Variational Mode Decomposition based Image Segmentation using Sine Cosine Algorithm

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***Abstract*—Multilevel thresholding utilizing the histogram-based method is the well-known and preferred technique of image segmentation. This method generally suffers from irregularities and sharp details in the histogram, which leads to stagnation. Along with this, the computational time of this method grows exponentially as the number of thresholds increments. In this study, a freshly developed Sine Cosine algorithm is coupled with variational mode decomposition (VSCA) to overcome these problems. VSCA employed Masi entropy as a cost function for image segmentation. The outcome of the proposed algorithm examined with Non-VMD based method using quantitative parameter such as structural similarity index (SSIM), feature similarity index (FSIM), quality index based on local variance (QILV), computational time, and mean square error(MSE). The outcomes confirm that the proposed algorithm performs more reliable results than Non-VMD based techniques.**

***Index Terms*—Image Segmentation, Sine Cosine Algorithm, Masi Entropy, Variational Mode Decomposition**

I. Introduction

Image segmentation means the partition of an image into various homogeneous regions which share similar pixel features for better understanding. It plays an important role in medical imaging [1], recognition tasks [2], satellite imagery and remote sensing [3], surveillance [4], machine vision, content-based image retrieval etc. There are several of segmentation techniques [5] such as Edge-based segmentation, Threshold-based segmentation, Fuzzy-based segmentation, region-based segmentation etc. However, a technique which can fulfil all the requirements of any imaging application is lacking.

Image Thresholding methods can be further classified into two essential approaches: parametric and non-parametric approaches. In the parametric approach, a statistical distribution of each segment is assumed, which further seeks to obtain the approximation of other parameters of the distribution. But this approach is very time-consuming. Whereas in Non-parametric approach, a cost function is utilized to achieve the thresholds. In the past, various such cost function has been introduced. Among different cost functions, an entropy-based function is one of the most common and effective techniques. In the literature, there exist various entropy models like maximum entropy, minimum cross-entropy etc.

The maximum entropy method seeks a threshold which maximized distribution. Tsallis and Renyi entropies are a generalized form of Shannon entropy which are proposed by C. Tsallis [6] and P. Sahoo et al. [7], respectively. Tsallis entropy has an entropic parameter q along with pseudo-additivity property which controls non-extensive information for statistically independent subsystems. Whereas, Renyi entropy has entropic parameter p, which can handle additive property for statistically independent subsystems. Nevertheless, Renyi and Tsallis entropies cannot control the additive and non-additive information at the same time. Recently, a new entropic measure which is based on analysis of the classical thermodynamic entropies has been presented by [8]. Essentially, Masi entropy unites the property of Renyi entropy and Tsallis entropy, i.e. additivity and non-extensivity.

Although, these histogram-based methods usually suffered from the low-quality segmentation results because of irregularities and sharp details present on the histogram. By studying other methods, variational mode decomposition (VMD) [9] gives better resolution in spectrum analysis as it has a various equivalent filter bank structure. With the use of VMD, a relevant frequency band is picked while discard remainder band which overcome the obstacle due to irregularities and sharp details.

Still, the computational efficiency of the threshold-based method is less and time increases exponentially as the number of the thresholds increase. To overcome this problem, a metaheuristic optimization algorithm is used. The metaheuristic optimization used entropy function as a cost function. It tried to maximise the value of this cost function to find out the threshold value. In the literature, there are several metaheuristic optimizations algorithm used with different cost function [10; 11; 12; 13]. Recently, a population-based optimization proposed called Sine Cosine Algorithm (SCA) [14]. Initially, SCA formulates multiple random solutions then shift them outwards or towards the optimum solution with the help of a mathematical model based on sine and cosine functions.

- The approach of VMD with Masi entropy function and SCA is employed for colour image segmentation the first time in the present research.
- The effect of VMD demonstrated by a comparison between the VMD and non-VMD method.
- An exhaustive state-of-the-art study has been conducted to examine the stated approach.
- The stated method can be examined for several problems in colour images in terms of precision and efficiency.

The rest of the paper organized as follows: Section II exhibits the problem formulation and explains the use of Masi entropy for multilevel thresholding. Moreover, this section describes the use of sine cosine algorithm and VMDmode decomposition. In section III, proposed methodology has been presented. Here forming of SCA-VMD has been reported along with Masi entropy function for multilevel segmentation. In Section IV, the outcome of the proposed algorithm is analyzed based on various quantitative parameters. Finally, in section V conclusion is provided.

