A Simulation Investigation of Modified FxLMS Algorithms for Feedforward Active Noise Control

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***Abstract*—In this paper, two modified FxLMS algorithms are proposed based on the post-masking-based LMS (PMLMS) algorithm. They are the PM*l*-FxLMS and the signed PM*l*-FxLMS (SPM*l*-FxLMS) algorithms. In both algorithms, *l* denotes the length of the error signal memory. The control filter coefficients are updated by the maximum absolute value in the error signal memory, instead of the immediate value. The difference between the two modified FxLMS algorithms is that the PM*l*-FxLMS algorithm keeps the sign of the error sample with the largest absolute value, while the SPM*l*-FxLMS algorithm uses the sign of the immediate error sample. The simulation results show that the SPM*l*-FxLMS algorithm converges faster than the standard FxLMS algorithm with the same step-size, and the PM*l*-FxLMS algorithm may be difficult to converge when *l* is large.**

I. Introduction

Since Lueg proposed the concept of the "process of silencing sound oscillations" in 1936 [1], active noise control (ANC) has been developed to become an efficient way to reduce low frequency noise [2]. In this decade, ANC has received an increasing number of reports on public media because of the prominence of the ANC headphones and other applications. Technically, there are two classes of ANC systems. They are the feedback and the feedforward ANC systems [3]. The feedback ANC system is often deployed to control the narrowband noise merely using the secondary sources and error sensors without any reference sensors. The feedforward ANC system can control the broadband noise efficiently by adding in the reference sensors [4]. The multi-channel ANC (MCANC) system aims to control noise in a relatively large area. For example, Fig. 1 shows a case (I, J, K) feedforward MCANC system, containing I reference sensors, J secondary sources and K error sensors. When I, J, K are equal to 1, the system is also known as the single-channel ANC (SCANC) system.

The adaptive algorithm is an important research topic of ANC. Numerous researchers have been attempting to improve the standard filtered-x least-mean-squares (FxLMS) algorithm [5, 6, 7]. Recently, Wang et al. proposed a single-channel post-masking-based LMS (PMLMS) algorithm for active sound-quality control. They suggested that this post-masking-based method be considered to modify the FxLMS algorithm [8]. Therefore, this paper explores this idea and develops the post-masking-based FxLMS algorithms.

The principle of the post-masking-based LMS method is to choose a larger error sample between the immediate and the previous samples. Based on this idea, in order to get a larger error sample to achieve better performance of ANC, we have enlarged the memory size and made the error signal memory reconfigurable. This modified FxLMS algorithm is called the PMl-FxLMS algorithm, where l denotes the length of the error signal memory. However, it is more likely to select an error sample which has the opposite sign of the immediate error sample when the length of the error signal memory is large. This may cause the divergence of the PMl-FxLMS algorithm.

To solve this problem, the signed PMl-FxLMS (SPMl-FxLMS) algorithm is further proposed, which uses the largest absolute value in the error signal memory and the sign of the immediate error sample to update the control filter. The simulation results show that this algorithm can achieve better noise control performance when the length of the error signal memory increases. Last but not the least, both the PMl-FxLMS and the SPMl-FxLMS algorithms are extended into the multi-channel case in this paper.

II. Modified FxLMS Algorithms

In Fig. 1, $\mathbf{R}_i$ denotes the reference paths, which is sometimes omitted for simplicity. $\mathbf{x}_i(n)$ denotes the reference signal vector of the i-th reference sensor, which is written as

$$\mathbf{x}_i\left(n, L\right) = \left[x_i\left(n\right), x_i\left(n-1\right), \cdots, x_i\left(n-L+1\right)\right]^T, \quad (1)$$

where L is the length of this vector and T denotes the transpose.

