A DOA Estimation Method of coherent and uncorrelated sources based on Nested Arrays

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***Abstract*—This paper presents a novel way to estimate the DOA of coherent and uncorrelated sources for nested array. At first, by using the vectorization of the covariance matrix of the received data, the LASSO algorithm with a modified dictionary matrix based on compressed sensing is applied to estimate a vector containing all signal powers. Then, the estimated vector is reconstructed into the covariance matrix of coherent and uncorrelated sources. A peak searching to estimate DOAs can be performed by using the diagonal elements of this reconstructed matrix. In this presented method, the advantage of fully utilizing the degree of freedom of nested arrays is preserved. Theoretical analysis and simulation results show the effectiveness of the proposed algorithm.**

Keywords—DOA estimation, uncorrelated signals, coherent signals, nested array, compressive sensing

I. INTRODUCTION

Direction of arrival(DOA) estimation is a major topic in phased arrays. Phased arrays have been successfully applied in radar, sonar and wireless communications. Over the past few decades, many DOA methods for uniform linear array(ULA) have been developed. These methods usually require that the number of signals should be less than the number of array elements. Nested array and co-prime array are two novel arrays proposed in recent years. DOA methods for them can greatly increase the degree of freedom (DOF) of the array. Hence, these two array structures have attracted more and more attention.

In the complex communication environment, due to the existence of multipath reflection and other factors, it often results in coherence between signal sources. Research on DOA estimation of coherent sources has always been a hot topic. The non-uniform array structure, nested array and co-prime array [1][2], makes it possible to resolve more signals. The superiority of nested array and co-prime array in degrees of freedom (DOF) promotes the development of the structure study and the DOA algorithm research for them. Yangg M et al[3] has introduced the redundant array to the co-prime array to enhancing degrees of freedom based on the original co-prime array, Pal P et al[4]has made some structural improvements via increasing the number of array sensors to improves the number of processing the signals. Wu C [5] has estimated more DOAs by reconstructing a TOEPLITZ matrix. Li J[6] has performed DOA estimation by means of real value interleaving covariance matrix. However, if there are coherent signals, those tradition methods become invalid. At present, some scholars have carried out in-depth research on this problem. Satoshi SHIRAI [7] has proposed a method by using the N-order root of the covariance matrix to estimate the DOA. However, the accuracy of the estimation error is very large when the signals are completely coherent. Y.Hu et al[8] has used the fourth-order cumulant to estimate the coherent signal based on expanded co-prime array. If the expanded co-prime array is not introduced, the error is still very large. Moreover, there will be some false peaks and the computational complexity is quite high. Qin S and Zhang Y D [9] has proposed a method to estimate DOAs of mixed coherent and uncorrelated signals exploiting a nested MIMO system. Firstly, they has used MUSIC to estimate the uncorrelated sources. Then, they has used compressive sensing techniques to estimate the remain correlated signals. Subsequently, they[10] has used Bayesian compressive sensing to estimate the DOA of coherent and uncorrelated targets exploiting collocated multiple-input multiple-output (MIMO) radar with transmit/receive co-prime arrays. However, these two algorithm is discussed in MIMO array system and is not suit for phased array system.

In this paper, a method to estimate DOAs of coherent and uncorrelated signal sources is proposed for the nested array with phased array system work mode. By the aid of the vectorization of the covariance matrix of received data, a vector containing all signal powers can be obtained by using the LASSO algorithm with a modified dictionary matrix based on CS. A covariance matrix of coherent and uncorrelated sources can be reconstructed by using the estimated vector. The final DOA estimate can be achieved by employing the diagonal elements of the reconstructed matrix. The degree of freedom of the nested array(NA) is fully utilized.

The rest of this paper is organized as follows: Section II reviews the signal model of NA. Section III presents the DOA estimation method based on CS. Simulation results are provided to validate the method in Section IV. Section V, concludes this paper.

II. SIGNAL MODEL

The NA concludes two or more ULA. At present, we use two ULA, where subarray 1 has M_1 elements with inter-element distance d and subarray 2 has M_2 elements with

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inter-element distance $(M_1+1)d$. The spacing between the subarray 1 and the subarray 2 is d, which usually equal to half-wavelength of the signal.

