Feedforward Active Noise Control with Coherence-Adjusting Filter for Improving Noise Reduction Performance under Low-Coherence Condition

†Kenta Iwai and ‡Takanobu Nishiura
Ritsumeikan University, Shiga, Japan
E-mail: † iwai18sp@fc.ritsumei.ac.jp, ‡nishiura@is.ritsumei.ac.jp

***Abstract*—We propose a feedforward active noise control (ANC) system with a coherence-adjusting filter. The feedforward ANC system can reduce the unwanted noise that is correlated with the reference signal picked up by the reference microphone. That is, the feedforward ANC system cannot reduce the unwanted noise when the unwanted noise and the reference signal are not correlated with each other. Hence, the coherence should be high to reduce the unwanted noise. However, only solution for the coherence problem is to place the reference microphone close to the error microphone. Under such a condition, the causality constraint is violated, and the noise reduction performance degrades. In this paper, we propose a feedforward ANC system with a coherence-adjusting filter that adjusts the reference signal to be similar to the unwanted noise. Then, the coherence between the reference signal and the unwanted noise increases, and the amount of noise reduction can be large even under the low-coherence condition. Simulation results show that the proposed ANC system can reduce the unwanted noise well compared with the conventional feedforward ANC system under different coherence conditions.**

***Index Terms*—Active noise control, feedforward control, coherence, adaptive digital filter**

I. Introduction

In recent years, active noise control (ANC) systems have been developed, for example, the multichannel ANC system with parametric array loudspeakers (PALs) [1], subband ANC systems based on a psychoacoustic feature [2], [3], virtual sensing ANC systems [4]–[6], and window ANC system [7]. These ANC systems are classified as feedforward ANC systems [8]–[11] or adaptive feedback ANC systems [12]–[15]. In this paper, we focus on the feedforward ANC system to improve its noise reduction performance.

Figure 1 shows the structure of the basic feedforward ANC system. The basic feedforward ANC system consists of a reference microphone, error microphone, and secondary loudspeaker. One of the problems of the feedforward ANC system is the lack of coherence between the signals obtained at the reference and error microphones [10], [16], [17]. This is because the feedforward ANC system can only reduce the unwanted noise that is correlated to the reference signal [16], [18]. Hence, the noise reduction performance degrades when the reference signal and the unwanted noise obtained at the error microphone are not correlated with each other, that is, the coherence is low.

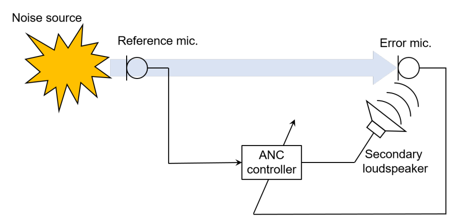


Fig. 1. Structure of basic feedforward ANC system.

As yet, the only solution for this problem is to place the reference microphone near the error microphone. Under this condition, the obtained signals of the reference and error microphones are highly correlated with each other. However, the noise reduction performance also degrades because the causality constraint is not satisfied [11], [16], [18]. That is, it is not better to place the reference microphone near the error microphone to solve the coherence problem because the generation and emission of anti-noise may not be completed before the unwanted noise reaches the error microphone.

In this paper, the feedforward ANC system with a coherence-adjusting filter (CAF) is proposed to increase the coherence between the reference signal and the unwanted noise. The proposed ANC system has an additional adaptive digital filter [19] for filtering the reference signal to estimate the unwanted noise. However, the proposed ANC system does not satisfy the causality constraint because the CAF is designed without considering the delay for the calculation of the ANC controller and propagation delay of the secondary path. Hence, some delays are added in the design of the CAF. Computer simulations were conducted to evaluate the capability of this system to increase the coherence between the reference signal and the unwanted noise and reduce the unwanted noise.

