A DOA Estimation Method in the presence of unknown mutual coupling based on Nested Arrays

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***Abstract*— A novel DOA method is proposed to deal with the DOA estimation in the presence of the unknown mutual coupling for nested arrays. By using a new expression of the steering matrix in the presence of mutual coupling, a novel expression of the receiving data vector in the virtual array field is available. Then, based on a modified direction matrix constructed with block matrix, which relates to space discretized sampling grid, the sparse Bayesian compressive sensing method applies to estimate a vector, which contains the signal powers information and the mutual coupling information. The problem of off-grid DOAs is also considered for sparse Bayesian compressive sensing. Based on the estimated vector, a peak searching is performed to estimate the initial DOA. Finally, the estimation of DOA is modified to initial estimate plus off-grid error value. The advantage of fully utilizing the degree of freedom of nested arrays is preserved in this proposed algorithm. Moreover, no complicated calculation is needed to obtain the mutual coupling coefficients or rearrange the position of array element. Theoretical analysis and simulation results show the effectiveness of the proposed algorithm.**

***Index Terms*—direction of arrival estimation, mutual coupling, nested array, off-grid model, sparse Bayesian learning method**

I. Introduction

The direction of arrival (DOA) estimation is a major topic in phased arrays. Phased arrays have been successfully applied in radar, sonar and wireless communications [1]-[4]. In recent years, high degree of freedom (DOF) and high resolution with finite array elements have attracted the attention of researchers. The sparse array structure, the nested array (NA) and the co-prime array [5]-[6], can be transformed into a virtual uniform array by vectorizing the covariance matrix of the received data. Then, the DOA estimation is performed in such virtual array field, whose array elements number is more than the number of the physical array elements. Therefore, the number of estimable sources for the NA and the co-prime array is larger than the number of real physical array elements. Subsequently, a series of sparse array structures are proposed, such as generalized NA, super NA, k-times extended co-prime arrays et al. Although NAs and co-prime array are sparse, the space between some array elements is not large enough to ignore the mutual coupling between them. The classical DOA estimation techniques, such as L1-svd[7], MUSIC[8] and ESPRIT[9], suffer from a model mismatch in the presence of the mutual coupling. Therefore, it is very meaningful to study DOA estimation in the presence of the mutual coupling.

To reduce the mutual coupling between array elements, a series of methods, devising a methodical step via rearranging the dense subarray of sparse array, have been proposed. Junpeng Shi [10] et al has proposed a generalized NA configuration. This method has designed two flexible co-prime factors and the inter-element spacing of two concatenated uniform linear subarrays can be enlarged. Thus, the mutual coupling effect of the array is reduced. Chun-Lin Liu[11] et al has proposed a Super NA. In this NA structure, the coarray remains unchanged but the number of array elements with small inter-elements spacing is reduced. The mutual coupling effect is naturally weakened compared to the traditional NA. Subsequently, a high-order hyper nested arrays structure is proposed by Chun-Lin Liu [12] et al. CHEN Lu[13] et al has optimized the NA structure and has proposed two translational NA structures. This structure greatly increases the sparsity of the NA and reduces the mutual coupling effect between array elements.

There are some researches tried to solve DOA estimation problem in the presence of the mutual coupling by modifying the estimation approach for some sparse array. Elie BouDaher[14] et al has proposed a method for co-prime arrays. This method needs to jointly estimate mutual coupling matrix, theirs DOAs and the source powers based on an optimization. The solution to the optimization problem is solved by the covariance matrix adaptation evolution strategy. However, it does not consider the case that the DOA is not fall on the space discrete grid. And his simulation results show that the estimate accuracy is not well. J.Dai [15] et al has proposed an iterative method to estimate DOA in the presence of mutual coupling based on sparse arrays. However, the method treats the sparse array as a subarray of a uniform linear array and cannot take full advantage of the increased DOFs for the sparse array. At present, no one has solved the DOA estimation problem of nested arrays with unknown mutual coupling from the perspective of estimation method.

The compressive sensing approach is highly applied in the DOA estimation. The result accuracy depends on the space-sampling grid. It is better to consider the case that real DOA is off-grid. Zai Yang[16] had proposed off-grid model, which can applies to both single snapshot and multi-snapshot cases. Subsequently, a series of off-grid model are proposed [17]-[19]. Anup Das[20] had demonstrated by analyzing passive sonar data from the SWellEx-96 ocean acoustic experiment. Yonghong Zhao[21] had proposed a method by using Bayesian learning based on an array covariance matrix for NA. However, no work is suited to the NA with unknown mutual coupling.

In this paper, a method to estimate the DOA in the presence of unknown mutual coupling for NA is proposed. By the aid of a new expression of the steering matrix in the presence of mutual coupling, a novel expression of the receiving data vector in the virtual array field can be obtained. Then, a sparse Bayesian method, considering the off-grid error, is employed to estimate a vector that includes the power information of signal sources and the mutual coupling information. The estimated vectors are used to estimate initial DOA via a peak searching. The off-grid error can be obtained by using sparse Bayesian method. The final DOA estimate is the sum of the initial DOA estimate and the off-grid error. The advantage of high DOF of NA is fully utilized, which makes the number of estimated signals exceed the aperture of physical array. The proposed algorithm does not need the mutual coupling matrix calibrating or rearrange the structure of the array. The simulation results show the effectiveness of the proposed algorithm.

The rest of this paper is organized as follows: Section II reviews the signal with mutual coupling model of NA. Section III presents the DOA estimation method based on sparse Bayesian learning method. Simulation results are shown in Section IV. Section V concludes this paper.

II. Signal Model

The NA concludes two or more ULA. At present, we use two ULAs, where the subarray 1 has M_1 elements with inter-element distance d and the subarray 2 has M_2 elements with inter-element distance $(M_1+1)d$. The NA has $M=M_1+M_2$ elements. The spacing between the subarray 1 and the subarray 2 is d, which usually equal to half-wavelength of the signal.