II. Problem formulation

Multilevel segmentation methods offer an efficient process to implement image analysis. Though, it is still a hurdle to choose the k-level optimal threshold value [11]. Multilevel thresholding can be expressed in mathematical form as:

$$\begin{gathered} S_0 = \{f(x,y) \in I \mid 0 \leq f(x,y) \leq t_1\} \\ S_1 = \{f(x,y) \in I \mid t_1 + 1 \leq f(x,y) \leq t_2\} \\ \vdots \\ S_k = \{f(x,y) \in I \mid t_k + 1 \leq f(x,y) \leq L-1\} \end{gathered} \tag{1}$$

where S_0, S_0,..., S_k are different classes, t_i (i = 1,..., k) is the i^{th} threshold value, and k is the number of thresholds. The probability of all grey value i is calculated as:

$$p_i = \frac{h_i}{N} \tag{2}$$

where i expresses intensity value, N indicates the total number of pixel value in the image. The probability of every class for multilevel thresholding for each band is represented as:

$$w_0 = \sum_{i=0}^{t_1} p_i, \quad w_1 = \sum_{i=\mathrm{t}_1+1}^{\mathrm{t}_2} p_i, \quad \ldots, \quad w_k = \sum_{i=\mathrm{t}_k}^{L-1} p_i \tag{3}$$

A. Masi Entropy

Masi entropy was introduced by [8] which is based on the study of the standard thermodynamic entropies. It couples the property of Renyi entropy and Tsallis entropy that is additivity and non-extensivity, respectively. The basic difference which differentiates Masi entropy from Renyi and Tsalli entropy is controlling parameter γ. In the case of Masi entropy, the probability function is raised to the power γ whereas state-probability distinctly is raised to a power p or q Renyi and Tsalli entropy, respectively [13]. The Masi entropy, which is utilised for image segmentation, is described as:

$$H_\gamma^{M_T} = H_\gamma^{M_0} + H_\gamma^{M_1} + H_\gamma^{M_2} + \cdots + H_\gamma^{M_k}, \tag{4}$$

where

$$\begin{aligned} H_\gamma^{M_0} &= \tfrac{1}{1-\gamma} \log\left[1 - (1-\gamma)\textstyle\sum_{i=0}^{\mathrm{t}_1} \left(\frac{h_i}{w_0}\right) \log\left(\frac{h_i}{\omega_0}\right)\right] \\ H_\gamma^{M_1} &= \tfrac{1}{1-\gamma} \log\left[1 - (1-\gamma)\textstyle\sum_{i=\mathrm{t}_1+1}^{\mathrm{t}_2} \left(\frac{h_i}{\omega_1}\right) \log\left(\frac{h_i}{\omega}\right)\right] \\ &\vdots \\ H_\gamma^{M_k} &= \tfrac{1}{1-\gamma} \log\left[1 - (1-\gamma)\textstyle\sum_{i=\mathrm{t}_k+1}^{L-1} \left(\frac{h_i}{\omega_n}\right) \log\left(\frac{h_i}{\omega_k}\right)\right] \end{aligned} \tag{5}$$

where $H_\gamma^{M_i}$ denotes the Masi entropy of i^{th} class, and the optimal multilevel segmentation problem is determined by considering the k-dimensional problem of optimization which utilised for the maximisation of cost function (ϕ):

$$\phi = \arg\ \max\left(\sum_{i=0}^{k} H_r^{M_i}\right) \tag{6}$$

B. Variational Mode Decomposition

Variational Mode Decomposition proposed by K. Dragomiretskiy and D. Zosso [9]. It is a non-recursive approach to extract the sub-modes of a signal concurrently. This method is more robust to noise and simple in comparison to initially introduced algorithms VMD evades the outcomes of irregularities and high-frequency fluctuations in a histogram signal to escape from sub-optimal solutions.

VMD disintegrates the histogram into various mode-signals termed intrinsic mode functions. All mode is band-limited and most compact around a centre frequency Ω_b. The reconstruction constrained variational problem now takes the form:

$$\begin{aligned} &\min_{\{s_b\},\{\Omega_k\}} \textstyle\sum_b \left\|\partial_t[(\delta(t) + \frac{j}{\pi t}) * s_t(t)]e^{-j\Omega_t t}\}\right\|_2^2, \\ &\text{s.t.} \textstyle\sum_b s_b = f, \end{aligned} \tag{7}$$

here, input data is defined by f, Dirac distribution represented by δ, index of decomposed mode is defined by b.