II. Relation to prior work and coherence problem of feedforward ANC system

In [1], the multichannel feedforward ANC system with PALs was proposed to create a quiet zone at the vicinity of the user's ears. A PAL has sharp directivity and this multichannel ANC system can create a quiet zone at the user's ears. Moreover, this ANC system can reduce the computational complexity because the crosstalk components are relatively small owing to the directivity of PALs. However, the noise reduction performance of this ANC system is almost the same as that of the conventional multichannel ANC system.

In [4]–[6], the use of the virtual sensing technique to reduce the unwanted noise at the appropriate area, for example, the ear drum, was reported. The ANC systems therein achieved the improved noise reduction performance. However, those ANC systems had improved noise reduction performance only at a certain area.

In [20], the multichannel feedforward ANC system with a sound source separation technique was proposed. This ANC system uses a microphone array to reduce the coherence between each reference signal obtained at the reference microphones and has improved the noise reduction performance in the two noise sources case. However, this ANC system requires the use of many reference microphones to reduce the unwanted noise.

Moreover, the above ANC systems do not focus on the coherence problem between the reference signal and the unwanted noise. Hence, they cannot reduce the unwanted noise components uncorrelated to the reference signal. This is because the coherence problem is generally solved by placing the reference microphone close to the error microphone. However, if the reference microphone is placed too close to the error microphone, the causality constraint [21], [22] is violated and the noise reduction performance of the feedforward ANC system degrades. Hence, other solutions are required for the coherence problem.

Figure 2 shows the block diagram of the basic feedforward ANC system. In Fig. 2, P, R, and S represent the primary path, reference path, and secondary path, respectively. $\hat{S}$ is the secondary path model that is the identified response of the secondary path. The unwanted noise $d(n)$ obtained at the error microphone is reduced by the anti-noise $y_{\mathrm{S}}(n)$. The anti-noise $y_{\mathrm{S}}(n)$ is generated using the noise control filter W and the reference signal $x(n)$; the noise control filter W is updated using the filtered reference signal $x_{\mathrm{S}}(n)$ and the error signal $e(n)$. The error signal $e(n)$ consists of $d(n)$ and $y_{\mathrm{S}}(n)$.

The ANC system minimizes the power of the error signal and the noise reduction performance can be estimated by the frequency analysis of the error power. Here, the power spectral density of the error signal $P_{ee}(k)$ is represented as

$$P_{ee}(k) = \mathbf{E}\left\{E^*(k)E(k)\right\}, \tag{1}$$

$$E(k) = D(k) - S(k)W(k)X(k), \tag{2}$$

where $\mathbf{E}$ is the expectation operator, $D(k)$ and $X(k)$ are the frequency spectra of $d(n)$ and $x(n)$, $S(k)$ and $W(k)$ respectively are the frequency responses of the secondary path and noise control filter, k is the frequency bin, and $*$ represents the complex conjugate. The ANC system reduces the unwanted noise $D(k)$ by minimizing the power of the error signal $P_{ee}(k)$. Then, the optimal noise control filter $W_{\mathrm{O}}(k)$ is obtained for the case that $\partial P_{ee}(k)/\partial W(k) = 0$ as

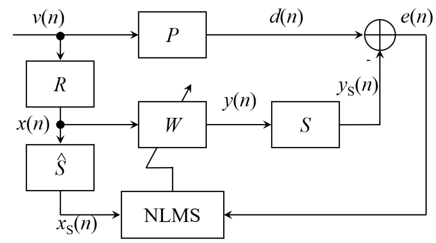


Fig. 2. Block diagram of basic feedforward ANC system.