Assume that there are K far-field narrowband signals impinging on the array and DOAs of the signal are $\{\theta_1,\theta_2,...,\theta_K\}$, respectively. When there is mutual coupling between the array elements, the received data is expressed as

$$\mathbf{x}(n)=\mathbf{CA}(\theta)\mathbf{s}(n)+\mathbf{e}(n), \mathrm{n}=1,2,\cdots,N \tag{1}$$

where $\mathbf{s}(n)=[s_1(n),s_2(n),\cdots,s_K(n)]^T$ is the signal vector. $s_k(n)$ represents the kth signal at the nth instant. $(\bullet)^T$ denotes transpose. $\mathbf{A}(\theta)=[\boldsymbol{a}(\theta_1),\boldsymbol{a}(\theta_2),\cdots,\boldsymbol{a}(\theta_K)]$ is the array manifold matrix and $\boldsymbol{a}(\theta_k)=\left[e^{-j(2\pi/\lambda)\mathrm{d}_1\sin\theta_k} \quad e^{-j(2\pi/\lambda)\mathrm{d}_2\sin\theta_k} \quad \cdots \quad e^{-j(2\pi/\lambda)\mathrm{d}_M\sin\theta_k}\right]$ is the steering vector of $s_k(n)$ in the idea case, respectively. $d_i\left(i=1,2,\cdots,M\right)$ is the position of the array. $\mathbf{C}$ is the mutual coupling matrix. $\mathbf{e}(n)$ is assumed to be temporally and spatially white noise, which is independent of the signals. The power of the noise is σ_n^2 . In this paper, we only consider that all signals are independent of each other.

The covariance matrix of the received data can be expressed as

$$\begin{aligned}\mathbf{R}_X &= E\left\{\mathbf{x}(n)\mathbf{x}^H(n)\right\} \\ &=\mathbf{CAR}_{ss}\mathbf{A}^H\mathbf{C}^H+\sigma_n^2\mathbf{I}_M\end{aligned} \tag{2}$$

where

$$\mathbf{R}_{ss}=\begin{bmatrix}\rho_1 & 0 & \cdots & 0\\ 0 & \rho_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \rho_K\end{bmatrix} \tag{3}$$

with $\rho_i=E\left(s_i(n)s_i^*(n)\right)$ $\left(i=1,2,\cdots,K\right)$, $(\bullet)^{\mathrm{H}}$ denotes conjugate transpose.

By vectorizing the covariance matrix $\mathbf{R}_X$, a vector $\mathbf{z}$ can be obtained as

$$\begin{aligned}\mathbf{z} &= vec(\mathbf{R}_X)=(\tilde{\mathbf{A}}^*\circledast\tilde{\mathbf{A}})\mathbf{p}_v+\sigma_n^2\mathbf{1}_M \\ &=\sum_{i=1}^{K}\left(\tilde{\boldsymbol{a}}_i^*\otimes\tilde{\boldsymbol{a}}_i\right)\rho_i+\sigma_n^2\mathbf{1}_M\end{aligned} \tag{4}$$

where $\tilde{\mathbf{A}}^*\circledast\tilde{\mathbf{A}}=\left[\tilde{\boldsymbol{a}}_1^*\otimes\tilde{\boldsymbol{a}}_1 \quad \tilde{\boldsymbol{a}}_2^*\otimes\tilde{\boldsymbol{a}}_2 \quad \cdots \quad \tilde{\boldsymbol{a}}_K^*\otimes\tilde{\boldsymbol{a}}_K\right]$ and $\tilde{\mathbf{a}}_i=\mathbf{C}\mathbf{a}(\theta_i)$. $\mathbf{p}_v=\left[\rho_1 \quad \rho_2 \quad \cdots \quad \rho_K\right]^T$. $\mathbf{1}_M=[\mathbf{e}_1^T,\mathbf{e}_2^T\cdots,\mathbf{e}_M^T]^T$ and $\mathbf{e}_m$ denotes a length-M column vector. The mth element in $\mathbf{e}_m$ is 1 and other elements in $\mathbf{e}_m$ is 0. $(\bullet)^*$ denotes conjugate. $\otimes$ is the Kronecker product and $\circledast$ is the Khatri-Rao product.

The vector $\mathbf{z}$ can be regarded as the received data expression of a virtual array field whose corresponding array manifold matrix is $\tilde{\mathbf{A}}^*\circledast\tilde{\mathbf{A}}$. $\mathbf{p}_v$ can be considered as the virtual signal vector with only one snapshot. When there is no mutual coupling, we get $\tilde{\boldsymbol{a}}_i=\boldsymbol{a}(\theta_i)$. A steering vector corresponding to a virtual ULA can be obtained after the duplicate removal and the rearrangement of $\tilde{\boldsymbol{a}}_i^*\otimes\tilde{\boldsymbol{a}}_i$. Therefore, the spatial smoothing processing (SSP) or the compressive sensing approach can be applied to deal with DOAs estimation problem by using only one snapshot data $\mathbf{z}$. When there is mutual coupling, $\tilde{\boldsymbol{a}}_i=\mathbf{C}\boldsymbol{a}(\theta_i)$ is different from $\boldsymbol{a}(\theta_i)$. If using the traditional method for the NA in the virtual array field, the DOA of the signal cannot be estimated without the mutual coupling calibrating.

III. The proposed method

In this section, we proposed a DOA estimation approach of the Bayesian compressive sensing in the presence of mutual coupling. In this paper, the NA is considered including two ULAs. The subarray 2 of the NA is a uniform sparse array with inter-element distance $(M_1+1)d$. The subarray 1 and the first element of the subarray 2 form a ULA with inter-element distance d. Ordinarily, the influence of the mutual coupling is related to the element distance of adjacent elements. We only consider the case that the mutual coupling has a presence in the ULA part and an absence from the sparse part.

