Classification of Seizure EEGs Based on Short-Time Fourier Transform and Hidden Markov Model

Yuwei Du[1*], Jing Jin[1], Yan Liu[2,3] and Qiang Wang[1]
[1]Department of Control Science and Engineering, Harbin Institute of Technology, Harbin, China
[2] Suzhou Institute of Biomedical Engineering and Technology, Chinese Academy of Science, Department of Medical Image, Suzhou 215163, China
[3]School of Automation, Harbin University of Science and Technology, Harbin 150080, China
E-mail: hitduyw@163.com, jinjinghit@hit.edu.cn, liuyan@sibet.ac.cn, wangqiang@hit.edu.cn
[*]Tel: +86-15776685483

***Abstract*— Epilepsy is a kind of disorder that has affected many people in the world. Electroencephalogram (EEG) is an effective tool in the diagnosis and treatment of epilepsy. The classification of EEG signals from different seizure stages is of great interest in this field. This paper proposes a seizure EEG classification method based on Short-Time Fourier Transform (STFT) and Hidden Markov Model (HMM). We construct feature sequences by STFT, and then 50% of the sequences are used to train HMMs. Finally, the other sequences are used to evaluate the HMMs. Experiments conducted on the dataset from University of Bonn are provided, with the accuracy for set D and set E reaching 97.18%, and the sensitivity and specificity reaching 98.54% and 95.82% respectively.**

I. INTRODUCTION

Epilepsy is one of the most common neurological diseases in the world of which incidence is just next to stroke. About 50 million people are suffering from it. Epilepsy is characterized by its recurrent and sudden seizures which are caused by the hyper-synchronous electrical discharges of the brain neurons. Electroencephalography (EEG) plays a significant role in the diagnosis and treatment of epilepsy, which can record the electrical activities of brain neurons. There are mainly two types of EEG which are scalp EEG (sEEG) and intracranial EEG (iEEG).

Traditional inspection of EEGs with trained neurologists is time-consuming, laborious and subjective. Hence, people have become interested in the automatic classification of seizure EEG signals. The main steps of this task are segmentation of EEG sequences, feature extraction of EEG signals and classification by proper classifiers. During the last decades, many methods are proposed. Methods based on features like power spectral density (PSD) of signals [1][2], feature based on wavelet transform [3] and some non-linear features such as Lyapunov exponent [4] are proposed. Machine learning algorithms, including support vector machine (SVM) and neural network algorithms like long short-term memory (LSTM), are widely used as classifiers. Ref. [5] proposed a method based on STFT and convolutional neural networks (CNN) using iEEG and sEEG. Ref.[6] proposed a method using a novel feature based on the Mahalanobis distance and discrete wavelet transformation (DWT), using extreme learning machine (ELM) as the classifier and evaluating the method on an iEEG dataset. Ref.[4] proposed a seizure EEG classification method based on Lyapunov exponent and support vector machine. Ref.[2] proposed a method based on spectral of EEG signals and SVM. Some other outstanding methods are also proposed.

In this paper, we propose a method based on STFT and HMM, which applies very concise feature extraction. We first extract features from EEG segments by STFT and then use these samples to solve the parameter identification problem of HMM. Then by solving the evaluation problem of HMM, we get a series of scores for a testing EEG segment according to HMM models of different EEG classes. By comparing the scores, we can finally determine the class that this segment belongs to. We show in our experiments that our method can handle the seizure EEG classification problem effectively with concise feature extraction.

The rest parts of this paper are organized as follows. The details of the proposed method and materials are described in Section II. The experimental setup and results are shown in Section III. Finally, Section IV concludes this paper.