$$W_{\mathrm{O}}(k) = \frac{P_{dx}(k)}{P_{xx}(k)S(k)}, \tag{3}$$

where $P_{dx}(k) = \mathbf{E}\left\{D^*(k)X(k)\right\}$ is the cross power spectral density of $d(n)$ and $x(n)$ and $P_{xx}(k) = \mathbf{E}\left\{X^*(k)X(k)\right\}$ is the power spectral density of $x(n)$. From (1)–(3), the minimum value of $P_{ee}(k)$ can be represented as

$$P_{ee,\min}(k) = P_{dd}(k) - \frac{P_{dx}^2(k)}{P_{xx}(k)}, \tag{4}$$

where $P_{dd}(k) = \mathbf{E}\left\{D^*(k)D(k)\right\}$ is the power spectral density of $d(n)$. From (4), the normalized version of $P_{ee,\min}(k)$ can be obtained as

$$\frac{P_{ee,\min}(k)}{P_{dd}(k)} = 1 - \mathrm{MSC}(k), \tag{5}$$

$$\mathrm{MSC}(k) = \frac{P_{dx}^2(k)}{P_{dd}(k)P_{xx}(k)}, \tag{6}$$

where $\mathrm{MSC}(k)$ is the magnitude-squared coherence (MSC) at frequency bin k. From (5), the power spectral density of the error signal $P_{ee}(k)$ is minimized when $\mathrm{MSC}(k) = 1$. That is, $\mathrm{MSC}(k)$ is equal to 1 when the reference signal $X(k)$ is exactly the same as the unwanted noise $D(k)$. If the unwanted noise $D(k)$ is accurately estimated from the reference signal $X(k)$, that is, $D(k)$ is accurately estimated by $X(k)$ and the additional filter $H(k)$, $\mathrm{MSC}(k)$ becomes 1 and the power spectral density $P_{ee,\min}(k)$ is minimized.

III. Feedforward active noise control system with CAF

As discussed in Secs. I and II, the noise reduction performance of the feedforward ANC system depends on the coherence between the reference signal $x(n)$ and the unwanted noise $d(n)$. From (5) and (6), the maximum noise reduction is realized when $\mathrm{MSC}(k)$ is equal to one for all frequencies. Under this condition, the reference signal $x(n)$ is exactly equal to the unwanted noise $d(n)$. The proposed feedforward ANC system aims to realize this condition by using the additional adaptive filter, called the CAF. The highest coherent condition [i.e., $x(n) = d(n)$] cannot be realized but a similar condition [i.e., $x(n) \approx d(n)$] can be realized by using the CAF.

Here, the output of the CAF in the z transform domain is defined as

$$X_{\mathrm{A}}(z) = H(z)X(z), \tag{7}$$

where $X_{\mathrm{A}}(z)$ is the z transform of the output of the CAF $x_{\mathrm{A}}(n)$, $H(z)$ is the frequency response of the CAF, and $X(z)$ is the z transform of $x(n)$. When the coherence is completely adjusted, the output of the CAF is $X_{\mathrm{A}}(z) = D(z)$. Under this condition, the optimal frequency response of CAF should be

$$H_{\mathrm{O}}(z) = \frac{D(z)}{X(z)} = \frac{P(z)}{R(z)}, \tag{8}$$

where $P(z)$ and $R(z)$ are the frequency responses of the primary and reference paths. However, the noise environment is generally a time-variant system and the optimal CAF $H_{\mathrm{O}}(z)$ cannot be uniquely identified. Therefore, the CAF should be updated by the adaptive filtering algorithm (e.g., normalized-least-mean-square (NLMS) algorithm).

The proposed ANC system with the CAF is shown in Fig. 3. Here, the CAF is updated by

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \frac{\alpha_{\mathrm{A}} e_{\mathrm{A}}(n)\mathbf{x}(n-n_{\mathrm{d}})}{\|\mathbf{x}(n-n_{\mathrm{d}})\|^2 + \beta_{\mathrm{A}}}, \tag{9}$$

where $\mathbf{h}(n)$ is the filter coefficient vector of the CAF, $\mathbf{x}(n)$ is the reference signal vector, α_{A} is the step size parameter ($0 < \alpha_{\mathrm{A}} < 2$), and β_{A} is the regularization parameter with a small positive value. It can be seen from Fig. 3 that the delay operator $z^{-n_{\mathrm{d}}}$ is used to update the CAF H. This is because the optimal noise control filter $W_{\mathrm{O}}(z)$ should satisfy the causality, that is,