The control signal vector of the j-th secondary source is written as

$$\mathbf{y}_j\left(n, L\right) = \left[y_j\left(n\right), y_j\left(n-1\right), \cdots, y_j\left(n-L+1\right)\right]^T. \quad (2)$$

where

$$y_j\left(n\right) = \sum_{i=1}^{I} \mathbf{x}_i{}^T\left(n, L_w\right)\mathbf{w}_{ji}\left(n\right) \quad (3)$$

and $\mathbf{w}_{ji}(n)$ denotes the control filter vector which calculates the output of the j-th secondary source based on the input of the i-th reference sensor. The control filter vector is written as

$$\mathbf{w}_{ji}\left(n\right) = \left[w_{ji}^{(0)}\left(n\right), w_{ji}^{(1)}\left(n\right), \cdots, w_{ji}^{(L_w-1)}\left(n\right)\right]^T, \quad (4)$$

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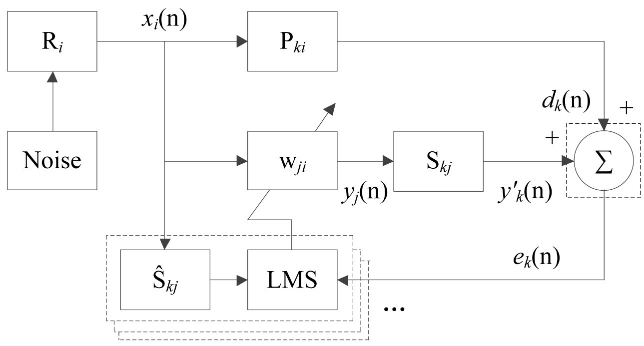

Fig. 1. Block diagram of standard multi-channel ANC system.

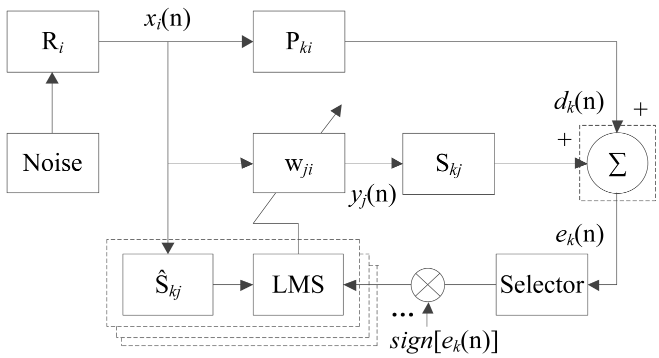

Fig. 2. Block diagram of SPMl-FxLMS algorithm.

where L_w is the tap length of control filter vector.

The error signal of the k-th error sensor is written as

$$e_k\left(n\right) = d_k\left(n\right) + \sum_{j=1}^{J} \mathbf{y}_j^T\left(n, L_s\right)\mathbf{S}_{kj}, \tag{5}$$

where the secondary path $\mathbf{S}_{kj}$ from the j-th secondary source to k-th error sensor is expressed as

$$\mathbf{S}_{kj} = \left[{s_{kj}}^{(0)}, {s_{kj}}^{(1)}, \cdots, {s_{kj}}^{(L_s-1)}\right]^T; \tag{6}$$

L_s is the length of the secondary path vector; and $d_k(n)$ denotes the noise signal received by the k-th error sensor. Furthermore, $d_k(n)$ is theoretically given by

$$d_k\left(n\right) = \sum_{i=1}^{I} {\mathbf{x}_i}^T\left(n, L_p\right)\mathbf{P}_{ki}, \tag{7}$$

where the primary paths $\mathbf{P}_{ki}$ from i-th reference sensor to k-th error sensor is expressed as

$$\mathbf{P}_{ki} = \left[{p_{ki}}^{(0)}, {p_{ki}}^{(1)}, \cdots, {p_{ki}}^{(L_p-1)}\right]^T, \tag{8}$$

and L_p is the length of the primary path vector.