C. Sine Cosine Algorithm

Sine cosine algorithm (SCA) is a population-based algorithm which has been introduced by S. Mirjalili in 2016 [14]. The optimization process starts with a random set of solution. This random set of population repeatedly explores the cost function which is governed by some rule. Fundamentally, in Stochastic optimization, two main phases are exploration and exploitation. In the exploration phase, search focus on investigating the nearby region of the elite solution while in exploitation stage the search focus on the unexplored area. A successful optimization algorithm must find the optimal balance between exploration and exploitation phase. Position updating equations are described for both phases:

$$B_i^{t+1} = \begin{cases} B_i^t + \rho_1 * \sin(\rho_2) * \left|\rho_3 P_i^t - B_i^t\right|, & \rho_4 < 0.5 \\ B_i^t + \rho_1 * \cos(\rho_2) * \left|\rho_3 P_i^t - B_i^t\right|, & \rho_4 \geq 0.5 \end{cases} \tag{8}$$

III. Proposed Methodology

In this section, proposed method to determine the optimal multilevel threshold values for a colour image, has been reported. Masi entropy, along with VMD is employed with SCA. The proposed method is simple to implement for image segmentation. Initially, the histogram of an input image separated into several sub-bands using VMD. Furthermore, the essential modes are chosen, and the rest band is rejected, which includes high-frequency fluctuation. Usually, the initial mode holds high amplitude and low frequency, which basically gives the trend of the histogram while higher modes comprise low amplitude and high frequency of the histogram and hold most noisy or inefficient fluctuations. In this study, total number of modes is set to six, where the first three modes contain a meaningful segment of the histogram whereas last three modes are influenced by notable variations and the intense details of the histogram thus eliminated from the histogram. Fig. 1 shows the graphical representation of the method. The details steps are described below:

- Select an image of size $M*N$ *3 for the segmentation.
- Initialise, the control parameters of SCA such as the number of search agent, number of iterations, threshold levels, the total number of modes, and number of significant modes for VMD.
- Find out the histogram of an image and calculate VMD of the histogram then merge significant modes.
- The optimum value of thresholds can be achieved by maximizing the cost function (Eq. 6) by SCA algorithm.
- The Best Destination point fitness and its position show the best cost function value with the set of optimal threshold values.
- The input image is segmented utilising the corresponding threshold values.

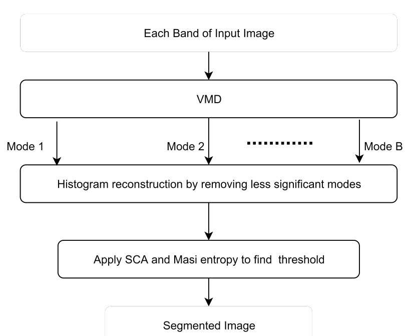


Fig. 1: Method of proposed algorithm based on VSCA.

IV. Results and Discussion

In this proposed technique, two principal parameters for VMD, i.e. α and B require to set in first. α is mainly responsible for the width of every mode of the histogram. Konstantin's theory state that the bandwidth of the separated modes is related to α. It means for a high value of α; it gets narrow bandwidth and tends to the finer separation of sub modal of the histogram. The reconstructed histogram can be obtained by summing all distinguish mode. α ranges from 200 to 3200. If α is very less, then there is no significant difference between original and reconstructed histogram. As α start rising ($\alpha > 1000$), a notable reduction is found. Thus in this paper, α is set to 1000. The second parameter which affects the characteristic of VMD is the number of modes (B). If the value of B is small ($B \leq 4$) high-frequency fluctuation cannot be appropriately segregated from the histogram. As B increases, high-frequency fluctuation separated significantly from the histogram but led to long convergence time. In this paper, for faster and significant separation of the histogram, B is set to 6 where the first 3 modes contain a meaningful segment of the histogram.Image with corresponding histogram and the reconstructed histogram are shown in Fig. 2.