$$W_{\mathrm{O}}(z) = \frac{P(z)}{S(z)H_{\mathrm{O}}(z)R(z)} = \frac{1}{S(z)}, \tag{10}$$

which should be a causal filter, where $S(z)$ is the frequency response of the secondary path. However, (10) is a noncausal filter because $S(z)$ is a nonminimum phase system. More simply, the maximum time delay of $S(z)$ is represented as $z^{-n_{\mathrm{S}}}$. Then, the optimal control filter becomes

$$W_{\mathrm{O}}(z) = \frac{1}{S(z)} = z^{n_{\mathrm{S}}}, \tag{11}$$

which is a noncausal filter. To avoid this condition, the delay operator $z^{-n_{\mathrm{d}}}$ is inserted and the CAF $H_{\mathrm{O}}(z)$ becomes

$$H_{\mathrm{O}}(z) = \frac{P(z)}{R(z)z^{-n_{\mathrm{d}}}}. \tag{12}$$

Then, the optimal noise control filter $W_{\mathrm{O}}(z)$ becomes

$$W_{\mathrm{O}}(z) = \frac{z^{-n_{\mathrm{d}}}}{S(z)} = z^{-n_{\mathrm{d}}+n_{\mathrm{S}}}. \tag{13}$$

To satisfy the causality, i.e., $-n_{\mathrm{d}} + n_{\mathrm{S}} < 0$, the additional delay n_{d} should be

$$n_{\mathrm{d}} > n_{\mathrm{S}}. \tag{14}$$

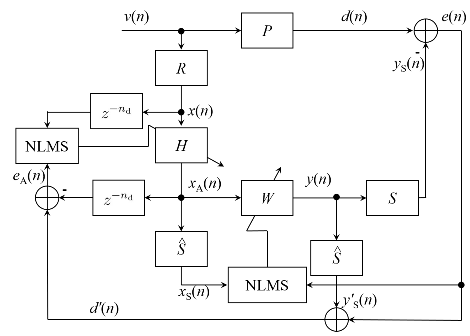


Fig. 3. Block diagram of proposed feedforward ANC system with CAF.

The noise control filter is updated by the filtered-x NLMS algorithm represented as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\alpha e(n)\mathbf{x}_{\mathrm{S}}(n)}{\|\mathbf{x}_{\mathrm{S}}(n)\|^2 + \beta}, \tag{15}$$

where $\mathbf{w}(n)$ and $\hat{\mathbf{s}}(n)$ are the coefficient vectors of the noise control filter W and the secondary path model, $\mathbf{x}_{\mathrm{S}}(n)$ is the filtered reference signal vector, α is the step size parameter ($0 < \alpha < 2$), and β is the regularization parameter of small positive value.

IV. Simulation Results

The computer simulations were conducted to confirm the increase in the coherence between the reference signal and the unwanted noise and the improvement of the noise reduction performance of the proposed ANC system. The performance of the proposed ANC system was compared with that of the conventional ANC system shown in Fig. 2. In these simulations, the identified impulse responses of each path were used and the reference paths were identified at three locations. These impulse responses were measured in a soundproof room (height : 2.2 m, width : 2.3 m, depth : 3.2 m) with the reverberation time of 100 ms. The measurement setup is shown in Fig. 4 and the identified impulse responses of each path are shown in Fig. 5. In this identification, the reference microphone was placed at 0.5 m, 1.0 m, and 1.5 m from the error microphone. These cases are indicated as case 1, case 2, and case 3, respectively. Here, case 3 has the lowest coherence condition. The coherence was evaluated by MSC of (6) using the converged CAF, and the amount of noise reduction was evaluated by

$$\mathrm{Reduction}(n) = 10\log_{10}\frac{\sum_{m=0}^{N-1} d^2(n-m)}{\sum_{m=0}^{N-1} e^2(n-m)}, \tag{16}$$

where N is the tap length of the noise control filter. Simulation conditions are shown in Table I.