A. A novel expression of received data in virtual array

Here, we consider that there is unknown mutual coupling. The mutual matrix $\mathbf{C}$ is unavailable now. The mutual coupling matrix of the uniform linear array is usually modeled with a banded symmetric toeplitz matrix. Suppose that the number of the non-zero coefficient of the mutual coupling for the NA is m and the coefficients of the mutual coupling are $c_i(\mathrm{i}=1,2,\cdots,\mathrm{m}-1)$. The mutual coupling matrix $\mathbf{C}$ for the NA can be expressed as

$$\mathbf{C}=\begin{bmatrix}1 & c_1 & \cdots & c_{m-1} & 0 & \cdots & 0 & \cdots & 0 & 0\\ c_1 & 1 & c_1 & \cdots & \ddots & 0 & 0 & \cdots & 0 & 0\\ \vdots & c_1 & 1 & \ddots & \cdots & c_{m-1} & 0 & \cdots & 0 & 0\\ c_{m-1} & \cdots & \ddots & \ddots & c_1 & \vdots & 0 & \cdots & 0 & 0\\ 0 & \ddots & \cdots & c_1 & 1 & c_1 & 0 & \cdots & 0 & 0\\ 0 & 0 & c_{m-1} & \cdots & c_1 & 1 & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & \cdots & \cdots & 0 & 1 & \cdots & 0 & 0\\ 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 0 & \cdots & 1\end{bmatrix}_{M\times M} \tag{5}$$

According to the banded symmetric toeplitz characteristics, we can re-express $\tilde{\boldsymbol{a}}_i$ in the following way

$$\tilde{\boldsymbol{a}}_i=\mathbf{C}\boldsymbol{a}(\theta_i)=\mathbf{T}(\theta_i)\tilde{\mathbf{c}} \tag{6}$$

where $\tilde{\mathbf{c}}$ is the non-zero mutual coupling vector

$$\tilde{\mathbf{c}} = \begin{bmatrix} 1 & c_1 & \cdots & c_{m-1} \end{bmatrix}^T \in \mathbb{C}^{m\times 1} \tag{7}$$

For NAs, $\mathbf{T}(\theta_i)$ is easily obtained using the known steering vector $\mathbf{a}(\theta_i)$. $\mathbf{T}(\theta_i)$ is a $M\times m$ matrix and can be constructed as

$$\mathbf{T}(\theta_i) = \begin{bmatrix} \hat{\mathbf{T}}_1(\theta_i) \\ \hat{\mathbf{T}}_2(\theta_i) \end{bmatrix} \tag{8}$$

where

$$\hat{\mathbf{T}}_1(\theta_i) = \mathbf{T}_1(\theta_i) + \mathbf{T}_2(\theta_i) \tag{9}$$

$$\begin{aligned} \left[\mathbf{T}_1(\theta_i)\right]_{j,k} &= \begin{cases} \left[\boldsymbol{a}(\theta_i)\right]_{j+k-1} & ,j+k\le M_1+2 \\ 0 & , \text{ otherwise} \end{cases} \\ \left[\mathbf{T}_2(\theta_i)\right]_{j,k} &= \begin{cases} \left[\boldsymbol{a}(\theta_i)\right]_{j-k+1} & ,j\ge k\ge 2 \\ 0 & , \text{ otherwise} \end{cases} \end{aligned} \tag{10}$$

and

$$\begin{aligned} \hat{\mathbf{T}}_2(\theta_i) &= \begin{bmatrix} \left[\boldsymbol{a}(\theta_i)\right]_{M_1+2} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \left[\boldsymbol{a}(\theta_i)\right]_{M} & 0 & \cdots & 0 \end{bmatrix} \\ &= \begin{bmatrix} \left[\boldsymbol{a}(\theta_i)\right]_{(M_1+2):\text{end}} & \mathbf{0}_{(M_2-1)\times(m-1)} \end{bmatrix} \end{aligned} \tag{11}$$

where $[\bullet]_{j,k}$ represents the element corresponding to the jth row and kth column of the matrix and $[\bullet]_j$ represents jth element, respectively.

Therefore, we can get

$$\begin{aligned} \tilde{\boldsymbol{a}}_i^* \otimes \tilde{\boldsymbol{a}}_i &= \left(\mathbf{T}(\theta_i)\tilde{\mathbf{c}}\right)^* \otimes \left(\mathbf{T}(\theta_i)\tilde{\mathbf{c}}\right) \\ &= \left(\mathbf{T}^*(\theta_i)\otimes\mathbf{T}(\theta_i)\right)\left(\tilde{\mathbf{c}}^*\otimes\tilde{\mathbf{c}}\right) \end{aligned} \tag{12}$$

Substituting (12) into (4), the vector of the received data in the presence of mutual coupling in the virtual array field can be expressed as

$$\begin{aligned} \mathbf{z} &= \sum_{i=1}^{K}\left(\mathbf{T}^*(\theta_i)\otimes\mathbf{T}(\theta_i)\right)\left(\tilde{\mathbf{c}}^*\otimes\tilde{\mathbf{c}}\right)\rho_i + \sigma_n^2\mathbf{1}_M \\ &= \tilde{\mathbf{T}}(\theta)\mathbf{p} + \sigma_n^2\mathbf{1}_M \end{aligned} \tag{13}$$

where

$$\tilde{\mathbf{T}}(\theta) = \begin{bmatrix} \mathbf{T}^*(\theta_1)\otimes\mathbf{T}(\theta_1) & \mathbf{T}^*(\theta_2)\otimes\mathbf{T}(\theta_2) & \cdots & \mathbf{T}^*(\theta_K)\otimes\mathbf{T}(\theta_K) \end{bmatrix} \in \mathbb{C}^{M^2\times m^2 K}$$

and $\mathbf{p} = \begin{bmatrix} \mathbf{p}_{c,1} & \mathbf{p}_{c,2} & \cdots & \mathbf{p}_{c,K} \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{m^2 K\times 1}$ with $\mathbf{p}_{c,i} = (\tilde{\mathbf{c}}^*\otimes\tilde{\mathbf{c}})^{\mathrm{T}}\rho_i$ $(i=1,2,\cdots,K)$.