A. Standard FxLMS Algorithm

In the multi-channel FxLMS algorithm, the cost function is the sum of the squared error samples [9], which is written as

$$J\left(n\right) = \sum_{k=1}^{K} e_k^2\left(n\right). \tag{9}$$

Therefore, the control filter is updated by

$$\mathbf{w}_{ji}\left(n+1\right) = \mathbf{w}_{ji}\left(n\right) - 2\mu\sum_{k=1}^{K} e_k\left(n\right)\mathbf{r}_{kji}\left(n\right), \tag{10}$$

where μ denotes the step-size and the leftmost element of the filtered reference signal vector $\mathbf{r}_{kji}\left(n\right)$ is given by

$$r_{kji}\left(n\right) = {\mathbf{x}_i}^T\left(n, L_{\hat{s}}\right)\hat{\mathbf{S}}_{kj}. \tag{11}$$

$\hat{\mathbf{S}}_{kj}$ is the estimate of the secondary path from j-th secondary source to k-th error sensor, which is written as

$$\hat{\mathbf{S}}_{kj} = \left[\hat{s}_{kj}^{(0)}, \hat{s}_{kj}^{(1)}, \cdots, \hat{s}_{kj}^{(L_{\hat{s}}-1)}\right]^T, \tag{12}$$

where $L_{\hat{s}}$ denotes the length of this vector

B. PMl-FxLMS Algorithm

The single-channel PMLMS algorithm provides that

$$\mathbf{w}\left(n+1\right) = \mathbf{w}\left(n\right) - 2\mu e'\left(n\right)\mathbf{x}\left(n\right), \tag{13}$$

where

$$e'\left(n\right) = \begin{cases} e\left(n\right), & \left|e\left(n\right)\right| \geq \left|e\left(n-1\right)\right| \\ e\left(n-1\right), & \left|e\left(n\right)\right| < \left|e\left(n-1\right)\right|. \end{cases} \tag{14}$$

Similarly, the single-channel post-masking-based FxLMS algorithm should update the control filter by

$$\mathbf{w}\left(n+1\right) = \mathbf{w}\left(n\right) - 2\mu e'\left(n\right)\mathbf{r}\left(n\right). \tag{15}$$

It is worth noting that the post-masking-based LMS and FxLMS algorithms keeps the sign of the selected error sample. If $e(n{-}1)$ and $e(n)$ have different signs, the algorithms stand a chance of divergence.

The PMl-FxLMS algorithm increases the length of the error signal memory and selects the error sample with the largest absolute value. Similar to the post-masking-based FxLMS algorithm, the PMl-FxLMS algorithm adopts the sign of the selected error sample. The control filters are therefore updated by

$$\mathbf{w}_{ji}\left(n+1\right) = \mathbf{w}_{ji}\left(n\right) - 2\mu\sum_{k=1}^{K} e_k\left(n-q\right)\mathbf{r}_{kji}\left(n\right) \tag{16}$$

and

$$\left|e_k\left(n-q\right)\right| = max\left[\left|e_k\left(n\right)\right|, \cdots, \left|e_k\left(n-l+1\right)\right|\right], \tag{17}$$

where q is the delay amount when the maximum absolute value is found in the error signal memory.

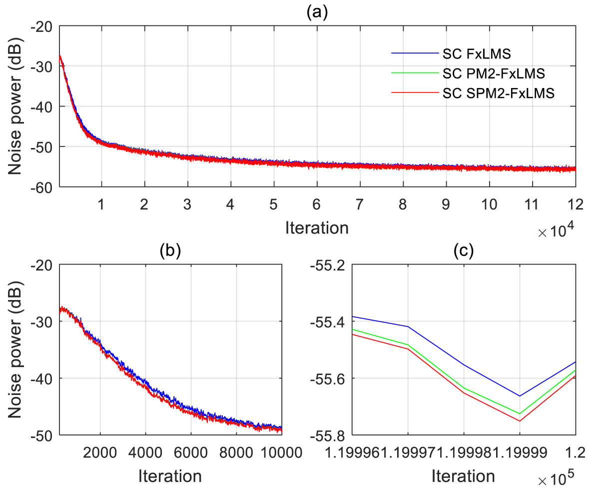

Fig. 3. Comparative results of the FxLMS, PM2-FxLMS and SPM2-FxLMS algorithms in the single-channel ANC system simulation: (a) all the iterations (b) the first 10000 iterations (c) the last 5 iterations.

