Joint Sparse Channel Estimation in Downlink NOMA System

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Abstract—Non-orthogonal multiple access (NOMA) is regarded as one of the most important techniques for future 5G systems. In the downlink general NOMA schemes, the received NOMA signal will be analyzed via two parallel channel state information (CSI) after sparse multiple path channel fading. In this paper, by exploiting the inherent sparsity of the channel, we proposed a low-complexity joint channel estimation in a single-input and multiple-output antennas system, based on the compressed sensing to detect each layer channel state information. As a comparison, the performance of compressed sensing is better than the conventional method Least-Square (LS) and Minimum Mean Square Error (MMSE).

Keywords—non-orthogonal multiple access, sparse channel estimation, distributed pilot, compressed sensing, orthogonal matching pursuit.

I. INTRODUCTION

Multiple access scheme is one of the key techniques for cellular mobile communication systems. Orthogonal frequency division multiple access (OFDMA) or single carrier frequency division multiple access (SC-FDMA) is adopted by 3GPP Long Term Evolution (LTE) and LTE-Advanced in 4th generation communication systems [1]. Although the orthogonal multiple access improves the throughput performance, it cannot satisfy the requirements for next generation cellular mobile communication systems.

In the future multiple access scheme, coverage, flexibility and massive multiple-input and multiple-output (massive-MIMO) should be considered in [2]. Non-orthogonal multiple access (NOMA) can adjust the power allocation of each user equipment (UE) to offer significant improvements for cellular-edge users and be combined with Massive MIMO, which is another key technique for 5th generation communication system [3]. In the NOMA system, multi-users can be composited on the same resources by allocating a fraction of the total power to the multiple users’ data flow, and the throughput of the cell-center and cell-edge users would be increased compared with OMA systems [4]. We proposed a downlink NOMA where multiple users are integrated into the power-domain on the transmitter side and multi-user data information on the receiver side is extracted via the successive interference cancellation (SIC) [5]. On the receiver side, the data information of weak channel gain user can be decoded based on SIC technique. By employing this way, the strongest channel gain user can remove interference from the weaker channel gain user [6]. However, the multiple user’s information can be separated by the channel gain, the effective signal-to-interference-plus-noise ratio (SINR) is influenced by the quality of the channel.

The impact of channel state information (CSI) under different channel gain and distance path loss would be impacted on the cancellation of the interference and the system-level user throughput. The SIC process is accomplished on the receiver side depends on the increasing the channel gain standardized by the noise and inter-cell interference power. Therefore, the performance of CSI at the cell-edge should be perfectly estimated. General Channel Estimation is implemented by interested known pilots using Least-Square (LS) or Minimum Mean Square Error (MMSE) method to collect the CSI from received signal on the receiver side. Sparse channel estimation based on the compressed sensing can overcome the noise interference which compares with the conventional scheme because of the inherence sparsity of sparse vector. On the other hand, the CS-based channel estimation is to achieve the reduction in the training overhead than what conventional methods require when the channel can be sparsely represented. Thus, in the NOMA systems, the impact of inter-interference could be decreased by compressed sensing and obtain a good performance of CSI from lower channel gain users with smaller amounts of pilots [7]. The joint pilot scheme and partial channel estimation has been investigated in [8][9]. However, the overhead of pilots and distributed scheme can be decreased and adopted by Compressed Sensing (CS) in OMA, because the features of the sparse channel [10].

In this paper, we propose a different size distributed pilot scheme in each system level and obtain the CSI by Orthogonal Matching Pursuit (OMP) under the frame of empirical compressed sensing algorithm on the receiver side. Specially, we use more pilots on the cell-edge user and small numbers of pilots on the cell-center user to obtain good channel state information of cell-edge to detect the cell-center user code and remove the successive interference cancellation. The remainder of this paper is organized as follows. In Section II, we describe the system model and the key features introduce the NOMA. In Section III, we discuss the performance of throughput and Bit Error Rate (BER) after the channel estimation by the compressed sensing and least square method. In Section IV, we conclude the paper and future work.