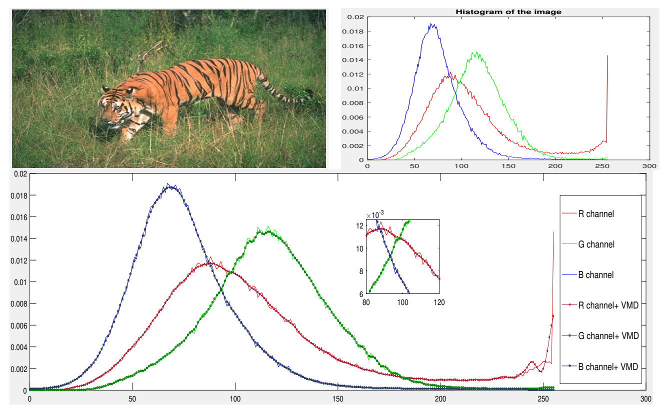
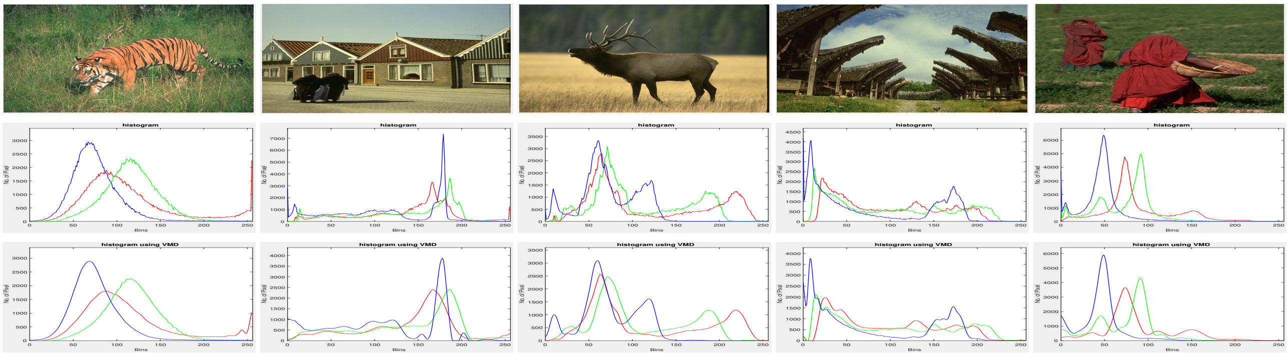


Fig. 2: Image with histogram and modified histogram.

In this section, experiments have led to estimate and validate the performance of the proposed thresholding method. The results obtained by the methodology are shortly addressed. For better perception, the segmented outcome of the proposed approach with existing methods, simulation results are presented. For performance evaluation, five images from the Barkley segmentation dataset [15], are utilised. Each image depicts a histogram and a histogram after the VMD operation, which is shown in Fig 3.

Stopping criteria, i.e. the number of iterations and population are set to 80 and 20, respectively. Each case runs for 50 times for taking mean value. The entropy parameter γ for Masi entropy is fixed to 0.25. Value is boldfaced when VMD based method performs better than non-VMD based method. This analysis has been conducted using Matlab R2018a on the Window 10 system with intel(R) Core(TM) Core i5-4570 @3.2 GHz processor with 20 GB RAM.

Performance is calculated with several quantitative parameters such as structural similarity index (SSIM), feature similarity index (FSIM), quality index based on local variance (QILV), computation time (CPU time in sec.), and mean square error (MSE) are used. MSE [16] index estimates the pixel variations, which is measured by averaging the squared intensity value of the input image and segmented image. A less significant value of MSE presents a more immeasurable



(1) (2) (3)

(a) (b) (c) (d) (e)

Fig. 3: (1) Sample test images, (2) corresponding histogram and (3) processed histogram (after applying VMD.)

TABLE I: The average quantitative value achieved by SCA methods with- and without-VMD for various thresholding level utilizing Masi entropy function after 50 runs for every case.