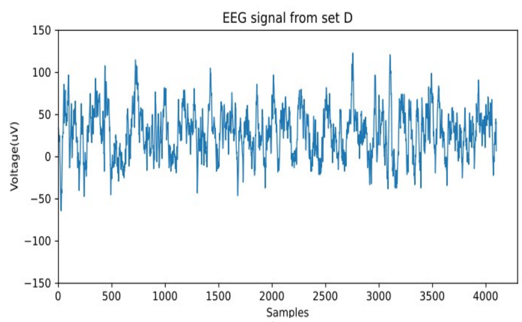
II. DATASET AND METHODS

In this section, we will introduce briefly the dataset from University of Bonn which will be used to evaluate our method first. And then, to realize seizure EEG classification, we need to solve the parameter identification problem and evaluation problem of HMM which are introduced later.

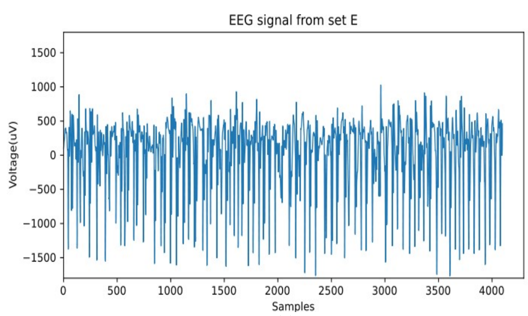
A. *Dataset*

This paper uses the dataset from the Department of Epileptology, University of Bonn [7]. The dataset consists of 5 parts which are set A, set B, set C, set D and set E. Each set contains 100 single-channel EEG segments of 23.6-sec duration without any artifacts. All signals are sampled at a sampling rate of 173.61Hz and converted by 12-bit analog-to-digital conversion. Band-pass filter setting is 0.53-40Hz (12dB/oct.). All the segments fulfill a stationary criterion [7]. Set A and set B consist of segments of scalp EEG taken from 5 healthy volunteers with eyes open (set A) and eyes close (set B) respectively. Set C, set D and set E consist of

(a)



(b)



(c)

Fig. 1 Electrode positions and EEG signals (a) electrode positions using by the dataset from University of Bonn (b) EEG signals from set D (interictal) (c) set E (ictal) respectively

intracranial EEG segments from 5 patients. Set C contains EEG signals from the hippocampal formation of opposite hemisphere of the brain during seizure-free periods (known as interictal). Signals in set D and E are gathered from patients using intracranial electrodes exactly localizing the seizure generating area called the epileptogenic zone [7]. Set D contains EEG signals during the seizure-free period (known as interictal) while set E contains EEG signals during seizure activity periods (known as ictal). This paper only uses the set D and set E for seizure EEG classification. Details about these 5 sets in the dataset are illustrated in Table 1. Fig.1 shows the electrodes implanted in patients' brains and signals from set D and set E respectively.

Table 1 Details about the dataset from University of Bonn

Set	Type of EEG	Subject	Detail
A	sEEG	normal	recorded with eyes open
B	sEEG	normal	recorded with eyes close
C	iEEG	patient	recorded during interictal from the hippocampal formation
D	iEEG	patient	recorded during interictal period from epileptogenic zone
E	iEEG	patient	recorded during ictal period from epileptogenic zone

B. Short-Time Fourier Transform

The main steps of our method are illustrated in Fig. 2, including feature sequence construction, model training and evaluation of the model.

There are many ways to imply time-frequency analysis on different kinds of signals. Some studies have proved that EEG is one kind of non-stationary signals [8]. But if the time interval is small enough, EEGs can be regarded as quasi-stationary signals. Hence, we choose STFT to calculate the spectrum of EEG. STFT is an effective and classic time-frequency analysis method. STFT multiplies the signal by a window function and then Fourier Transform (FT) is carried out. By the shift of the window, we will get a series of results of FT. Finally, we can get a 2-dimension time-frequency image of the signal.

The formula of STFT can be expressed as

$$STFT(t,\omega)=\int_{-\infty}^{\infty}x(\tau)w^{*}(\tau-t)e^{-j\omega\tau}d\tau \tag{1}$$

where $w(t)$ is the window function. For discrete signals, Discrete Fourier Transform (DFT) replace FT and usually Fast Fourier Transform (FFT) is carried out.