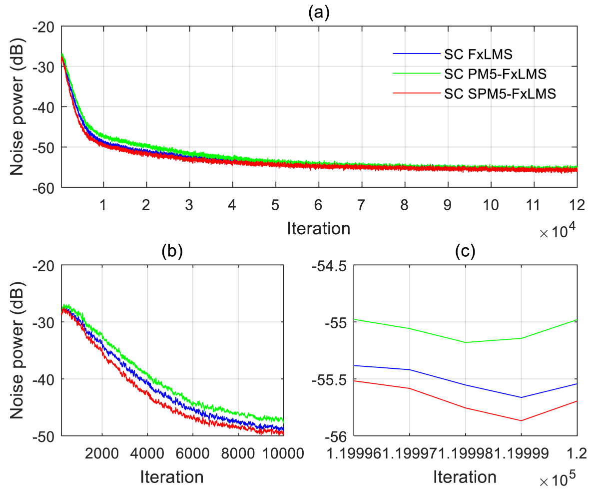

Fig. 4. Comparative results of the FxLMS, PM5-FxLMS and SPM5-FxLMS algorithms in the single-channel ANC system simulation: (a) all the iterations (b) the first 10000 iterations (c) the last 5 iterations.

C. SPMl-FxLMS Algorithm

The block diagram of the SPMl-FxLMS algorithm is shown in Fig. 2. As compared to the PMl-FxLMS algorithm, the SPMl-FxLMS algorithm adopts the sign of the immediate error sample and the maximum absolute value in the error signal memory to update the control filter, which can be written as

$$\mathbf{w}_{ji}(n+1) = \mathbf{w}_{ji}(n) - 2\mu \sum_{k=1}^{K} \beta_k \mathbf{r}_{kji}(n), \tag{18}$$

and

$$\beta_k = \begin{cases} e_k(n-q), & e_k(n) > 0 \\ -e_k(n-q), & e_k(n) < 0. \end{cases} \tag{19}$$

Apparently, when $l = 1$, both the proposed PMl-FxLMS and SPMl-FxLMS algorithm are identical to the standard FxLMS algorithm.

III. Simulation Results

Case $(1, 1, 1)$ and case $(1, 4, 4)$ feedforward ANC systems are configured for simulations [10]. The primary noise signal is generated as a band-limited random white noise, whose frequency ranges from 300 Hz to 1300 Hz. In order to omit the feedback path, the reference signal is set to be the same as the primary noise. The sampling frequency is set to 16 kHz. The tap length of the secondary path models and control filters is 400. Each simulation takes 120,000 iterations for the control filter to converge.

A. Case $(1, 1, 1)$ ANC System Simulation

The step-size of the case $(1, 1, 1)$ ANC system simulation is set to $4.0e-4$. Fig. 3(a) shows the noise power curve achieved by three different algorithms (single-channel standard FxLMS algorithm, single-channel PM2-FxLMS algorithm and single-channel SPM2-FxLMS algorithm). The convergence speeds

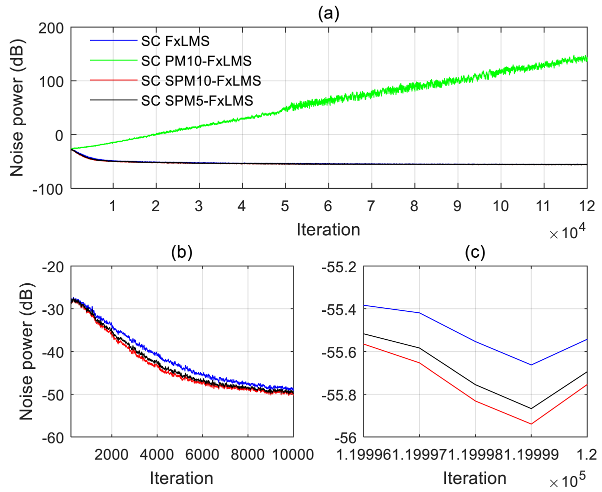

Fig. 5. Comparative results of the FxLMS, PM10-FxLMS and SPM10-FxLMS algorithms in the single-channel ANC system simulation: (a) all the iterations (b) the first 10000 iterations (c) the last 5 iterations.