A. Evaluation of coherence

Figure 6 shows the MSC between the reference signal and the unwanted noise and Table II shows the average MSC and its improvement. From Fig. 6, MSCs of the proposed ANC

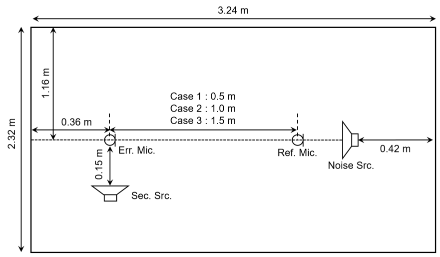
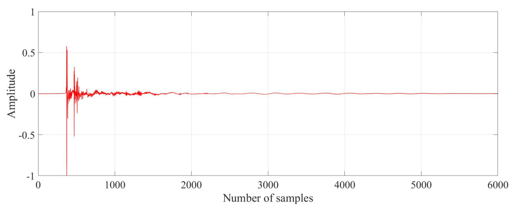
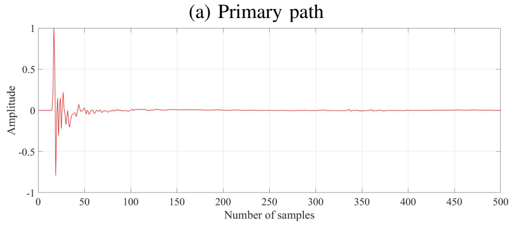


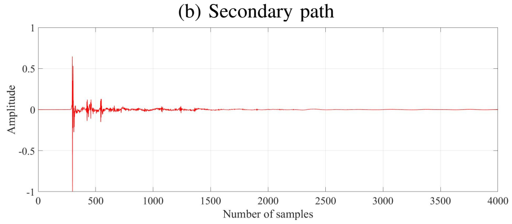
Fig. 4. Measurement setup.



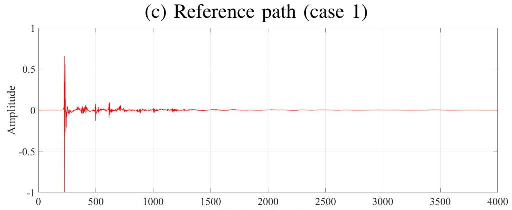
(a) Primary path



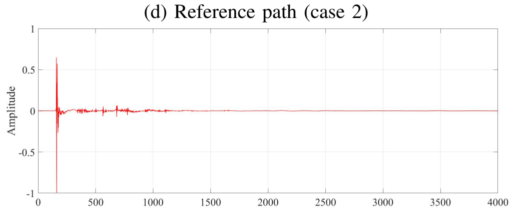
(b) Secondary path



(c) Reference path (case 1)



(d) Reference path (case 2)



(e) Reference path (case 3)

Fig. 5. Impulse responses of each path.

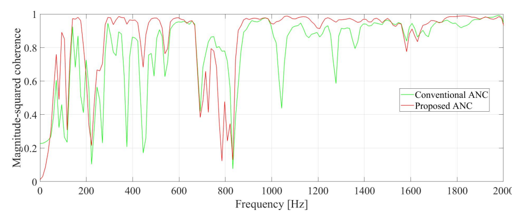
system are seen to be greater than that of the conventional ANC system in all cases. As seen in Table II, MSCs are improved by about 0.07 using the proposed ANC system. In particular, the MSC significantly increases at some frequencies with the use of the CAF. Hence, it can be said that the CAF can adjust the coherence between the reference signal and the unwanted noise.

TABLE I
SIMULATION CONDITIONS.