After STFT, the base-10 logarithm of the result is computed and multiplied by 10 to get the ultimate feature sequences. Fig. 3 shows an STFT of a 23.6s EEG window from the dataset. All the EEG segments in the dataset have passed through a band-pass filter which ranges from 0.53Hz to 40Hz, so only the first half of the STFT result(the part of 0~40Hz) is useful. We need time sequences to train HMM, so we consider the result of STFT as a time sequence with several features from different frequency bands.

EEG signal is a time sequence itself, so we use raw EEG signals as the input of HMM as a comparison to STFT sequences. After STFT, raw EEG signals become shorter in the time dimension and larger in feature dimension (frequency dimension) while raw EEG signal is quite long in the time dimension and has only one dimension in feature dimension.

C. Hidden Markov Model

Hidden Markov Model is one type of stochastic signal model which was initially introduced in the late 1960s and has worked well in machine recognition of speech, which shows

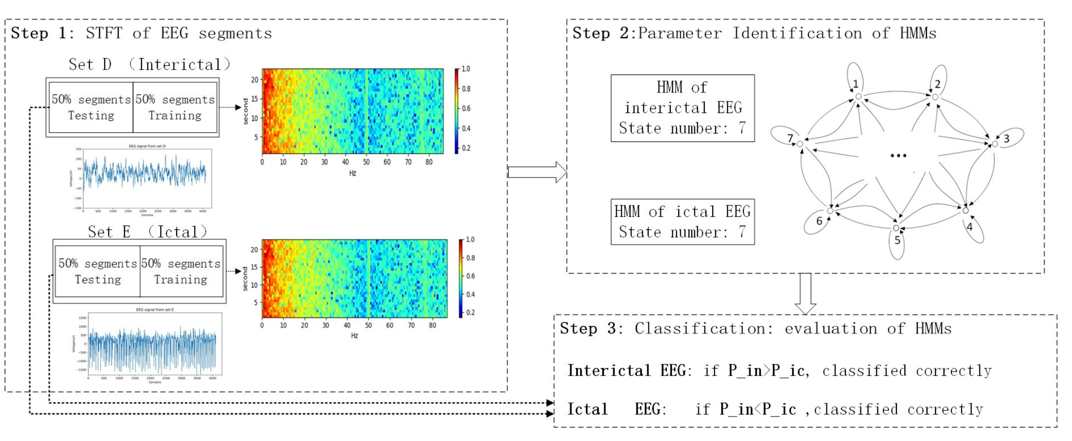


Fig. 2 Feature sequence construction, HMM model training and evaluation. P_in means probability evaluated given the EEG segment and HMM for interictal EEGs while P_ic means probability evaluated given the EEG segment and HMM for ictal EEGs

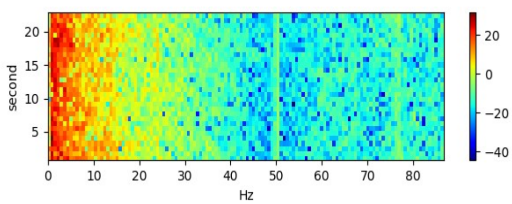


Fig. 3 STFT of EEG segment

its proficiency in ‘characterizing real-world signals in terms of signal models’ [9].

HMM is an extension of Markov Process which contains a number of states and the probabilistic description of the current state only depends on the current and the predecessor states, that is

$$P\{q_t = S_j \mid q_{t-1} = S_i, q_{t-2} = S_k, \cdots\} = P\{q_t = S_j \mid q_{t-1} = S_i\} \quad (2)$$

Each state of Markov Process corresponds to an observable event, which limits its application to some problems of interest. Researchers extend Markov Process to Hidden Markov Model where states become no more observable (that is hidden). The state at a certain time generates its observation according to some probabilistic function. It can be described that HMM is a model where the observation is a probabilistic function of the state.