of these algorithms are highlighted in Fig. 3(b). The noise power in the last 5 iterations is shown in Fig. 3(c). The two proposed algorithms perform slightly better than the standard FxLMS algorithm. The SPM2-FxLMS algorithm results in the fastest convergence and the best noise reduction, although this observation is not prominent.

When $l = 5$, the comparative noise reduction performance is demonstrated in Fig. 4. The SPM5-FxLMS algorithm still achieves the fastest convergence speed and the best noise reduction. However, the PM5-FxLMS algorithm performs worse than the standard FxLMS algorithm. This is likely due to the

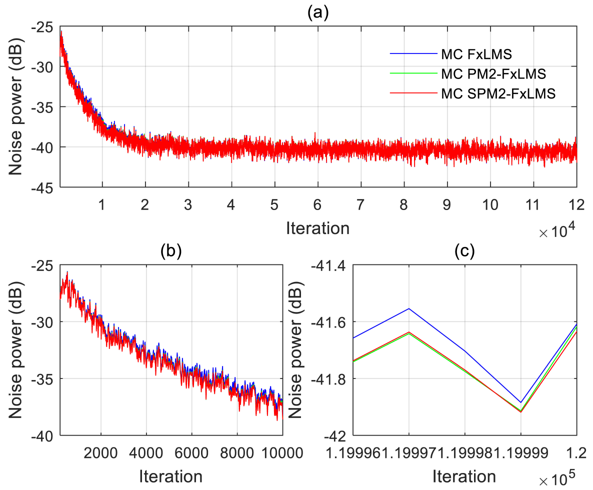

Fig. 6. Comparative results of the FxLMS, PM2-FxLMS and SPM2-FxLMS algorithms in the multi-channel ANC system simulation: (a) all the iterations (b) the first 10000 iterations (c) the last 5 iterations.

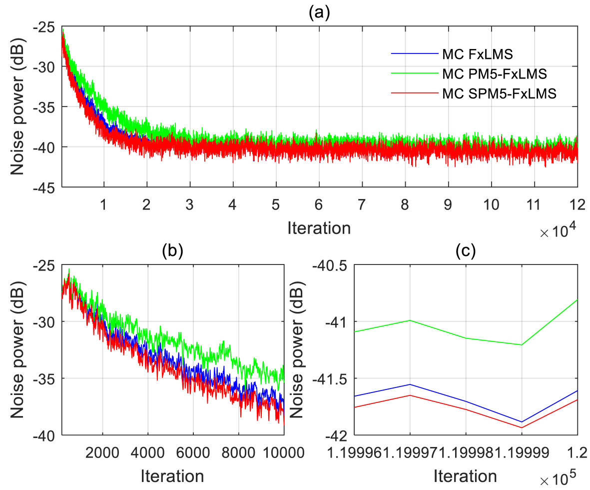

Fig. 7. Comparative results of the FxLMS, PM5-FxLMS and SPM5-FxLMS algorithms in the multi-channel ANC system simulation: (a) all the iterations (b) the first 10000 iterations (c) the last 5 iterations.

wrong sign incurred in the PMl-FxLMS algorithm.