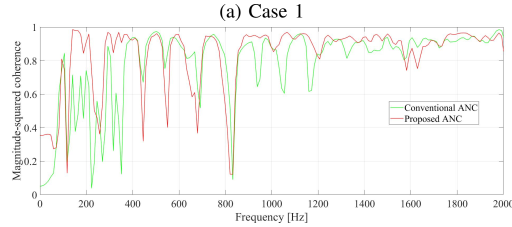
Unwanted noise	White noise
Sampling frequency	48000 Hz
Frequency range	70 – 2000 Hz
Tap length of primary path P	6000
Tap length of reference path R	4000
Tap length of secondary path S	500
Tap length of secondary path model $\hat{S}$	500
Tap length of noise control filter W	1024
Tap length of coherence adjusting filter H	6000
Delay n_{d}	74 (case 1), 145 (case 2), 212 (case 3)
Step size parameters α and α_{A}	0.01
Regularization parameters β and β_{A}	1.0×10^{-6}

TABLE II
AVERAGE IMPROVEMENT OF MSC.

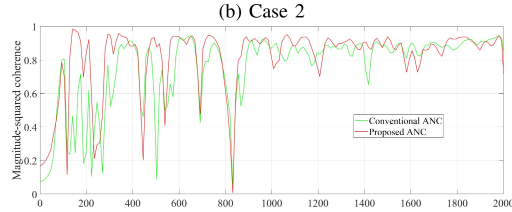
	w/o CAF	w/ CAF	Improvement
Case 1	0.79	0.86	0.07
Case 2	0.78	0.84	0.06
Case 3	0.75	0.81	0.06



(a) Case 1



(b) Case 2

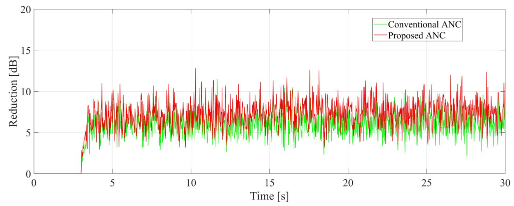


(c) Case 3

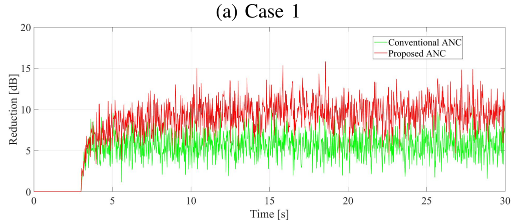
Fig. 6. Magnitude-squared coherence between reference signal and unwanted noise.

B. Evaluation of amount of noise reduction

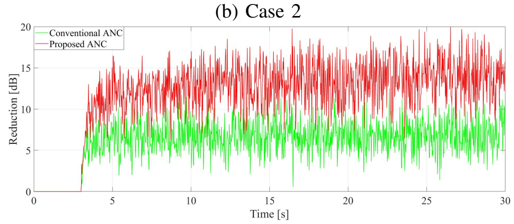
Figure 7 shows the amount of noise reduction for each case. The proposed ANC system can reduce the unwanted noise more than the conventional ANC system in each case.