Elements of HMM are: the number of states N, the number of distinct observation symbols per state M, the state transition probability distribution $A = \{a_{ij}\}$, where

$$a_{ij} = P\{q_{t+1} = S_j \mid q_t = S_i\}, \qquad 1 \le i, j \le N, \quad (3)$$

the observation symbol probability distribution in the state j, $B = \{b_j(k)\}$, where

$$b_j(k) = P\{v_k \text{ at } t \mid q_t = S_j\} \quad (4)$$

and the initial state distribution $\pi = \{\pi_i\}$, where

$$\pi_i = P\{q_1 = S_i\},\ 1 \le i \le N \quad (5)$$

We usually use a compact notation for convenience

$$\lambda = (A, B, \pi) \quad (6)$$

Besides, we denote states as $S = \{S_1, S_2, \cdots, S_N\}$, state at time t as q_t and observation symbols as $V = \{v_1, v_2, \cdots, v_M\}$

There are three main problems in HMM [9]:

1) Evaluation: Given the observation sequence, and an HMM model $\lambda = (A, B, \pi)$, how to efficiently compute the probability of the observation sequence $P(\mathrm{O} \mid \lambda)$.

2) Decode: Given the observation sequence, and the model λ, how to choose a corresponding state sequence which is optimal in some meaningful sense.

3) Parameter identification: how to adjust model parameters to maximize $P(\mathrm{O} \mid \lambda)$.

This paper only needs to solve Problem 1) and Problem 3). So the solutions to these two problems are described as follows.

The solution to Problem 1): Forward-Backward Procedure is the main method to solve Problem 1), usually only the forward part is used. We introduce the algorithm as follows:

Given a fixed observation sequence $O = \{\mathrm{O}_1, \mathrm{O}_2, \cdots, \mathrm{O}_t\}$ and a model λ, we can evaluate the probability of the occurrence of this observation sequence using the forward-backward procedure.

First, define the forward variable $\alpha_t(i)$ as

$$\alpha_t(i) = P(\mathrm{O}_1 \mathrm{O}_2 \cdots \mathrm{O}_t, q_t = S_i \mid \lambda) \quad (7)$$

which is the probability of partial observation sequence $\mathrm{O}_1\ \mathrm{O}_2 \cdots \mathrm{O}_t$ and state S_i at time t given the model λ. Then we can compute $P(\mathrm{O} \mid \lambda)$ inductively, as follows:

1) Initialization:

$$\alpha_1(i) = \pi_i b_i(\mathrm{O}_1), \qquad 1 \le i \le N \quad (8)$$

2) Induction

$$\alpha_{t+1}(j)=\left[\sum_{i=1}^{N}\alpha_t(i)a_{ij}\right]b_j(\mathrm{O}_{t+1})$$
$$1\le t\le T-1, 1\le j\le N \qquad (9)$$

3) Termination

$$P(O\mid\lambda)=\sum_{i=1}^{N}\alpha_T(i) \qquad (10)$$

Algorithm 1 summarizes the whole process of the solution of the evaluation problem.

The solution to Problem 3): Baum-Welch algorithm.

To adjust the model parameter, Baum-Welch algorithm is the most used algorithm. We define $\xi_t(i,j)$ as the probability of being in state S_i at time t and state S_j at time t+1, given observation sequence O and model λ , i.e.

$$\xi_t(i,j)=P(q_t=S_i,q_{t+1}=S_j\mid O,\lambda) \qquad (11)$$

We define the forward variable as (7) and backward variable $\beta_t(i)$ as

$$\beta_t(i)=P(\mathrm{O}_{t+1}\mathrm{O}_{t+2}\cdots\mathrm{O}_T,q_t=S_i\mid\lambda) \qquad (12)$$