Assume that the whole DOA space is uniformly divided into D grid, marked as $\boldsymbol{\theta} = \begin{bmatrix} \tilde{\theta}_1 & \tilde{\theta}_2 & \cdots & \tilde{\theta}_D \end{bmatrix}$. The DOA estimation can be obtained by using the following optimization problem:

$$\arg\min_{\tilde{\mathbf{p}}} \|\tilde{\mathbf{p}}\|_0 \quad s.t. \quad \left\|\mathbf{z} - \tilde{\mathbf{T}}_D\tilde{\mathbf{p}} - \sigma_n^2\mathbf{1}_M\right\|_2 < \varepsilon \tag{14}$$

where

$$\tilde{\mathbf{T}}_D = \begin{bmatrix} \mathbf{T}^*(\tilde{\theta}_1)\otimes\mathbf{T}(\tilde{\theta}_1) & \mathbf{T}^*(\tilde{\theta}_2)\otimes\mathbf{T}(\tilde{\theta}_2) & \cdots & \mathbf{T}^*(\tilde{\theta}_D)\otimes\mathbf{T}(\tilde{\theta}_D) \end{bmatrix}$$

Replacing θ_i with $\tilde{\theta}_i$ in (8), $\mathbf{T}(\tilde{\theta}_i)$ can be obtained. ε is a preset number with a small value. $\tilde{\mathbf{p}}$ can be expressed as $\tilde{\mathbf{p}} = \begin{bmatrix} \tilde{\mathbf{p}}_{c,1}, \tilde{\mathbf{p}}_{c,2}, \cdots, \tilde{\mathbf{p}}_{c,D} \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{m^2 D\times 1}$,where $\tilde{\mathbf{p}}_{c,i}(i=1,2,\cdots,D)$ is a vector with dimension $m^2\times 1$.

This optimization problem can be solved by using LASSO method or the orthogonal matching pursuit (OMP) algorithm based on the block matching dictionary matrix. However, both of them cannot deal with the problem that DOAs is off grid of the dictionary matrix. We intend to adopt the approach of the Bayesian compressive sensing to estimate DOAs.

B. The off-grid model for Bayesian compressive sensing

In fact, the true covariance matrix $\mathbf{R}_X$ is replaced with the sample covariance matrix $\hat{\mathbf{R}}_X$, which is calculated as

$$\hat{\mathbf{R}}_X = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{15}$$

Vectorizing $\hat{\mathbf{R}}_X$ yields the following vector

$$\breve{\mathbf{z}} = vec\left(\hat{\mathbf{R}}_X\right) = \mathbf{z} + \Delta\mathbf{z} = \tilde{\mathbf{T}}(\theta)\mathbf{p} + \sigma_n^2\mathbf{1}_M + \Delta\mathbf{z} \tag{16}$$

Due to the limited number of snapshots, there is an error in the result of $\breve{\mathbf{z}}$. As it in [22], the error $\Delta\mathbf{z}$ is an asymptotic normal distribution with $\Delta\mathbf{z}\sim \mathrm{AsN}(0,\mathbf{W})$,where $\mathbf{W} = (\mathbf{R}_x^{\mathrm{T}}\otimes\mathbf{R}_x)/N$ .By using $\hat{\mathbf{W}} = (\hat{\mathbf{R}}_x^{\mathrm{T}}\otimes\hat{\mathbf{R}}_x)/N$ instead of W, a prewhitening procedure is allowed to be performed to yield the single measurement vector as

$$\mathbf{y} = \hat{\mathbf{W}}^{-1/2}\breve{\mathbf{z}} = \hat{\mathbf{W}}^{-1/2}\mathbf{z} + \Delta\tilde{\mathbf{z}} \tag{17}$$

where $\Delta\tilde{\mathbf{z}} = \hat{\mathbf{W}}^{-1/2}\Delta\mathbf{z}\sim \mathrm{AsN}(0,\mathbf{I})$ denotes a complex Gaussian white noise vector.

When the DOA of the signal is not on the dividing grid, it will inevitably cause the grid mismatch and the estimate error will appear. If the real DOA is θ_k , it can be expressed by using the corresponding estimated value on the grid $\tilde{\theta}_{l_k}$ as

$$\theta_k = \tilde{\theta}_{l_k} + \delta_{l_k} \tag{18}$$

where δ_{l_k} is the error between them and is generally supposed to be uniformly distributed in the siding-to-siding block $[-l/2, l/2]$ with $l = \tilde{\theta}_2 - \tilde{\theta}_1$. By using Taylor formula and ignoring the expansion terms above first order, $\mathbf{T}^*(\theta_k)\otimes\mathbf{T}(\theta_k)$ can be rewritten as

$$\begin{aligned} \mathbf{T}^*(\theta_k)\otimes\mathbf{T}(\theta_k) &\approx \mathbf{T}^*(\tilde{\theta}_{l_k})\otimes\mathbf{T}(\tilde{\theta}_{l_k}) + \left(\mathbf{T}^*(\tilde{\theta}_{l_k})\otimes\mathbf{T}(\tilde{\theta}_{l_k})\right)'(\theta_k - \tilde{\theta}_{l_k}) \\ &= \mathbf{T}^*(\tilde{\theta}_{l_k})\otimes\mathbf{T}(\tilde{\theta}_{l_k}) + \left(\mathbf{T}^*(\tilde{\theta}_{l_k})\otimes\mathbf{T}(\tilde{\theta}_{l_k})\right)'\delta_{l_k} \end{aligned} \tag{19}$$

where $k\in\{1,2,\cdots,\mathrm{K}\}$ and $l_k\in\{1,2,\cdots,D\}$. $\left(\mathbf{T}^*(\tilde{\theta}_{l_k})\otimes\mathbf{T}(\tilde{\theta}_{l_k})\right)'$ is the first derivative of $\left(\mathbf{T}^*(\tilde{\theta}_{l_k})\otimes\mathbf{T}(\tilde{\theta}_{l_k})\right)$,which can be labeled as $\boldsymbol{b}(\tilde{\theta}_{l_k})$. Therefore, $\tilde{\mathbf{T}}(\theta)$ can be expressed as

$$\tilde{\mathbf{T}}(\theta) = \tilde{\mathbf{T}}_e + \mathbf{B}\boldsymbol{\Delta} \tag{20}$$

where

$$\tilde{\mathbf{T}}_e = \begin{bmatrix} \mathbf{T}^*(\tilde{\theta}_{l_1})\otimes\mathbf{T}(\tilde{\theta}_{l_1}) & \mathbf{T}^*(\tilde{\theta}_{l_2})\otimes\mathbf{T}(\tilde{\theta}_{l_2}) & \cdots & \mathbf{T}^*(\tilde{\theta}_{l_K})\otimes\mathbf{T}(\tilde{\theta}_{l_K}) \end{bmatrix} \tag{21}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{b}(\tilde{\theta}_{l_1}) & \mathbf{b}(\tilde{\theta}_{l_2}) & \cdots & \mathbf{b}(\tilde{\theta}_{l_K}) \end{bmatrix} \tag{22}$$

$$\boldsymbol{\Delta} = blkdiag\left(\begin{bmatrix} \delta_{l_1}\boldsymbol{I}_{m\times m} & \delta_{l_2}\boldsymbol{I}_{m\times m} & \cdots & \delta_{l_K}\boldsymbol{I}_{m\times m} \end{bmatrix}\right) \tag{23}$$