(a) Case 1

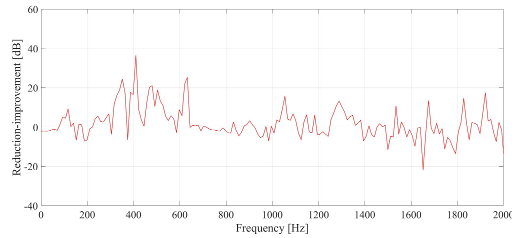


(b) Case 2

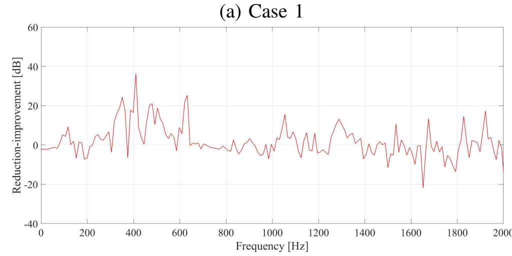


(c) Case 3

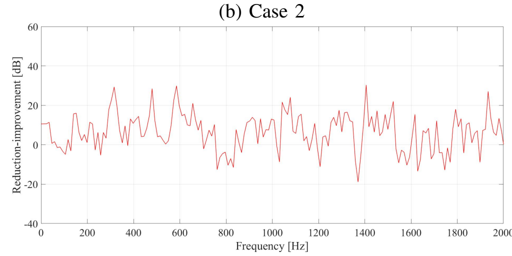
Fig. 7. Amount of noise reduction.



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 8. Improvement in amount of noise reduction.

TABLE III
AVERAGE IMPROVEMENT OF AMOUNT OF NOISE REDUCTION.

	Average	Maximum
Case 1	2.2 dB	36.4 dB
Case 2	4.4 dB	43.5 dB
Case 3	6.4 dB	30.3 dB

In particular, the amount of noise reduction is markedly increased by using the proposed ANC system in case 3. Hence, the proposed ANC system has an improved noise reduction performance because of adjusting the coherence between the reference signal and unwanted noise.

Figure 8 and Table III respectively show the improvement of the amount of noise reduction and the average improvement in the frequency domain for each case. The results shown in Fig. 8 and Table III indicate that the proposed ANC system can improve the amount of noise reduction by about 2–6 dB. At a certain frequency, the proposed ANC system can improve the amount of noise reduction by over 30 dB. Figures 6 and 8 show that the amount of noise reduction increases by over 10 dB at the frequency where the coherence is over 0.9. In particular, the amount of noise reduction greatly increases at the frequency where the coherence improves to more than 0.3. However, the amount of noise reduction is lower than 10 dB under 200 Hz although the coherence is more than 0.9. This is because the noise control filter is too short to reduce the unwanted noise under 200 Hz.

From the above results, the proposed ANC system is found to be effective for solving the coherence problem in the feedforward ANC system. The stationary noise condition was considered in the simulations and we will study the proposed ANC system under the non-stationary noise condition. Moreover, in the simulations, each path was assumed to be a the time-invariant system. In a real system, each path is a time-variant system and the proposed ANC system should be modified for this condition.

V. CONCLUSION

In this paper, a feedforward ANC system with a CAF is proposed. To solve the coherence problem of the feedforward ANC system, the proposed ANC system uses the additional adaptive filter to adjust the reference signal to be similar to the unwanted noise. Simulation results show that the proposed ANC system can increase the coherence between the reference signal and the unwanted noise, and the system shows improved noise reduction performance compared with the basic feedforward ANC system. In the simulations, we only considered the stationary noise condition with time-invariant paths. In the future, we will study this ANC system under nonstationary noise and time-variant paths conditions. Also, the proposed ANC system uses the secondary path model to obtain the CAF, and the fluctuation of the secondary path directly affects the noise reduction performance of the proposed ANC system. Hence, we will study solutions for degradation of the noise reduction performance under fluctuation of the secondary path.

Acknowledgment

This work is supported by KDDI Foundation Research Grant Program 2019, and JSPS KAKENHI (19H04142).

References

[1] K. Tanaka, C. Shi, and Y. Kajikawa, “Binaural active noise control using parametric array loudspeakers,” *Appl. Acoust.*, vol. 116, pp. 170–176, Jan. 2017.

[2] H. Bao and I. Panahi, “Psychoacoustic active noise control based on delayless subband adaptive filtering,” in *IEEE Int. Conf. Acous. Speech Signal Process.*, Mar. 2010, pp. 341–344.

[3] V. Belyi and W. S. Gan, “Psychoacoustic subband active noise control algorithm,” in *2017 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC), Kuala Lumpur, Malaysia*, Dec. 2017, pp. 457–463.