In this paper, assume that there are K far-field narrowband signals impinging on the array and the angles of the signal are $\{\theta_1,\theta_2,...,\theta_K\}$. We consider the situation when the all signals are coherent with each other. The received data are expressed as

$$\mathbf{x}(n)=\mathbf{A}(\theta)\mathbf{s}(n)+\mathbf{e}(n), n=1,2,\cdots,T \tag{1}$$

where $\mathbf{s}(n)=[s_1(n),s_2(n),\cdots,s_K(n)]^T$ is the signal vector. $s_k(n)$ represents the number of k signal at the nth instant. The mean value of $s_k(n)$ $(n=1,2,\cdots,T)$ is assumed to be 0. $(\bullet)^T$ denote transpose. $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1)\ \ \mathbf{a}(\theta_2)\ \ \cdots\mathbf{a}(\theta_K)]$ is the array manifold matrix and $\mathbf{a}(\theta_k)$ denotes the steering vector of $s_k(n)$, respectively. $\mathbf{e}(n)$ is assumed to be temporally and spatially white noise, which is independent of the signals. The power of the noise is σ_n^2.

The covariance matrix of the receive data can be expressed as

$$\begin{aligned}\mathbf{R}_X&=\mathrm{E}\{\mathbf{x}(n)\mathbf{x}^H(n)\}\\&=E\{(\mathbf{A}\mathbf{s}(n)+\mathbf{e}(n))(\mathbf{A}\mathbf{s}(n)+\mathbf{e}(n))^H\}\\&=\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H+\sigma_n^2\mathbf{I}_M\end{aligned} \tag{2}$$

where $\mathbf{I}_M$ denotes unit matrix and $\mathbf{R}_{ss}=E\{\mathbf{s}(n)\mathbf{s}^H(n)\}$. As for $\mathbf{R}_{SS}$, we can know that the element on its main diagonal are the autocorrelation of signals, and the elements on the off-diagonal are cross correlation of signals. If signals are independent of each other, all elements on the off-diagonal line are 0. If there is coherence between the signals, elements on the off-diagonal line have a non-zero value. In this way, when signal is independent, it can be regarded as a special case of signal coherence, in which signal cross correlation is 0.

If all signals are coherent with each other, the above equation can be transformed into

$$\begin{aligned}\mathbf{R}_X&=\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H+\sigma_n^2\mathbf{I}_M\\&=\sum_{i=1}^{K}\rho_{ii}\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)+\sum_{i=1}^{K}\sum_{\substack{j=1\\i\neq j}}^{K}\rho_{ij}\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_j)+\sigma_n^2\mathbf{I}_M\end{aligned} \tag{3}$$

where $(\bullet)^H$ denote conjugate transpose. ρ_{ij} denotes correlation between the i signal and the j signal. If $i=j$, ρ_{ij} denote the number of i signal power. If $s_i(n)$ and $s_j(n)$ is coherent, ρ_{ij} is not equal to zero. Otherwise, ρ_{ij} is zero.

III. THE PROPOSED ALGORITHM

Because physical arrays are non-uniform, spatial smoothing method cannot be directly used to the whole physical array. Because of the sparsity of the array, the physical array cannot use the CS method.

A. Reconstruction of Covariance Matrix $\bar{\mathbf{R}}$

In order to process the coherent signals and the covariance matrix is guaranteed to be full-rank, we will restructure covariance matrix $\mathbf{R}_X$, and the iteration process is given as follow [11]:

TABLE.1 The Iterative Algorithms for Reconstruction of Covariance Matrix $\bar{\mathbf{R}}$

Initialisation:

$\hat{\mathbf{s}}_k(n)=\mathbf{a}_k^H\mathbf{x}(n)/(M_1+M_2), n=1,\ldots,N, k=1,\cdots,D$

$\hat{P}_k(n)=\frac{1}{N}\sum_{n=1}^{N}\left|\hat{\mathbf{s}}_k(n)\right|^2, k=1,\cdots,D$

Iteration:

$\bar{\mathbf{R}}=\mathbf{A}_D\hat{\mathbf{P}}\mathbf{A}_D^H$

for $k=1,\cdots,m$

$\mathbf{w}_k=\frac{\bar{\mathbf{R}}^{-1}\mathbf{a}_k}{\mathbf{a}_k^H\bar{\mathbf{R}}^{-1}\mathbf{a}_k}, \hat{P}_k=\mathbf{w}_k^H\mathbf{R}_X\mathbf{w}_k$

end

Where, $\mathbf{A}_D=[\mathbf{a}_1^H\ \ \mathbf{a}_2^H\ \ \cdots\ \ \mathbf{a}_D^H]$ is the dictionary matrix, m is the number of iterations, D is the number of the angle area of interest signals.

