A Proposal toward Standardization of Design Examples for IIR Filter Design Methods

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***Abstract*—Design methods of IIR filters have been proposed with design examples to present the effectiveness of the design methods. However, we point out that the evaluation of the design methods is biased toward a particular class of IIR filter design problem. Therefore, we propose benchmark problems of frequency selection IIR filters based on the design examples chosen from IEEE Xplore.**

I. Introduction

Digital filers are used in many applications such as acoustic equipments, speech processing, image processing, biomedical systems. Digital filters are classified into FIR (Finite impulse response) filters and IIR (Infinite Impulse Response) filters by the impulse response length. Especially, IIR filters can realize the steep roll-off and the high attenuation characteristic with the less filter order compared with FIR filters. However, the design of IIR filters with the rational transfer function and the unrealizable perfect linear phase response is a non-convex optimization problem under the complex approximation criterion. Moreover, nonlinear constraint conditions for filter coefficients must be considered such as the stability and restricting the excess ripple in the transition band. Thus, it is difficult to obtain the global optimal solution for IIR filter design problem formulated as a nonlinear constrained non-convex optimization problem.

For this challenging problem, a lot of the design methods of IIR filters have ever been proposed. The framework of the design methods is roughly categorized into mathematical programming and metaheuristics. In the design methods of mathematical programming, iterative methods under the L_∞ norm criterion based on LP (Linear Programming) method [1], SOCP (Second Order Cone Programming) method [2], SDP (Semidefinite Programming) method [3] are proposed. The iterative method of LP and SOCP could obtain an approximation solution under a sufficient condition of the stability by the positive realness. In addition, the method of SOCP clarified the guarantee of convergence and the convergence rate. The iterative method of SDP could calculate the approximation solution under the sufficient and necessary stability condition based on Lyapunov theorem. However, the method based on SDP does not declare the guarantee of convergence in the process of iteration, the stability condition is limited to sufficient conditions. From the above, for a particular class of IIR filters, these methods may not converge the local optimal solution and failed the design.

On the other hand, in the design methods using metaheuristics, RCGA (Real Coded Genetic Algorithm) [4] under the multiple criterion (the linear combination of L_2 norm and L_∞ norm), CS (Cuckoo Search) [5] under the multiple criterion (the weighted linear combination of L_∞ norm for the amplitude response and the group delay response) and is included the constraint of the excess ripple in the transition band. DE (Differential Evolution) [6] and PSO (Particle Swarm Optimization) [7] under the L_∞ norm criterion or the L_2 norm criterion are proposed. Moreover, a two-stage design method combined with iterative QP (Quadratic Programming) and PSO under the L_∞ norm criterion is proposed as a two framework [8]. These methods impose the sufficient and necessary stability condition and can design the IIR filter under the complex constraints compared to mathematical programming methods.

However, the design examples are not unified between the papers and are limited because only a few design examples in a paper when the performance of the proposed method is evaluated. In addition, it is unknown what kind of purposes and criterions these design examples were determined for the performance evaluation in many cases. Therefore, we can't decide how to evaluate the performance of the design method of IIR filter and compare with the others exactly.

To tackle the above difficulty, we propose to set the benchmark problems of frequency selection IIR filters based on the design examples chosen from IEEE Xplore. Specifically, we searched for the word "IIR filter design" on IEEE Xplore and investigated 597 desired responses in 224 papers. The benchmark problems are set up by the investigated results. From the above, The benchmark problems may contribute to standardize the performance comparison of the design method of IIR filter.

This paper is organized as following : in section 2, IIR filter design problem under the arbitrary error criterion is described. in section 3, the study results of design examples in many papers is shown. IIR filter design benchmark problems is proposed in section 4. The conclusion of this work are drawn in section 5.

II. IIR Filter Design Problem

A. Formulation

The frequency response of direct form or cascade form IIR filter used frequently in the filter design problem are described as

$$H(\omega) = \frac{\sum_{n=0}^{N} a_n e^{-jn\omega}}{1 + \sum_{m=1}^{M} b_m e^{-jm\omega}} \tag{1}$$

$$= a_0 \cdot \frac{\prod_{n_1=1}^{N_1} (1+a_{0,n_1} e^{-j\omega})}{\prod_{m_1=1}^{M_1} (1+b_{0,m_1} e^{-j\omega})} \cdot \frac{\prod_{n_2=1}^{N_2/2} (1+\sum_{l=1}^{2} a_{l,n_2} e^{-jl\omega})}{\prod_{m_2=1}^{M_2/2} (1+\sum_{l=1}^{2} b_{l,m_2} e^{-jl\omega})} \tag{2}$$

where $\omega \in [0, \pi]$ is a normalized angular frequency, $N = (N_1 + N_2) \in \mathbb{N} \cup 0$ is a numerator order, $M = (M_1 + M_2) \in \mathbb{N} \cup 0$ is a denominator order, a_n, $b_m \in \mathbb{R}$ are direct form filter coefficients, a_{k,n_k}, b_{k,m_k}, $k \in \{0, 1, 2\}$ are cascade form filter coefficients, $\mathbb{N}$ is a set of natural numbers and $\mathbb{R}$ is a set of real numbers.