When $\theta_k = \tilde{\theta}_{l_k}$, there is $\delta_{l_k} = 0$. Otherwise, $\delta_{l_k}\ne 0$. Substituting (20) into (16), $\breve{\mathbf{z}}$ can be expressed as

$$\breve{\mathbf{z}} = \left(\tilde{\mathbf{T}}_e + \mathbf{B}\boldsymbol{\Delta}\right)\mathbf{p} + \sigma_n^2\mathbf{1}_M + \Delta\mathbf{z} \tag{24}$$

Naturally, $\mathbf{y}$ can be re-expressed as

$$\mathbf{y} = \mathbf{G}\mathbf{p} + \sigma_n^2\hat{\mathbf{W}}^{-1/2}\mathbf{1}_M + \Delta\tilde{\mathbf{z}} \tag{25}$$

where

$$\mathbf{G} = \hat{\mathbf{W}}^{-1/2}(\tilde{\mathbf{T}}_e + \mathbf{B}\boldsymbol{\Delta}) \tag{26}$$

Noted that $\tilde{\theta}_{l_k}$ and δ_{l_k} ($1,2,\cdots,K$) are the ones to be estimated. According to (26), the complete dictionary matrix for Bayesian compressive sensing can be constructed as

$$\tilde{\mathbf{G}} = \hat{\mathbf{W}}^{-1/2}\left(\tilde{\mathbf{T}}_D + \mathbf{B}_D\mathbf{\Delta}_D\right) \tag{27}$$

where

$$\mathbf{B}_D = \left[\mathbf{b}(\tilde{\theta}_1) \quad \mathbf{b}(\tilde{\theta}_1) \quad \cdots \quad \mathbf{b}(\tilde{\theta}_D)\right] \tag{28}$$

$$\Delta_D = blkdiag\left(\left[\delta_1 \boldsymbol{I}_{m\times m} \quad \delta_2 \boldsymbol{I}_{m\times m} \quad \cdots \quad \delta_D \boldsymbol{I}_{m\times m}\right]\right) \tag{29}$$

In the theory of compressive sensing, (25) can be expressed as

$$\mathbf{y} = \tilde{\mathbf{G}}\tilde{\mathbf{p}} + \sigma_n^2\hat{\mathbf{W}}^{-1/2}\mathbf{1}_M + \Delta\tilde{\mathbf{z}} \tag{30}$$

Here, the DOA estimation is based on block matching. When $\tilde{\theta}_{l_k}$ $\left(l_k \in \{1,2,\cdots,D\}\right)$ is the estimated value for θ_k $\left(k \in \{1,2,\cdots,\mathrm{K}\}\right)$, the l_k th $m^2\times 1$ dimension vector in $\tilde{\mathbf{p}}$ will approximately equal to $(\tilde{\mathbf{c}}^*\otimes\tilde{\mathbf{c}})^{\mathrm{T}}\rho_k$ .The other elements in $\tilde{\mathbf{p}}$ is approximately to be 0.

C. Sparse Bayesian learning method for the DOA estimation in the presence of unknown mutual coupling

Since the angle of the target signal with mutual coupling does not exist in the angle region we divide, we can adopt we adopt the sparse reconstruction idea based on sparse Bayesian learning method. Therefore, the iteration process is given as follow

$$\boldsymbol{\mu} = \alpha_0\mathbf{\Sigma}\tilde{\mathbf{G}}^H\tilde{\mathbf{y}} \tag{31}$$

$$\mathbf{\Sigma} = (\mathbf{\Lambda}^{-1} + \alpha_0\tilde{\mathbf{G}}^H\tilde{\mathbf{G}})^{-1} = \mathbf{\Lambda} - \mathbf{\Lambda}\tilde{\mathbf{G}}^H(\alpha_0^{-1}\mathbf{I} + \tilde{\mathbf{G}}\mathbf{\Lambda}\tilde{\mathbf{G}}^H)^{-1}\tilde{\mathbf{G}}\mathbf{\Lambda} \tag{32}$$

$$\alpha_i^{new} = \frac{-1 + \sqrt{1 + 4\rho([\boldsymbol{\mu}]_i^{\mathrm{H}}\bullet[\boldsymbol{\mu}]_i + [\mathbf{\Sigma}]_{i,i})}}{2\rho} \tag{33}$$

$$\alpha_0^{new} = \frac{\bar{M} + (\mathrm{c}-1)}{d + \left\|\tilde{\mathbf{y}} - \tilde{\mathbf{G}}\boldsymbol{\mu}\right\|_2^2 + \alpha_0^{-1}\sum_{i=1}^{m^2 D}(1 - \alpha_i^{-1}[\mathbf{\Sigma}]_{i,i})} \tag{34}$$

$$\begin{aligned} &\mathbf{F} = \mathrm{Re}\{(\hat{\mathbf{B}}_W^H\hat{\mathbf{B}}_W)^* \odot (\boldsymbol{\mu}\bullet\boldsymbol{\mu}^H + \mathbf{\Sigma})\} \\ &\mathbf{v} = \mathrm{Re}\{diag(\boldsymbol{\mu}^*)\hat{\mathbf{B}}_W^H(\mathbf{y} - \hat{\boldsymbol{T}}_W\boldsymbol{\mu})\} \\ &-\mathrm{Re}\{\mathrm{diag}(\hat{\mathbf{B}}_W^H\hat{\boldsymbol{T}}_W\mathbf{\Sigma})\} \\ &\hat{\mathbf{B}}_W = \hat{\mathbf{W}}^{-1/2}\mathbf{B}_D \\ &\hat{\boldsymbol{T}}_W = \hat{\mathbf{W}}^{-1/2}\tilde{\mathbf{T}}_D \end{aligned} \tag{35}$$

$$\tilde{\delta}_n^{new} = \frac{[\mathbf{v}]_n - (\mathbf{F}(n,:))_{-n}^T\boldsymbol{\delta}_{-n}}{[\mathbf{F}]_{n,n}} \tag{36}$$

$$\hat{\delta}_n^{new} = \begin{cases} \tilde{\delta}_n^{new}, & \tilde{\delta}_n^{new} \in \left[-\frac{l}{2}, \frac{l}{2}\right] \\ -\frac{l}{2}, & \tilde{\delta}_n^{new} < -\frac{l}{2} \\ \frac{l}{2}, & otherwise \end{cases} \tag{37}$$

