case that some terminals are nonactive \((N_a < N, N > M, N_a < M)\)

Here, we evaluate the performance of the proposed approach when the number of IoT terminals \(N\) is greater than the number of antennas at the BS \(M\), but the number of active terminals \(N_a\) is less than \(N\) and \(M\). Since the evaluation of BER is difficult in this case due to the existence of the symbol “0”, we evaluate the symbol error rate (SER) for the case with some non-active terminals. In the simulations, number of all terminals \(N\) and number of active terminals \(N_a\) are assumed to be known to the BS, and \(N_a\) active terminals are randomly selected in each simulation trial.

Figures 10 - 13 show the SER performance of the proposed approach using frequency domain received signals both in the small system \((M = 6, N = 8)\) and the large system \((M = 60, N = 80)\) for the cases with and without precoding. We have set the number of active terminals \(N_a = 1\) in the small system and \(N_a = 10\) in the large system. In this case, we can observe the same tendency as in the case when all terminals are active. Specifically, the proposed method with the DFISTA has good performance in the small system if precoding is employed, while the performance of the proposed method with the DAMP is greatly deteriorated in the small system regardless of the precoding. Moreover, in the large system, the proposed method with the DFISTA has good performance regardless of the precoding, but the proposed method with the DAMP and precoding has better SER performance than the case with DFISTA.

C. SER performance for the case that some terminals are active \((N_a < N, N < M)\)

Finally, we consider more challenging scenario, where not only the number of all terminals \(N\) but also the number of active terminals \(N_a\) are greater than that of antennas at the BS \(M\).

Figures 14 - 17 show the SER performance of the proposed approach using frequency domain received signals both in the small system \((M = 6, N = 8)\) and the large system \((M = 60, N = 80)\) for the cases with and without precoding. We have set the number of active terminals \(N_a = 7\) in the small system and \(N_a = 70\) in the large system. Basic tendency of the performance is the same as the previous cases, however, the performance of the proposed method with the DFISTA is significantly degraded even for the case with the i.i.d. Gaussian random channel matrix. On the other hands, we don’t so
Fig. 10. SER performance with frequency domain received signals and w/o precoding ($M = 6$, $N = 8$, $N_a = 1$)

Fig. 11. SER performance with frequency domain received signals and with precoding ($M = 6$, $N = 8$, $N_a = 1$)

much performance degradation in the proposed method with the DAMP if the system size is large and the precoding is employed.

V. CONCLUSION

In this paper, we have proposed uplink signal detection schemes for overloaded MU-MIMO OFDM system assuming IoT applications. The proposed method utilizes the SOAV optimization to obtain the estimate of the transmitted symbols, where it takes advantage of the discreteness and the sparsity of the transmitted symbols, assuming the non-active terminals transmit the symbol of “0”. From the numerical experiments, we have found that the proposed method with the DFISTA works well if the system size is large or the precoding by using a common Hadamard matrix is employed. The proposed method with the DAMP has much better performance than the proposed method with the DFISTA especially when the number of active terminals is greater than the number of receiving antennas at the BS, while it requires both the large system size and the precoding.

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REFERENCES


Fig. 14. SER performance with frequency domain received signals and w/o precoding ($M = 6, N = 8, N_a = 7$)

Fig. 15. SER performance with frequency domain received signals and with precoding ($M = 6, N = 8, N_a = 7$)

Fig. 16. SER performance with frequency domain received signals and w/o precoding ($M = 60, N = 80, N_a = 70$)

Fig. 17. SER performance with frequency domain received signals and with precoding ($M = 60, N = 80, N_a = 70$)