In this paper, IIR filter design problem under the arbitrary error criterion with the stability condition constraint and the constraint of an excess ripple of the amplitude response in the transition band is considered to formulate. IIR filter design problem is formulated as following,

$$\min_{\boldsymbol{x}} ||W(\omega)\{D(\omega) - H(\omega, \boldsymbol{x})\}||_p, \ \omega \in \Omega_a \tag{3}$$

$$\text{s.t. } \boldsymbol{x} \in S \tag{4}$$

$$|H(\omega, \boldsymbol{x})| \leq \delta_t, \ \omega \in \Omega_t, \tag{5}$$

$$\boldsymbol{x} = [a_0 \ a_1 \ a_2 \cdots a_N \ b_1 \ b_2 \cdots b_M]^{\mathrm{T}} \in \mathbb{R}^{N+M+1} \tag{6}$$

$$(\boldsymbol{x} = [a_0 \ \underbrace{a_{0,1} \cdots a_{0,N_1}}_{N_1} \ \underbrace{a_{1,1} \ a_{2,1} \cdots a_{1,N_2/2} \ a_{2,N_2/2}}_{N_2}$$

$$\underbrace{b_{0,1} \cdots b_{0,M_1}}_{M_1} \ \underbrace{b_{1,1} \ b_{2,1} \cdots b_{1,M_2/2} \ b_{2,M_2/2}}_{M_2}]^{\mathrm{T}} \in \mathbb{R}^{N+M+1}).$$

where $\boldsymbol{x}$ is a design parameter, $D(\omega) \in \mathbb{C}$ is a desired frequency response, $||\cdot||_p$ is L_p, $p \in \mathbb{N} \cup \infty$ norm, $W(\omega) \in \mathbb{R}$ is a weight function, $S \in \mathbb{R}^{N+M+1}$ is a set of stability region, $\delta_t \in [0, \infty)$ is a specified maximum amplitude in the transition band, $\Omega_a \subset [0, \pi]$ is a set of approximation band, $\Omega_t \subset [0, \pi]$ is a set of transition band and $\mathbb{C}$ is a set of complex numbers. For instance, the desired frequency response of the low pass filter can be described as following.

$$D(\omega) = \begin{cases} e^{-j\omega\tau_d}, & \omega \in [0, \omega_p] \subset \Omega_a \\ 0, & \omega \in [\omega_s, \pi] \subset \Omega_a \end{cases} \tag{7}$$

where τ_d is a desired pass band group delay, ω_p is a pass band edge normalized angular frequency and ω_s is a stop band edge normalized angular frequency.

B. Stability Condition

Necessary and sufficient conditions of the stability of IIR filter are enumerated following

- Lyapunov theorem [1]
- stability triangle [4]

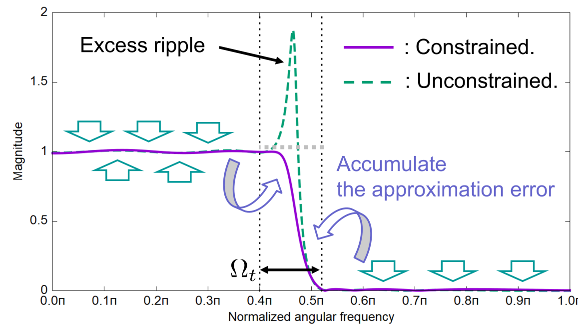

Fig. 1. Example of the excess ripple

- the condition based on the existence of the pole inside the unit circle on z plane [7]
- Jury's stability test [9]

From the above the conditions, a set of the stability region S is determined. For the direct form, all the above stability conditions are formulated as the nonlinear constraint. If the positive realness condition is used as shown in [1][2], the condition becomes a linear constraint, but the condition is not a necessary and sufficient condition for the stability. On the other hand, For the cascade form, only the stability triangle can be presented as a linear constraint.