[4] S. Hirose and Y. Kajikawa, “Effectiveness of headrest anc system with virtual sensing technique for factory noise,” in *2017 APSIPA Ann. Summit Conf., Kuala Lumpur, Malaysia*, Dec. 2017, pp. 464–468.

[5] H. Deng, C. Chang, and S. M. Kuo, “Active noise control with virtual sensing technology for headrest,” in *2018 APSIPA Ann. Summit Conf., Hawaii, USA*, Nov. 2018, pp. 1272–1275.

[6] C. Shi, R. Xie, N. Jiang, H. Li, and Y. Kajikawa, “Selective virtual sensing technique for multi-channel feedforward active noise control systems,” in *2019 IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP 2019), Brighton, United Kingdom*, May 2019, pp. 8489–8493.

[7] C. Shi, N. Jiang, H. Li, D. Shi, and W. Gan, “On algorithms and implementations of a 4-channel active noise canceling window,” in *2017 Int. Symp. Intell. Signal Process. Commun. Syst. (ISPACS), Xiamen, China*, Nov. 2017, pp. 217–221.

[8] S. Elliott, *Signal Processing for Active Control*. San Diego, CA: Academic Press, 2001.

[9] S. Elliott and P. Nelson, “Active noise control,” *IEEE Signal Process. Mag.*, vol. 10, no. 4, pp. 12–35, Oct. 1993.

[10] S. M. Kuo and D. R. Morgan, *Active Noise Control Systems: Algorithms and DSP Implementation*. New York: John Wiley & Sons, 1996.

[11] Y. Kajikawa, W. Gan, and S. Kuo, “Recent advances on active noise control: open issues and innovative applications,” *APSIPA Trans. Signal Inform. Process.*, vol. 1, pp. 1–21, Aug. 2012.

[12] S. Kuo, X. Kong, and W. Gan, “Applications of adaptive feedback active noise control system,” *IEEE Trans. Control Syst. Technol.*, vol. 11, no. 2, pp. 216–220, Mar. 2003.

[13] T. Meurers, S. Veres, and S. Elliott, “Frequency selective feedback for active noise control,” *IEEE Control Syst.*, vol. 22, no. 4, pp. 32–41, Nov. 2002.

[14] H. Sakai and S. Miyagi, “Analysis of the adaptive filter algorithm for feedback-type active noise control,” *Signal Process.*, vol. 83, no. 6, pp. 1291–1298, June 2003.

[15] W. Gan, S. Mitra, and S. Kuo, “Adaptive feedback active noise control headset: Implementation, evaluation and its extensions,” *IEEE Trans. Consum. Electron.*, vol. 51, no. 3, pp. 975–982, Aug. 2005.

[16] S. M. Kuo and D. R. Morgan, “Active noise control: a tutorial review,” *Proc. IEEE*, vol. 87, no. 6, pp. 943–973, June 1999.

[17] P. Nelson and S. Elliott, *Active Control of Sound*. San Diego, CA: Academic Press, 1992.

[18] C. Fuller, S. Elliott, and P. Nelson, *Active Control of Vibration*. San Diego, CA: Academic press, 1996.

[19] S. O. Haykin, *Adaptive Filter Theory*, 4th ed. Upper Saddle River, NJ: Prentice Hall, 2001.

[20] K. Iwai, S. Kinoshita, and Y. Kajikawa, “Multichannel feedforward active noise control system combined with noise source separation by microphone arrays,” *Journal of Sound and Vibration*, vol. 453, pp. 151–173, Aug. 2019.

[21] H. Janocha and B. Liu, “Simulation approach and causality evaluation for an active noise control system,” *Proc. IEE Control Theory Appl.*, vol. 145, pp. 423–426, July 1998.

[22] X. Kong and S. Kuo, “Study of causality constraint on feedforward active noise control systems,” *IEEE Trans. Circuits Syst.*, vol. 46, no. 2, pp. 183–186, Feb. 1999.