Through $\bar{\mathbf{R}}$, the TOEPLITZ algorithm, the spatial smoothing processing(SSP) algorithm and the orthogonal matching pursuit (OMP) algorithm can successfully estimate the DOA of the signal. However, those algorithms are limited to the reconstructed matrix $\bar{\mathbf{R}}$. If the $\bar{\mathbf{R}}$ is inaccurate, it will inevitably produce some errors, which will affect the accuracy of the DOA of the signals. Next, we will propose an algorithm to avoid the above problems.

B. Reconstruction of Matrix $\hat{\mathbf{R}}_{sz}$

In order to make full use of the DOF and avoid the error caused by matrix $\bar{\mathbf{R}}$, the traditional NA processing method will vectorize the covariance matrix. Compared with the case that there are no correlated or coherent signals, the signal form has changes when there are correlated or coherent signals.

By vectorizing the covariance matrix of (3), a new vector can be formed as

$$\mathbf{z}=vec(\mathbf{R}_X)=vec(\sum_{i=1}^{K}\rho_{ii}\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)+\sum_{i=1}^{K}\sum_{\substack{j=1\\i\neq j}}^{K}\rho_{ij}\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_j)+\sigma_n^2\mathbf{I}_M) \tag{4}$$

Compared with the case that there are no correlated or coherent signals, there are additional term $\sum_{i=1}^{K}\sum_{\substack{j=1\\i\neq j}}^{K}\rho_{ij}\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_j)$. Hence, the traditional method for NA in virtual array field is invalid for the DOA estimate of coherent signal.

Fortunately, the LASSO method based on CS can be used. We translate the above equation into the following way:

$$\mathbf{z}=\hat{\mathbf{A}}\hat{\mathbf{p}}+\sigma_n^2 vec(\mathbf{I}_M) \tag{5}$$

where $\hat{\mathbf{p}}=[\rho_{11},\rho_{12},\cdots,\rho_{1K},\cdots,\rho_{K1},\cdots,\rho_{KK}]^T$,

$$\begin{aligned}\hat{\mathbf{A}}=[&\mathbf{a}^*(\theta_1)\otimes\mathbf{a}(\theta_1)\ \ \mathbf{a}^*(\theta_1)\otimes\mathbf{a}(\theta_2)\ \ \cdots\ \ \mathbf{a}^*(\theta_1)\otimes\mathbf{a}(\theta_K)\ \ \cdots\\&\mathbf{a}^*(\theta_K)\otimes\mathbf{a}(\theta_1)\ \ \cdots\ \ \mathbf{a}^*(\theta_K)\otimes\mathbf{a}(\theta_K)]\end{aligned}.$$

In order to estimate the DOAs of coherent signals, we adopt the sparse reconstruction idea based on CS. The angle of the objective signal is assumed to exist in the angle area of our division and we will not consider grid error of dividing area. Therefore, the received data matrix under the virtual array is

$$\tilde{\mathbf{z}} = \tilde{\mathbf{A}}\tilde{\mathbf{p}} + \sigma_n^2 vec(\mathbf{I}_M) \tag{6}$$

where

$$\tilde{\mathbf{A}} = \big[\mathbf{a}^*(\theta_{d_1}) \otimes \mathbf{a}(\theta_{d_1}) \quad \mathbf{a}^*(\theta_{d_1}) \otimes \mathbf{a}(\theta_{d_2}) \quad \cdots \quad \mathbf{a}^*(\theta_{d_1}) \otimes \mathbf{a}(\theta_{d_D}) \quad \cdots \quad \mathbf{a}^*(\theta_{d_D}) \otimes \mathbf{a}(\theta_{d_1}) \quad \cdots \quad \mathbf{a}^*(\theta_{d_D}) \otimes \mathbf{a}(\theta_{d_D})\big]$$

To estimate DOAs, the following optimization problem is established:

$$\begin{aligned} &\min \|\tilde{\mathbf{p}}\|_1 \\ &s.t. \\ &\left\|\mathbf{z} - \tilde{\mathbf{A}}\tilde{\mathbf{p}} - \sigma_n^2 vec(\mathbf{I}_M)\right\|_2^2 \le \beta \end{aligned} \tag{7}$$

where β is a preset parameter. We can convert it into the LASSO method for the above model of DOA estimation. We have

$$\min_{\tilde{\mathbf{p}}} \left\|\mathbf{z} - \tilde{\mathbf{A}}\tilde{\mathbf{p}} - \sigma_n^2 vec(\mathbf{I}_M)\right\|_2 + \lambda_t \|\tilde{\mathbf{p}}\|_1 \tag{8}$$

where $\tilde{\mathbf{p}} \in \mathbb{C}^{D^2 \times 1}$ and λ_t is regular parameter.