Where $(\bullet)^{\mathrm{i}}$ denote absolute operation and the ith iteration, $\boldsymbol{\delta}$=$diag(\Delta_D)$, $\mathbf{\Lambda} = diag(\boldsymbol{\alpha})$, c and d are fixed parameters which are required to approximate to 0. For convenience, we replace $\mathbf{y}$ with $\tilde{\mathbf{y}} = \mathbf{y} - \sigma_n^2\hat{\mathbf{W}}^{-1/2}\mathbf{1}_M$=$\tilde{\mathbf{G}}\tilde{\mathbf{p}} + \Delta\tilde{\mathbf{z}}$.

Observing (27) and (30), it is easy to find that the proposed method is not to estimate the power of the signal directly, but to estimate the vector $\tilde{\mathbf{p}}_{c,i}\left(i = 1,2,\cdots,D\right)$. Therefore, the final power to estimate the DOA should be calculated as

$$\hat{\xi}_j = \left\|\varsigma((\mathrm{m}^2\bullet(j-1)+1):(\mathrm{m}^2\bullet j))\right\|^2, (j = 1,2,\cdots,D) \tag{38}$$

where

$$\begin{aligned} \varsigma_i &= \mathrm{E}\{\left\|[\tilde{\mathbf{p}}]_i\right\|^2\} = \left\|E\{[\tilde{\mathbf{p}}]_i\}\right\|^2 + E\{\left\|[\tilde{\mathbf{p}}]_i - E\{[\tilde{\mathbf{p}}]_i\}\right\|^2\} \\ &= \left\|[\boldsymbol{\mu}]_i\right\|^2 + Tr\{[\mathbf{\Sigma}]_{i,i}\} \end{aligned}$$

In fact, due to finite snapshot, there is a big value for $\hat{\xi}_j$ when there is a DOA in $\tilde{\theta}_i$. Otherwise, there is a small non-zero value for $\hat{\xi}_j$. We perform spectral peaking search on $\hat{\xi}_j$. Finally, the estimated K DOAs will be calculated as $\hat{\theta}_k = \tilde{\theta}_{l_k} + \delta_{l_k}$, $k = 1,2,\cdots,K$, where $\tilde{\theta}_{l_1},\tilde{\theta}_{l_2},\cdots,\tilde{\theta}_{l_K}$ are the DOAs corresponding to K Maximum Peak Points of $\hat{\xi}_j$.

IV. SIMULATION RESULTS

In this section, we consider some simulations that demonstrate the DOA estimation of signals in the presence of unknown mutual coupling by using the proposed approach. Here, a NA with $M = 6$ ($M_1 = 3, M_2 = 3$) omnidirectional sensors with $d = \lambda/2$ is considered. The position of array is [1,2,3,4,8,12]d. The number of the non-zero coefficient of the mutual coupling $m = 4$ and $c = \left[1, 0.2121 + 0.2121i, -0.0882 + 0.1214i, -0.0588 + 0.0809i\right]$. The signal is independent of each other. The scanning angle area is $(-90°, 90°)$. The array's additive noise is a Gaussian zero-mean spatially and temporally white process and $\sigma_n^2 = 1$.

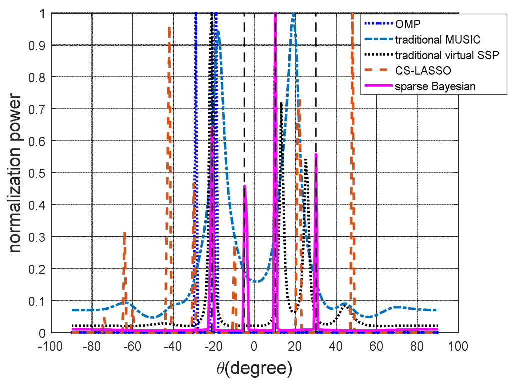

Fig.1 Resolution ability of DOA estimation

A. The DOA estimation of the signal number less than the number of the array elements

Because the traditional MUSIC in physical array field is limited by the DOF, we consider the case that the source number is smaller than the element number of the physical array. Results of the traditional MUSIC in the physical array field and the MUSIC with SSP in the virtual array field without the mutual coupling calibration are compared with results of three compressive sensing methods, the LASSO with prewhitening procedure, the OMP algorithm[23] and the sparse Bayesian learning method.

Assumed that SNR is 10dB and the number of snapshot is 1000. For the sparse Bayesian learning method, we set $l = 1, \rho = 0.01, \tau = 0.001, c = d = 10^{-4}$, with $Q = 2000$ being the maximum number of iteration. We can initialize $\alpha_0, \boldsymbol{\alpha}$ and Δ_D as $\alpha_0 = 100/\mathrm{var}(\mathbf{y})$, $\boldsymbol{\alpha}$=$\left|\hat{\boldsymbol{T}}_W^{\mathrm{H}}\mathbf{y}\right|/M^2$, and $\Delta_D = \mathbf{0}_{m^2 D\times 1}$,where D is the number of grid. For compressive sensing method of LASSO, we set $\lambda_t = 1.28$. The DOAs of signals are $[-21.1°, -5.2°, 10.1°, 30.1°]$. As shown in Fig.1, the traditional MUSIC in the physical array field, the MUSIC with SSP in the virtual array field without the mutual coupling calibration, the compressive sensing method of LASSO with prewhitening procedure and the OMP algorithm fails to estimate the DOA, while the sparse Bayesian learning method works well. The performance of the traditional MUSIC in the physical array field and the MUSIC with SSP in the virtual array field without the mutual coupling calibration suffers from the mutual coupling effect. The OMP algorithm is affected by the estimated angle and it also constrained by Least Squares Principle. The performance of the OMP algorithm has degraded when the DOA of signals is not very far apart. The prewhitening procedure may be the cause of the LASSO algorithm failure.