II. SYSTEM MODEL

This section describes the system model and distributed pilot scheme in NOMA for multi-user at the transmitter side with SIC at UE. In this paper, we assume a single-input and
multiple-output (SIMO) systems where the number of antennas is one \(N_r = 1\), and the numbers of UE is two used single antennae in different distance in the same cellular \(N_c = 2\). We assume the multi-users is defined as \(U_s = \{s=1,2\}\), the time index is \(t\), the subcarrier index is \(f\), and the power allocation is \(P_s = \{i=1,2\}, P_i = 1\).

### A. Basic NOMA System

The transmitted signal \(x_t\) is a coded modulation symbol, \(d_f\) of the \(s\)-th user’s data information by modulation. We combine the multiple users together by distributing into the different power locations. Thus, the \(d_f\) of all users are superposed as

\[
X = \sqrt{P_i}d_S + \sqrt{P_f}d_S \tag{1}
\]

Where the \(E[d] = 1\). The \(N_r\) dimensional received signal of users \(y\) is represented by

\[
y = h\sqrt{E}\cdot x_t + n \tag{2}
\]

where \(a\) means the different distance path loss from a transmitter to the cell-center user and cell-edge user. \(h\) is the \(N_r\) dimensional channel matrix from the transmitter to the user side whose elements represent Rayleigh fading channel gain \([9]\), and \(n\) means the additive white Gaussian noise (AWGN) with zero mean and variance \(\sigma^2\). We define the channel gain as \(h/n\). In the NOMA downlink, the SIC process is implemented on the receiver side, and user can correctly decode based on the channel gain, and the UE with lower channel gain can eliminate the inter-user interference and allocated higher level of transmitter power than the user with higher channel gain. We assume the cell-edge user (UE2) with lower channel gain than cell-center user (UE1), \(h_2/f/n_2 < h_1/f/n_1\), UE1 with higher channel gain is treated as noise when decoding the UE2 signal with no decoding, nor cancellation of UE1. Thus, we can decode the UE2 signal by eliminating the UE1 inter-interference. According to Shannon formula, the UE1 and UE2 throughput can be shown as

\[
R_1 < \log (1 + \frac{P_1h_1^P}{P_1f_1^P + n_0}) \tag{3}
\]

\[
R_2 < \log (1 + \frac{P_2h_2^P}{n_0}) \tag{4}
\]

We can observe system-level throughput depended on the contribution of channel gain and the condition of lower channel gain user, the lower channel gain UE2 (cell-edge) can be decoded without SIC, and the higher channel gain UE1 (cell-center) is influenced by the SINR \([11]\). On the other hand, the transmitted power ratio also influences the performance of throughput and it has considered in \([6]\). The condition of CSI affects the quality of channel gain on the receiver side, we have to collect the CSI as good as possible.

### B. Sparse Channel Estimation

Sparsity of a signal is defined as the number of non-zero elements in the certain domain. It means that sparse signal is merely linear combination with few atoms from a certain domain. General channel estimation transmit known pilot signals and we receive the pilot signal on the receiver that has gone through the fading channel. The scattering, multipath fading, and etc. will destroy the transmitted signal. In order to remain the accuracy of channel estimation, we have to insert amounts of pilot signals into the subcarriers to counteract the destruction from the frequency-selective fading channel. Since the pilot signals only occupy the subcarriers but do not convey the data information, the spectrum efficiency will be below. On the other hand, the performance of channel impulse respond in the time domain is sparse, or at least approximately sparse on another basis \([12]\). That means the fading channel represent a sparsity in the time domain and there are few nonzero channel coefficients would impact the transmitted signal. Therefore, the ultra-wide band signals received over multipath fading channels can be approximately considered as a linear combination of several non-zeros channel coefficients in the time domain.