However, the above processes can't estimate the DOA of the signals, we need to further deal with the estimated power vector $\tilde{\mathbf{p}}$. In other words, we need to rearrange $\tilde{\mathbf{p}}$ to an $D \times D$ matrix $\hat{\mathbf{R}}_{sz}$ according to the column's order of $\hat{\mathbf{p}}$. Extract the main diagonal element of $\hat{\mathbf{R}}_{sz}$. If θ_{d_i} is one of signal DOAs, the value of its corresponding elements in the main diagonal element of $\hat{\mathbf{R}}_{sz}$ should be big. Otherwise, it should be very small, even close to 0. In this way, we can obtain the DOA via a spectral peaking search.

The change of the dictionary matrix leads to a change for the number of estimable sources compared with the case that there are no coherent sources. We will analyze the maximum number of signals can be estimated at the following part.

B.1 The case that there are only correlated or coherent signal sources

In the proposed algorithm, the maximum number of distinguishable signals is determined by both the physical array and the coherence number of the signal. As we know, each pair of coherent signals will affect $\mathbf{R}_{SS}$. Elements on the off-diagonal line of $\mathbf{R}_{SS}$ are cross correlation and the elements on the diagonal line of $\mathbf{R}_{SS}$ are autocorrelation. $\mathbf{R}_{SS}$ can be constructed by using $\hat{\mathbf{p}}$. Because of conjugation symmetry of $\mathbf{R}_{SS}$, the number of elements in $\hat{\mathbf{R}}_{sz}$ that we need to estimate can be reduced from K^2 to $K(K+1)/2$. Notice that these elements form the lower triangle of the matrix $\mathbf{R}_{ss}$. Therefore, we can find that our proposed algorithm require $L_{PA} \ge 2L_{VA}$ [12], where L_{PA} is the number of the physical arrays. And $L_{VA} = K + P$, where K is the number of the signals and P is the least number of cross correlation which we need use to judge which signals are coherent or correlated. For example, if there are three far-field narrowband coherent signals impinging on the array, we can consider lower triangle elements of

$$\mathbf{R}_{ss} = \begin{bmatrix} \rho_{11} & \rho_{12}^* & \rho_{13}^* \\ \rho_{12} & \rho_{22} & \rho_{23}^* \\ \rho_{13} & \rho_{23} & \rho_{33} \end{bmatrix}$$

Therefore, $L_{VA} = K + P = 3 + 2 = 5$, $L_{PA} \ge 2L_{VA} = 2 \times 5 = 10$. In order to estimate DOA, we need 10 arrays at least.

B.2 The case that there are coherent and uncorrelated signal sources

The virtual array is also a ULA, and it also needs to satisfy the relationship under the ULA. This can also be analyzed from the perspective of actual physical arrays. For the NA, we complement the non-uniform array into a uniform array with $\mathrm{d} = \lambda/2$ by vectorizing the covariance matrix.

Assume that there are G coherent signals groups. Each of group is independent of each other, and the signal in each group is coherent with each other. Group g ($g = 1, 2, \cdots, G$) has K_g coherent signals. The number of the coherent signals in each group also needs to satisfy the physical array conditions $L_{PA} \ge 2L_{VAg}$, where $2L_{VAg}$ is the number of coherent conditions in group g. The final value of $2L_{VA}$ in this case is obtained by adding the coherent condition numbers of all groups, where $2L_{VA} = 2L_{VA1} + 2L_{VA2} + \cdots + 2L_{VAg} + \cdots + 2L_{VAG}$. Since we solve it by the NA, the DOF of the array is increased when it is transformed into a virtual array. When the number of the NA is odd, the maximum DOF of the virtual array is $L_{VIR} = (M^2 - 1)/2 + M$. When the number of the NA is even, the maximum DOF of the virtual array is $L_{VIR} = (M^2 - 2)/2 + M$. Therefore, the maximum number of signals identified is $L_{VIR} \ge 2L_{VA}$.