We rewrite $\xi_t(i,j)$ as

$$\xi_t(i,j)=[\alpha_t(i)a_{ij}b_j(\mathrm{O}_{t+1})\beta_{t+1}(j)]/P(O\mid\lambda)$$
$$=\frac{\alpha_t(i)a_{ij}b_j(\mathrm{O}_{t+1})\beta_{t+1}(j)}{\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_t(i)a_{ij}b_j(\mathrm{O}_{t+1})\beta_{t+1}(j)} \qquad (13)$$

By summing over j, we can get $\gamma_t(i)$, the probability of being in state S_i at time t

$$\gamma_t(i)=\sum_{j=1}^{N}\xi_t(i,j) \qquad (14)$$

Then we can adjust the values of λ as

$$\overline{\pi}_i=\gamma_1(i) \qquad (15)$$

$$\overline{a}_{ij}=\sum_{t=1}^{T-1}\xi_t(i,j)\Big/\sum_{t=1}^{T-1}\gamma_t(i) \qquad (16)$$

$$\overline{b}_j(k)=\sum_{\substack{t=1\\ s.t.\ O_t=v_k}}^{T}\gamma_t(j)\Big/\sum_{t=1}^{T}\gamma_t(j) \qquad (17)$$

We have talked about discrete observation above, but the EEG signal is not discrete. Hence continuous observation density is introduced.

Given a hidden state, the observation is assumed to have a Gaussian distribution:

$$b_j(\mathrm{O})=\Pi[\mathrm{O},\mu_j,\mathrm{U}_j],$$
$$1\le j\le N \qquad (18)$$

where μ_j is the mean vector and U_j is the covariance matrix of the Gaussian distribution respectively

The re-estimation of μ_j and U_j is as follows:

$$\overline{\mu}_j=\sum_{t=1}^{T}\gamma_t(j)\cdot O_t\Big/\sum_{t=1}^{T}\gamma_t(j) \qquad (19)$$

$$\overline{U}_j=\sum_{t=1}^{T}\gamma_t(j)\cdot\left(O_t-\mu_{jk}\right)\left(O_t-\mu_{jk}\right)'\Big/\sum_{t=1}^{T}\gamma_t(j) \qquad (20)$$

Algorithm 1 Evaluation of HMM: Forward-Backward Procedure

Input: HMM $\lambda=(A,B,\pi)$ and an observation sequence $O=\{\mathrm{O}_1,\mathrm{O}_2,\cdots,\mathrm{O}_t\}$

Output: $P(\mathrm{O}\mid\lambda)$

Forward variable $\alpha_t(i)$ as (7)

1. Initialization : $\alpha_1(i)=\pi_i b_i(\mathrm{O}_1),\quad 1\le i\le N$
2. **for** t=1 to T-1, do
3. $\alpha_{t+1}(j)=\left[\sum_{i=1}^{N}\alpha_t(i)a_{ij}\right]b_j(\mathrm{O}_{t+1})$, $1\le j\le N$
4. **end for**
5. **return** $P(O\mid\lambda)=\sum_{i=1}^{N}\alpha_T(i)$

Algorithm 2 Parameter identification of HMM

Input: HMM $\lambda=(A,B,\pi)$ where B represented as

$$b_j(\mathrm{O})=\Pi[\mathrm{O},\mu_j,\mathrm{U}_j],\quad 1\le j\le N$$

and training sequence $O=\{\mathrm{O}_1,\mathrm{O}_2,\cdots,\mathrm{O}_t\}$

Output: Re-estimation of HMM $\overline{\lambda}=(\overline{A},\overline{B},\overline{\pi})$