Furthermore, when $l = 10$, the comparative noise reduction performance is demonstrated in Fig. 5. The SPM10-FxLMS algorithm outperforms the standard FxLMS and the PM10-FxLMS algorithms in terms of the convergence speed. This is because, when the error signal memory is large, there is a high chance to find the largest absolute value significantly greater than the absolute value of the immediate error sample. In the end, this leads to a much larger effective step-size. Moreover, the PM10-FxLMS algorithm diverges in this case. Summarizing the results in Figs. 3, 4 and 5, we confirm the importance of the sign in the adaptive algorithm. The problem of the post-masking-based LMS and FxLMS algorithms has been revealed.

B. Case $(1, 4, 4)$ *ANC System Simulation*

The step-size the case $(1, 4, 4)$ ANC system simulation is set to $3.0e{-}5$. Similar trends are observed in the multi-channel simulations. The comparison among the FxLMS, PM2-FxLMS and SPM2-FxLMS algorithms are shown in Fig. 6. When the error signal memory is short, both proposed algorithms perform slightly better than the standard FxLMS algorithm.

The comparison between the FxLMS, PM5-FxLMS and SPM5-FxLMS algorithms are shown in Fig. 7. The SPM5-FxLMS algorithm is able to achieve a faster convergence and a better noise reduction in the case $(1, 4, 4)$ ANC system simulation. The PM5-FxLMS algorithm performs worse than the standard FxLMS algorithm again.

When l increases to 10, the problem of the PMl-FxLMS algorithm is confirmed. As illustrated in Fig. 8, the PM10-FxLMS algorithm cannot converge in the multi-channel ANC system simulation. The SPM10-FxLMS algorithm remains to be the fastest algorithm, but the noise reduction is not as good as that of the SPM5-FxLMS algorithm. This implies that when the computational power allows, the error signal memory should be configured to a suitable size.

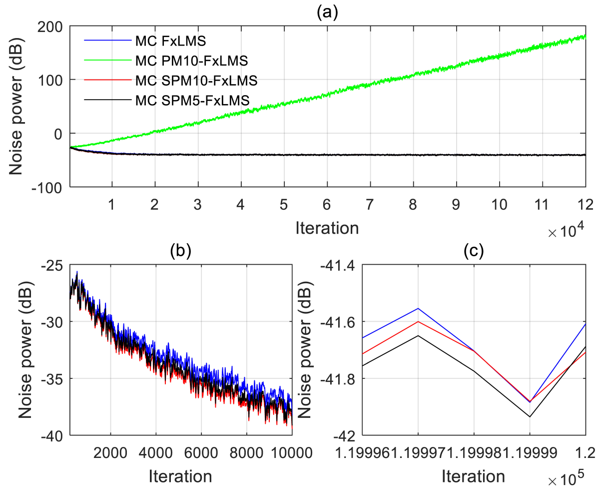

Fig. 8. Comparative results of the FxLMS, PM10-FxLMS and SPM10-FxLMS algorithms in the multi-channel ANC system simulation: (a) all the iterations (b) the first 10000 iterations (c) the last 5 iterations.

In summary, the simulation results demonstrate that the SPMl-FxLMS algorithm has the fastest convergence speed in both single-channel and multi-channel ANC system simulations. When the length of the error signal memory l increases, the performance of the PMl-FxLMS algorithm can become worse than that of the standard FxLMS algorithm and difficult to converge when l is relatively large.

IV. Conclusions

This paper proposes two modified FxLMS algorithms based on the post-masking-based LMS method. The proposed algorithms are the PM*l*-FxLMS and the SPM*l*-FxLMS algorithms. In both the single-channel and multi-channel simulations, with the error signal memory size increases, the SPM*l*-FxLMS algorithm is able to accelerate the convergence speed and improve the noise reduction performance. In contrast, when the error signal memory size is large, the PM*l*-FxLMS algorithm performs worse than standard FxLMS algorithm. Especially when the error signal memory is sufficiently large, the PM10-FxLMS algorithm cannot converge. This is explained by that a larger error signal memory causes a higher chance for the selected error sample to have the opposite sign of the immediate error sample. Therefore, the problem of the post-masking-based LMS and FxLMS algorithms has been confirmed.

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