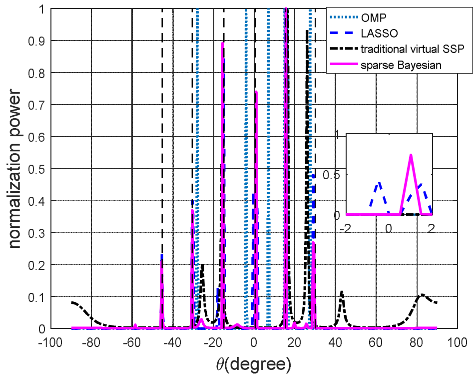

Fig.2 Resolution ability of DOA estimation performance analysis of the proposed algorithm

B. The DOA estimation of the signal number more than the number of the array apertures

In this part, we consider the case that the number of signals is more than the element number of the physical array. Results of the MUSIC with SSP in the virtual array field without the mutual coupling calibration and two compressive sensing methods, the LASSO without prewhitening procedure and the OMP algorithm, are compared with results of the sparse Bayesian learning method.

The DOAs of signals are [$-45.3°$, $-30.51°$, $-14.98°$, $0.5°$, $15.5°$, $30.1°$] . $l = 0.5$. Simulation results are shown in Fig.2. Although the compressive sensing method of LASSO without prewhitening procedure can estimate the DOA of the signal, there are many false peaks, whose peak values are nearly equal to the true peak value (see local enlargement in Fig. 3). It's not easy for us to distinguish the real peaks. The MUSIC with SSP in the virtual array field without the mutual coupling calibration fails to work due to the presence of the mutual coupling. The OMP still fails to estimate the DOA. The sparse Bayesian learning method works best.

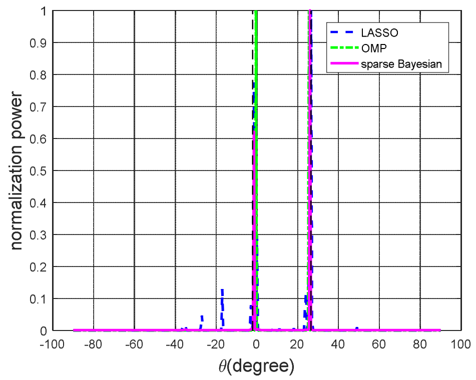

Fig.3 simulation results of the OMP algorithm, the LASSO algorithm and the Bayesian algorithm

C. The DOA estimation of the signal number much less than the number of the array apertures

The OMP algorithm, the LASSO algorithm without prewhitening procedure and the sparse Bayesian learning method are all based on compressive sensing. Among the three algorithms, the OMP algorithm has the greatest influence on the angle measurement results. In this part, only two sources at -2.15° and 26.38° are considered to validate these algorithms effectiveness for the case with small signal number. The number of snapshots is 1000 and SNR=10dB. Results of three algorithms are shown in Fig.3. The LASSO algorithm without prewhitening procedure still has many false pesks. Fortunately, two peaks correspond to the true estimates of LASSO algorithm are higher than the false peaks. The OMP algorithm only has two peaks. However, the estimate accuracy of the OMP algorithm is worse than the sparse Bayesian learning method.

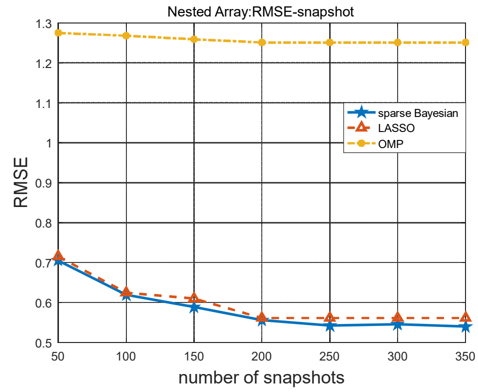

Fig.4 RMSE versus number of snapshot

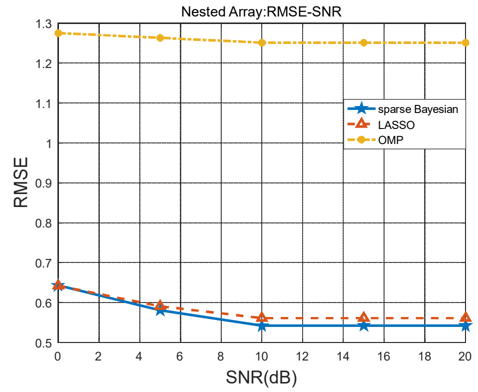

Fig.5 RMSE versus number of SNR

D. The DOA estimation performance

In order to observe the performance of the OMP algorithm, the LASSO algorithm without prewhitening procedure and the sparse Bayesian learning method better, Root Mean Square Errors (RMSE) varied with signal-to-noise (SNR) and number of snapshots are studied. Two sources at $-5.24°$ and $16.15°$ are considered.

In Fig.4, the number of snapshots varies from 50 to 350 and SNR=10dB. As shown in Fig.4, RMSE for all algorithms decreased as the number of snapshots increased at the beginning. When the snapshot number is greater than 200, RMSE for all algorithms gradually converges. When snapshot is small, the sample matrix $\mathbf{y}$ estimated using finite snapshots is imprecise. Therefore, the result with small snapshot is not good as the estimate one with large snapshot. The performance of the OMP algorithm is worst and the performance of the sparse Bayesian learning is best.