IV. SIMULATION RESULTS

In this part, we consider some simulations that demonstrate the DOA estimation of coherent and uncorrelated signals by using the proposed approach. Firstly, we compare the performance of the proposed algorithm with the one of the SSP and N-order root algorithm. Secondly, Root Mean Square Errors (RMSE) varied with signal-to-noise(SNR) and number of snapshots are studied.

A. The estimation of coherent and uncorrelated signals

Here, a NA with $M = 10$ ($M_1 = 5, M_2 = 5$) omnidirectional sensors with $d = \lambda/2$ is considered. SNR is assumed to be 10dB and the number of snapshot is 1000. The scanning angle area is $(-90^o, 90^o)$. The array's additive noise is a Gaussian zero-mean spatially and temporally white process. 10 narrowband coherent and uncorrelated signals with DOA uniformly distributed in $[-60^o, 60^o]$ is considered. The coherent signals are divided into two groups. The No.1, No.2 and No.3 signal are coherent with each other. The No.4 and No.5 signal are coherent with each other. The rest of signals are independent of each other. Results of the SSP and N-order root algorithm are also shown here. Notice that

N-order root algorithm is performed after the operation of SSP for covariance matrix in the virtual array. It is obviously that SSP and N-order root algorithm fails to estimate the coherent signals DOA in this case. Fig.1 shows that our proposed method can work well.

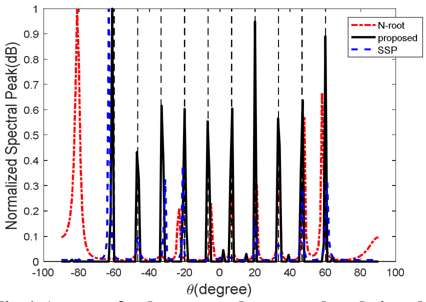

Fig.1 A case of coherent and uncorrelated signals

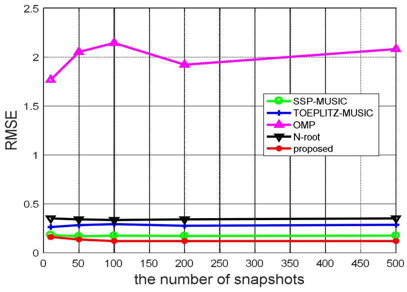

Fig.2 RMSE versus number of snapshot

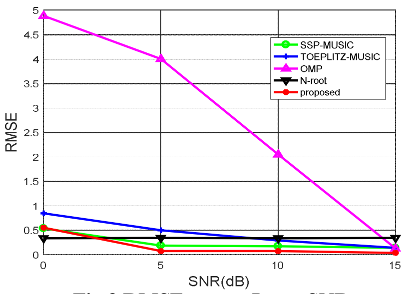

Fig.3 RMSE versus Input SNR

B Performance analysis of the proposed algorithm

In this part, we briefly study the RMSE of DOA estimates for the proposed method, the SSP-MUSIC method, the TOEPLITZ-MUSIC, the OMP method, and the N-order root method. In Fig.2 and Fig.3, two coherent sources at -10° and 20° are considered. In Fig.2, SNR=10dB. In Fig.3, T=1000. As for the proposed algorithm, RMSE decreased as the number of snapshots increased at the beginning for the proposed. When the snapshot number is greater than 100, RMSE gradually converges. When snapshot is small, the sample matrix estimated using finite snapshots is imprecise. Hence, results of small snapshots are worse than the case of large snapshots. In Fig.2 and Fig.3, the performance advantage of the proposed algorithm has great advantage. However, as for the N-order root method, SSP-MUSIC method, TOEPLITZ-MUSIC and OMP method, the performance is very poor. Especially, the dictionary matrix of the OMP algorithm is susceptible to local convergence, which makes the result inaccurate, which its performance is very poor. Of course, for OMP algorithm, the error caused by reconstruction covariance matrix also needs to be considered. As described in Chapter III.A.

V. CONCLUSIONS

In this paper, a novel way to DOA estimation of coherent signals and uncorrelated signals is proposed for NA. When there are coherent or correlated signals, the signal expression of the vectorization of the received signal covariance matrix has changed. Therefore, the LASSO algorithm with a modified dictionary matrix based on compressed sensing is used here to obtain a vector estimation, which contains all signal powers and cross correlation between signals. This estimated vector is used to reconstruct a matrix. The final DOA estimation can be acquired by peak searching based on the diagonal elements of this reconstructed matrix. The requirement of element number for algorithm is discussed. The degree of freedom of the NA is fully utilized. Simulation results demonstrate the effectiveness of the propose algorithm.

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