### C. Basic principle of CS

CS has been developed a few years ago, and it is widely used in the imaging, data compression, and also in wireless communication. The most feature of CS is the capacity to solve the underdetermined system. The CS can wisely recovery the sparse signal by greedy iteration algorithms. The greedy iteration algorithms can identify the support set in a greedy iterative manner, such as orthogonal matching pursuit (OMP). The most benefit of the CS-based channel estimation is to achieve the reduction in the training overhead when the channel can be sparsely represented, the channel can be required by the CS technique using a much smaller number of pilots than what the conventional method require. We assume multipath fading with few k-nonzero channel coefficients as \(x \in \mathbb{R}^K\) is the k-sparse vector. The key of CS theory is to find out a stable measurement matrix \(A \in \mathbb{R}^{M \times N}\) (\(M < N\)), to acquire exact recovery of the sparse signal with length \(N\) from \(M\) measurement. Since the \(M < N\), it is an undetermined problem, we can acquire the solution when and only when the measurement matrix satisfies the Restricted Isometry Property (RIP) under the following condition:

\[
(1 - \delta_k)\|x\|^2_2 \leq \|Ax\|^2_2 \leq (1 + \delta_k)\|x\|^2_2 \tag{5}
\]

where \(\delta_k \in (0,1)\). The measurement \(A\) matrix acts almost like an isometry for sparse vectors having more than \(K\) non-zeros entries when the measurement matrix satisfies the RIP \([13]\). In other words, the RIP ensures the numbers of non-zeros entries in the reconstruct sparse vector \(\hat{x}\) will not be larger than \(K\).

In the greedy iteration algorithm, we use a Fourier matrix as the measurement matrix since the UWB signal represent the sparsity in the frequency domain. Because the fading channel only correspond at \(K\) taps in the time domain, we assume the numbers of the pilot signals at least larger than the \(2K\) and insert into the subcarriers by certain pilot space. In this paper, we consider the typical and one of the most
popular greedy iteration algorithms in CS, that is orthogonal matching pursuit algorithm to recovery the channel coefficients. The most feature of OMP algorithm is, it will optimize the local optimal solution in each iteration. The accuracy is considered in this algorithm, and OMP can be represented as following:

**Algorithm 1 Orthogonal Matching Pursuit**

**Require:** $y \in \mathbb{R}^n$ ( observed vector), $A$ is a measurement Matrix, $K$ is the number of nonzero taps

**Ensure:** $x \in \mathbb{R}^n$ (sparse vector)

**Start:** $x[0] = 0$

$y[0] = 0$

$\Lambda = []$

$K = 0$

**repeat**

$S = \text{arg max} \langle y, a_j \rangle$

$\Lambda = \Lambda \cup [S]$

$x'[k+1] = \text{arg min} ||A x-y||^2$

$r[k+1] = y- A x[k+1]$

$k = k+1$

**until** $r=0$ or $k = K$

**return** $x = x'[k]$

**Step 1:** Calculate and select the maximum inner products among the columns of normalized measurement matrix $a_j$ and observed pilot vector:

$$s = \frac{(a_i^T y)}{(a_i^T a_i)} \tag{6}$$

**Step 2:** Update the recovery sparse $x$ and $a_j$ into the recovery vector and submatrix of measurement matrix $A$.

**Step 3:** Evaluate the $x$ with measurement $A$ submatrix and observed vector $y$ by using the $l^2$-loss function and update the more accurate $x$ instead of the pervious:

$$x_{new} = \text{arg min}_{i \in \Lambda} \| y-A_i x_i \|^2 \tag{7}$$

**Step 4:** Calculate the residual by subtraction of the linear combination with measurement submatrix $A$ and recovery sparse vector $x$.

The iterative process stops when the residual equals zero or achieve the sparsity $K$.