1. compute forward variable and backward variable as (7) and (12)
2. denote $\xi_t(i,j)$ as (13), and $\gamma_t(i)$ as (14)
3. Re-estimation of $\pi:\overline{\pi}_i=\gamma_1(i)$
4. Re-estimation of A :
$$\overline{a}_{ij}=\sum_{t=1}^{T-1}\xi_t(i,j)\Big/\sum_{t=1}^{T-1}\gamma_t(i)$$
5. Re-estimation of $b_j(\mathrm{O})=\Pi[\mathrm{O},\mu_j,\mathrm{U}_j],\quad 1\le j\le N$:
$$\overline{\mu}_j=\sum_{t=1}^{T}\gamma_t(j)\cdot O_t\Big/\sum_{t=1}^{T}\gamma_t(j)$$
$$\overline{U}_j=\sum_{t=1}^{T}\gamma_t(j)\cdot\left(O_t-\mu_{jk}\right)\left(O_t-\mu_{jk}\right)'\Big/\sum_{t=1}^{T}\gamma_t(j)$$

Algorithm 2 summarizes the whole process of the parameter identification of HMM.

The reason why we can use HMM to set up models for seizure EEG signals is: hidden states can be seen as some certain physiological situations of the human brain which don't have to contain specific meanings. EEG can be regarded as an observation sequence generated by hidden states. EEG signals generated during different stages of seizure correspond to distinctive HMMs. By solving the evaluation problem of HMM, the classification of EEGs can be realized.

For the classification of seizure EEG, we need to solve Problem 3) using EEG segments and then evaluate the HMM model by solving Problem 1). For EEG sequences from each kind of seizure stage, we need to train HMM models respectively. Given an EEG sequence with an unknown label,

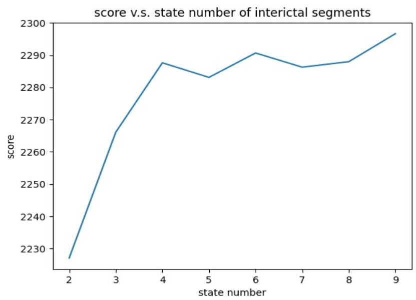


Fig. 4 Performance of HMM of different state numbers for set D (interictal EEG signals)

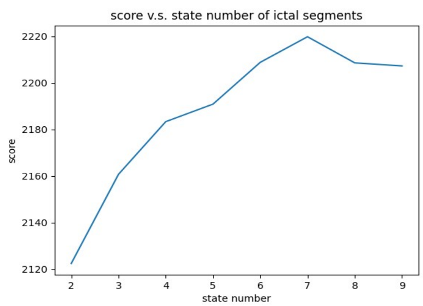


Fig. 5 Performance of HMM of different state numbers for set E (ictal EEG signals)

all the HMM models should be evaluated and probabilities according to each HMM will be given. This EEG sequence will be classified into a certain class of which model gets the largest probability.

III. Experiments and Results

In this section, we first initialize the HMM and then train HMMs of the optimal state numbers using 50% of the EEG segments from the dataset. Finally, we evaluate the HMMs using the last 50% EEG segments from the dataset. Because raw EEG signals are time sequences themselves, we also use raw EEG signals to train HMMs as a comparison.

A. Parameter initialization

We initiate HMM by $A=\{1/N\}_{N\times N}$ and $\pi=\{1/N\}_{1\times N}$, because the uniform initial estimate is adequate in almost all cases [9]. For the initialization of *B*, segmental k-means segmentation with clustering is used for the initialization of the means vector for the continuous observation densities. The specific process is as follows:

Initialization of μ_j: Given the number of states *N* and the set of training sequences, k-means algorithm can give *N* cluster centers and we use these centers as the initial estimate of μ_j according to *N* states.

Initialization of U_j : Given the set of training sequences

$$\mathbf{O}=[\mathbf{O}_1,\mathbf{O}_2,\cdots,\mathbf{O}_L] \quad (21)$$

Table 2 Results using different sequences

Sequences	Sensitivity	Specificity	Accuracy
Raw EEG	95.8%	87.22%	91.52%
STFT	98.54%	95.82%	97.18%

where the shape of $\mathbf{O}_j$, $1\le j\le L$ is $T_j\times n_f$ (T_j is the length of the sequence and n_f is the number of features). We initialize U_j by

$$U_j=\mathrm{cov}(\left[\mathbf{O}_1^T\mathbf{O}_2^T\cdots\mathbf{O}_L^T\right])+\varepsilon\cdot\mathbf{I}_{n_f\times n_f} \quad (22)$$

where ε is the least covariance, which we set ε=10^{-3} there, and $\mathbf{I}_{n_f\times n_f}$ is the $n_f\times n_f$ identity matrix.