In Fig.5, the SNR varies from 0dB to 20dB and T=1000. As shown in Fig.5, we can know that the estimated accuracy of high SNR is better than the one of low SNR. Besides, the LASSO algorithm can't deal with the problem of off-grid. The estimate results are worsening than the proposed algorithm. The orthogonal complete basis matrix of the OMP algorithm is susceptible to local convergence, which makes the result inaccurate. It can also be clearly seen from the Fig.4 and Fig.5 that the OMP algorithm is slightly bad than LASSO algorithm and the proposed al gorithm. The proposed algorithm can estimate the DOA very well.

V. Conclusions

A DOA estimation method based on sparse Bayesian learning method in the presence of unknown mutual coupling is proposed. This novel algorithm is able to solve problem that there are unknown mutual coupling and off-grid mismatch together. Besides, no complicated calculations are needed to obtain the mutual coupling coefficients or rearrange the position of array element. The DOF of the NA is fully utilized in this proposed method. Simulation results demonstrate the effectiveness of the propose algorithm.

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References

[1] Zhang, Y.P., Li, W.J., Chen, Q., et al.: ‘A robust adaptive beamformer based on desired signal covariance matrix estimation’. 2016 IEEE Int. Conf. on Signal Processing, Communications and Computing (ICSPCC), Hong Kong,China, August 2016, pp. 1–4

[2] Krim H , Viberg M . Two decades of array signal processing research: the parametric approach[J]. IEEE Signal Processing Magazine, 1996, 13(4):67-94.

[3] Tie-Jun Shan, Wax, M, Kailath, T. On spatial smoothing for direction-of-arrival estimation of coherent signals[J]. IEEE Trans.acoust.speech Signal Processing, 1985, 33(4):806-811.

[4] Tuncer T E , Friedlander B . Classical and Modern Direction-of-Arrival Estimation[J]. 2009.

[5] Pal P, Vaidyanathan P P. Nested arrays: a novel approach to array processing with enhanced degrees of freedom[J]. IEEE Transactions on Signal Processing, 2010, 58(8):4167-4181.

[6] Vaidyanathan, P.P. Sparse Sensing With Co-Prime Samplers and Arrays[J]. IEEE Transactions on Signal Processing, 2011, 59(2):573-586.

[7] Malioutov D , Çetin, Müjdat, Willsky A . A sparse signal reconstruction perspective for source localization with sensor arrays[J]. IEEE Transactions on Signal Processing, 2005, 53(8):3010-3022.

[8] Schmidt R , Schmidt R O . Multiple emitter location and signal parameters estimation[J]. IEEE Transactions on Antennas & Propagation, 1986, 34(3):276-280.

[9] Roy R , Kailath T . ESPRIT-estimation of signal parameters via rotational invariance techniques[J]. IEEE Transactions on Acoustics, Speech, and Signal Processing, 2002, 37(7):984-995.

[10] J. Shi, G. Hu, X. Zhang and H. Zhou, "Generalized Nested Array: Optimization for Degrees of Freedom and Mutual Coupling," in IEEE Communications Letters, vol. 22, no. 6, pp. 1208-1211, June 2018.

[11] C. Liu and P. P. Vaidyanathan, "Super Nested Arrays: Linear Sparse Arrays With Reduced Mutual Coupling—Part I: Fundamentals," in IEEE Transactions on Signal Processing, vol. 64, no. 15, pp. 3997-4012, 1 Aug.1, 2016.

[12] C. Liu and P. P. Vaidyanathan, "Super Nested Arrays: Linear Sparse Arrays With Reduced Mutual Coupling—Part II: High-Order Extensions," in *IEEE Transactions on Signal Processing*, vol. 64, no. 16, pp. 4203-4217, 15 Aug.15, 2016.

[13] Chen, Lu; Bi, Daping; Pan, Jifei, A Direction of Arrial Estimation Algorithm for Translational Nested Array Besed on Sparse Bayesian Learning[J]. Dianzi Yu Xinxi Xuebao/Journal of Electronics and Information Technology, v 40, n 5, p 1173-1180, May 2018.

[14] E. BouDaher, F. Ahmad, M. Amin and A. Hoorfar, "DOA estimation with co-prime arrays in the presence of mutual coupling," 2015 23rd European Signal Processing Conference (EUSIPCO), Nice, 2015, pp. 2830-2834.

[15] J.Dai, D. Zhao, and Z. Ye,” DOA estimation and self-calibration algorithm for nonuniform linear array” in Proc. International Symposium on Intelligent Signal Processing and Communications Systems,Chengdu,China,2010

[16] Yang Z , Xie L , Zhang C . Off-Grid Direction of Arrival Estimation Using Sparse Bayesian Inference[J]. IEEE Transactions on Signal Processing, 2013, 61(1):38-43.

[17] Liu H , Zhao L , Li Y , et al. Off-grid DOA estimation[C]// IEEE International Conference on Digital Signal Processing. IEEE, 2015.

[18] Cui H , Duan H , Liu H . Off-grid DOA estimation using temporal block sparse Bayesian inference[C]// IEEE International Conference on Digital Signal Processing. IEEE, 2017.

[19] J. Dai, X. Bao, W. Xu and C. Chang, "Root Sparse Bayesian Learning for Off-Grid DOA Estimation," in *IEEE Signal Processing Letters*, vol. 24, no. 1, pp. 46-50, Jan. 2017.

[20] A. Das, "Theoretical and Experimental Comparison of Off-Grid Sparse Bayesian Direction-of-Arrival Estimation Algorithms," in IEEE Access, vol. 5, pp. 18075-18087, 2017.doi: 10.1109/ACCESS.2017.2747153

[21] Y. Zhao, L. Zhang and Y. Gu, "Array covariance matrix-based sparse Bayesian learning for off-grid direction-of-arrival estimation," in *Electronics Letters*, vol. 52, no. 5, pp. 401-402, 3 3 2016.

[22] Yin J.H, Chen T.Q. Direction-of-Arrival Estimation Using a Sparse Representation of Array Covariance Vectors[J]. IEEE Transactions on Signal Processing, 2011, 59(9):4489-4493.

[23] J. A. Tropp and A. C. Gilbert, "Signal Recovery From Random Measurements Via Orthogonal Matching Pursuit," in *IEEE Transactions on Information Theory*, vol. 53, no. 12, pp. 4655-4666, Dec. 2007. doi: 10.1109/TIT.2007.909108