I	K	SSIM		FSIM		QILV		CPU time		MSE	
		Normal	VMD	Normal	VMD	Normal	VMD	Normal	VMD	Normal	VMD
1	3	0.8036	0.8019	0.8920	**0.8951**	0.9785	**0.9808**	0.1982	**0.1221**	415.976	420.432
	5	0.8892	**0.8917**	0.9417	**0.9439**	0.9931	**0.9943**	0.2455	**0.1497**	171.042	**162.604**
	8	0.9352	**0.9416**	0.9631	**0.9692**	0.9964	**0.9973**	0.3580	**0.1891**	95.935	**83.144**
	10	0.9493	**0.9551**	0.9715	**0.9751**	0.9974	**0.9978**	0.4098	**0.2159**	73.983	**62.872**
	12	0.9596	**0.9618**	0.9772	**0.9794**	0.9976	**0.9985**	0.4483	**0.2418**	58.174	**53.762**
2	3	0.6924	**0.7597**	0.8559	0.8545	0.9590	**0.9600**	0.2280	**0.1189**	387.513	**333.050**
	5	0.8465	**0.8532**	0.9202	**0.9203**	0.9732	**0.9741**	0.2651	**0.1433**	158.101	**152.949**
	8	0.8918	**0.8945**	0.9455	**0.9482**	0.9836	**0.9850**	0.3386	**0.1825**	94.621	**93.636**
	10	0.8906	**0.9141**	0.9571	**0.9596**	0.9843	**0.9892**	0.3796	**0.2071**	92.162	**69.117**
	12	0.9199	**0.9256**	0.9632	**0.9654**	0.9889	**0.9905**	0.4429	**0.2334**	63.804	**57.883**
3	3	0.5776	**0.7072**	0.8483	**0.8537**	0.9856	**0.9860**	0.1726	**0.1166**	340.071	**276.633**
	5	0.8010	**0.8383**	0.8650	**0.8675**	0.9902	**0.9903**	0.2140	**0.1417**	152.178	**129.869**
	8	0.8443	**0.8683**	0.8952	**0.8986**	0.9930	**0.9936**	0.2593	**0.1798**	89.858	**73.857**
	10	0.8753	**0.8877**	0.9077	**0.9111**	0.9953	0.9951	0.3131	**0.2047**	69.199	**60.348**
	12	0.8874	**0.9047**	0.9178	**0.9228**	0.9957	**0.9961**	0.3464	**0.2309**	57.958	**49.233**
4	3	0.7135	**0.7206**	0.8229	0.8217	0.9680	**0.9692**	0.1599	**0.1178**	369.870	**363.146**
	5	0.7997	**0.8026**	0.8836	**0.8862**	0.9867	**0.9876**	0.1946	**0.1436**	166.311	**160.322**
	8	0.8704	0.8703	0.9339	**0.9382**	0.9924	**0.9942**	0.2464	**0.1820**	90.525	**86.572**
	10	0.8949	**0.8968**	0.9479	**0.9512**	0.9945	**0.9945**	0.2828	**0.2083**	71.057	**66.234**
	12	0.9123	**0.9175**	0.9552	**0.9579**	0.9953	**0.9961**	0.3161	**0.2348**	56.401	**50.996**
5	3	0.8200	**0.8844**	0.7753	**0.8331**	0.9737	0.9734	0.1603	**0.1157**	254.074	**197.684**
	5	0.8856	**0.9080**	0.8697	**0.8818**	0.9856	**0.9883**	0.1958	**0.1408**	130.079	**106.560**
	8	0.9208	**0.9425**	0.9122	**0.9338**	0.9912	**0.9934**	0.2491	**0.1789**	82.696	**60.302**
	10	0.9402	**0.9547**	0.9324	**0.9467**	0.9945	**0.9953**	0.2895	**0.2051**	59.814	**46.510**
	12	0.9488	**0.9594**	0.9417	**0.9569**	0.9954	**0.9965**	0.3192	**0.2317**	52.549	**40.380**

quality of the processed image. SSIM is developed by [17] which combines luminance comparison, contrast comparison and structure comparison of an image. FSIM [18] is based upon on a concept which is governed by the human visual system defines an image essentially by its low-level characteristics. QILV estimates structural information of the image [16]. This is based on a theory, i.e. a massive amount of structural data is coded in its local variance, which can be utilised to analyse two images suitably. The higher value of these three parameters shows the more reliable quality of the segmented image.

Table I shows outcome acquired by SCA optimization algorithm for Masi entropy as cost function. The boldfaced value displayed in every table confers the best outcomes achieved after analysing VMD based algorithm to Non-VMD based algorithm. These outcomes indicate that most of the cases are outperformed by VMD-based method when compared with non-VMD based method. It is also noted that VMD based method provides faster result when compared to non-VMD based method.

V. Conclusions

In this article, proposed method has been examined for image segmentation. It has been used SCA as metaheuristic optimization and Masi entropy as a cost function to find out the threshold. Along with this, VMD has been used to eliminate artefacts existing in the histogram, which manages more reliable segmentation presented in results shown. The proposed algorithm has been compared with Non-VMD based method. For comparison, five different images have been taken from Barkley Dataset. The outcome has been compared using various quantitative parameter such as feature similarity index, structural similarity index, quality index based on local variance, computational time, and mean square error. The outcomes implied that the VMD based method had outperformed the non-VMD based method and also has provided faster results. The outcomes are assuring, which motivate to perform this method for image processing application like image enhancement, image classification, and pattern recognition problems.

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