After model initialization, training observation sequences can be segmented into states, based on the current initial model λ. λ will be adjusted by the set of training sequences according to the Baum-Welch algorithm in Section II.

B. Experimental Setup and Result

STFT is implemented with the Tukey window, 128 points overlapped between two segments and 256 points for Fast Fourier Transform.

50 percent of segments from set D and set E are chosen randomly to train HMMs. To find the best state number of the models, this paper use leave-one-out validation. Fig.4 and Fig.5 illuminate the mean score for the validation sequences of interictal and ictal respectively, with state number ranging from 2 to 9. It can be seen that scores become higher as state numbers increase and reach a platform after some state number. For interictal segments, the score tends to reach a platform after state number 4 while for ictal segments, the score reaches the peak at state number 7. Though the larger state number may lead to a higher score, the time cost will increase accordingly. Thus, 7 is a proper number for both kinds of EEG signals.

We train HMMs by Baum-Welch algorithm using 1000 iterations which can ensure the convergence of the model.

We have used accuracy, sensitivity and specificity to evaluate the proposed method. Accuracy is the proportion of the correctly classified test samples to all the test samples. Sensitivity is the proportion of correctly classified ictal samples to all the ictal samples used for testing. Specificity is the proportion of correctly classified interictal samples to all the interictal samples used for testing

The last 50 percent of segments are used to evaluate the models. We did the experiment 100 times and the mean results are shown in Table 2. The mean sensitivity, specificity, accuracy of the 100 experiments we did are 98.54%, 95.82% and 97.18%, which prove the effectiveness of the method. Compared to raw EEG signals as input (sensitivity 95.8%, specificity 87.22%, accuracy 91.52%), STFT has improved the performance of HMM, and the efficiency of the algorithm has been enhanced substantially.

Table 3 Comparison between our method and state-of-art-work

Sequences	Sensitivity	Specificity	Accuracy
Our method	98.54%	95.82%	97.18%
Ref.[6]	98.97%	82.60%	90.60 %

C. Comparison between the Proposed Method and Other Existing Methods

We compare our method with the method from [6], the result is shown in Table 3. Our method has a relative high specificity which means less interictal samples are wrongly classified as ictal samples.

Ref.[6] proposed a novel feature based on Mahalanobis distance and discrete wavelet transform. Based on the dataset from University of Bonn, the algorithm has a performance of 96.18% sensitivity, 98.89 % specificity and 97.53% accuracy. Our method has a relatively higher sensitivity and similar accuracy. Ref.[5] proposed a seizure EEG classification method based on STFT and CNN which has sensitivities of 89.8% and 89.1% respectively for data sets from Freiburg Hospital (iEEG) and Children's Hospital Boston with Massachusetts Institute of Technology (sEEG). However, CNN has a relatively complex structure which is time-consuming and hard to train. The method proposed in this paper constructs feature sequences by STFT and no further feature extractions is taken which promises the simplicity of our method. And after STFT, shorter sequences than raw EEG signals are gotten and HMMs can be trained more efficiently.

IV. Conclusions

In this paper, we use the result of STFT as the input of HMM rather than further feature extraction, which can reduce the complexity of the algorithm. And we determine the proper states of HMM by cross-validation. After training the HMMs, EEGs of different seizure stages are classified with a satisfactory result. We prove the seizure detection capability of the proposed method by classifying ictal and interictal EEGs from the dataset from University of Bonn. In the future, we need more data sets to evaluate the seizure prediction capability of our method.

Acknowledgment

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