**III. SIMULATION RESULT**

In this section, we investigate the joint channel estimation performance of the OMP algorithm in the SIMO wireless-communication system. The main parameters are shown as following. The number of users is $K=2$, the number of non-orthogonal subcarrier is $N=2048$, we use different power allocation, $P_1 = 0.1$, $P_2 = 0.9$ and $P_1 = 0.4$, $P_2 = 0.6$, to investigate the influence from the fairness of NOMA. The low channel gain user allocates 16 pilot signals into the subcarriers and strong channel gain user allocates 32 pilot signals into the subcarriers. We assume a FIR filter with 7-taps multipath fading in addition to the AWGN channel in this simulation. We compare the BER performance of the OMP algorithm and perfect channel state information, where the QPSK modulation is considered. We will compare the BER and investigate the influence of different power allocation with the OMP algorithm in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency</td>
<td>28[GHz]</td>
</tr>
<tr>
<td>User Number</td>
<td>2</td>
</tr>
<tr>
<td>Subcarrier</td>
<td>2048</td>
</tr>
<tr>
<td>Digital Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Channel Tap</td>
<td>7</td>
</tr>
<tr>
<td>FFTsize</td>
<td>2048</td>
</tr>
<tr>
<td>Celluar Radius</td>
<td>200[m]</td>
</tr>
<tr>
<td>Power1 allocation</td>
<td>0.1, 0.4</td>
</tr>
<tr>
<td>Power2 allocation</td>
<td>0.9, 0.6</td>
</tr>
<tr>
<td>Distance of UE1</td>
<td>20[m]</td>
</tr>
<tr>
<td>Distance of UE2</td>
<td>200[m]</td>
</tr>
</tbody>
</table>

Fig. 1 shows the BER between high power allocation user and low power allocation user. The performance of high power allocation user is better than low power allocation. There are two main reasons for this. Firstly, the low power allocation user should overcome the interference from other users. We have to cancel the interference of high power allocation user before the detection of low power allocation user. The CIR of high power allocation user will affect the performance of cancellation. Secondly, the ratio of power allocation also affects the capacity of robustness. The high power allocation user can greatly overcome the influence from noise and fading channel.

Fig.2 displays the normalized mean square error and SNR for CS channel estimation with 16 and 32 pilot space. Generally, the weaker channel gain user should be allocated with the large pilot space, and the strong channel gain user could be allocated small pilot space to guarantee the performance of CIR on the high channel gain user side. On the other hand, the performance of channel estimation based on the OMP is also affected by the correction that received pilot signals align with the measurement matrix. When the pilot space becomes larger, the assurance about selecting the columns of measurement matrix $A$ randomly would be lower. Therefore, the investigation of appropriate measurement submatrix in the NOMA system is also an important condition and it should be considered.

Fig.3 shows the channel spectrum in the frequence domain between the OMP channel estimator and perfect channel. The OMP channel estimator could recovery channel in the time domain since we choose the Fourier matrix as the measurement matrix. Therefore, we no longer need do interpolation comparing with the conventional channel estimation. On the other hand, we should convert the channel into the frequency domain before the channel equilization. As the result shown, the recovery channel is nearly same as the perfect channel to ensure a good channel equilization.
This paper investigates a proposal of joint user pairing channel estimation based on CS on the downlink NOMA and investigates the interference for the channel estimation by different power ratio. We use the OMP algorithm to obtain the channel state information. Furthermore, the different system-level power allocation can affect the performance of signal detection on the weaker channel gain user side. The simulation results reveal that proposed channel estimation is better than the conventional channel estimation, and we can also see the error of channel estimation will destroy the SIC on the strong channel gain user side.

IV. FUTURE WORK

Through this experiment, we know the CS is still potential in the non-orthogonal multiple access system and can observe a better result than the conventional method by using the less known signals. On the other hand, the performance of channel estimation based on the CS also depends on the decision of measurement submatrix and the prior sparsity. In the future, I would like to investigate the performance of CSI in the NOMA system with the Massive MIMO technique, and combine the knowledges of deep learning to decide a appropriate measurement submatrix and without prior sparsity.

REFERENCES