C. Restricting The Excess Ripple

In the design of IIR filters, the approximation error in the approximation band tends to accumulate in the transition band and an excessive amplitude ripple occurs in the transition band [5]. Example of the excess ripple is shown in Fig.**??**. Generally speaking, the amplitude ripple in transition band is undesirable for filter behavior. The condition is regarded with occurring the amplitude ripple is defined as

$$\max_{\omega \in \Omega_t} |H(\omega, \boldsymbol{x})| > \delta_t. \tag{8}$$

Therefore, the constraint in (5) is imposed for the IIR filter design problem.

III. Study of Design Examples

Comparing performance of the filter design methods, the design examples are shown to evaluate. The design examples are not unified between the papers and are limited due to only a few design examples in a paper. For example, the well known Deczky's design example [10] whose conditions are $N = 12$, $M = 6$, the desired pass band group delay $\tau_d = 9$, the pass band edge normalized angular frequency $\omega_p = 0.5\pi$ and the stop band edge normalized angular frequency $\omega_s = 0.6\pi$ are used in [3][7][8][11]. However, it is not clear whether the design example is an appropriate condition setting from the viewpoint of the design difficulty and it is not enough clarified how good performance the method applied to the filter design. Thus, we searched for the word "IIR filter design" on IEEE Xplore and picked up 597 desired responses in 224 papers to review the design difficulty of IIR filter design problem. In

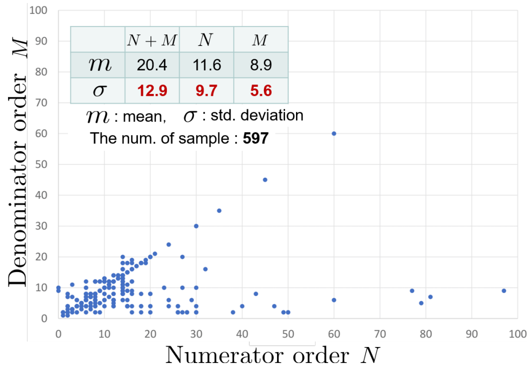

Fig. 2. The scatter plot of the filter orders

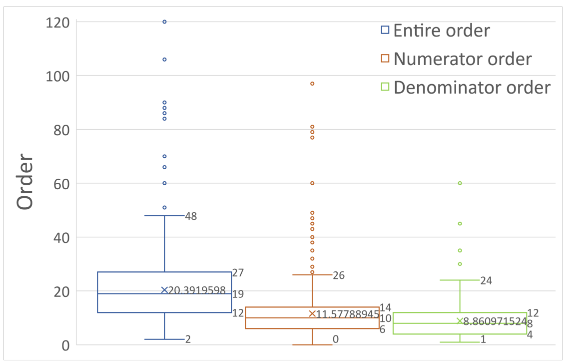

Fig. 3. The box plot of the filter orders

this section, to simplify the discussion, we assume that the design difficulty depends only on the filter order. The scatter plot of the filter orders as one of the investigated results is shown in Fig.**??**. In Fig.**??**, the plots are a pair of numerator order and denominator order in the design example. The design difficulty of IIR filter design problem depends on the filter order decided by the dimension of the design problem. Therefore, the difficulty can be defined by the statistics of the filter orders in Fig.**??**.

The box plot of the filter orders in Fig.**??** is shown in Fig.**??** when the data outside the range of 1.5 times the quartile range are regarded as a singular value. In Fig.**??**, a cross point is a mean value of entire, numerator and denominator filter order. The three-stage difficulties correspond to the quartile. The design difficulty of IIR filter design problem based on the entire filter order can be classified the three-stage: low, middle and high order presented by the summation of the numerator order and the denominator order respectively. Specifically, the low order is about 10, the middle order is up to about 20 and the high order is about 30 (strictly 12, 19, 27 respectively). From the above, Deczky's design example is the middle difficulty approximately. Therefore, the design difficulties of the design example can be classified approximately based on the investigated results. However, we must consider not only the filter order but also the other factors to understand the more exactly design difficulty.

IV. Proposed Benchmark Problems

To tackle the difficulties described in section 3, we propose to set benchmark problems of frequency selection IIR filters based on the design examples chosen from IEEE Xplore. The benchmark problems are set by conditions following as

1) The design difficulties can be classified into three-stage.
2) The ratio of the numerator order and the denominator order in the entire order is considered.
3) The setting of the transition band is classified into the stop band attenuation and the pass bandwidth.
4) The desired pass band group delay is set to a half of the entire filter order $(N+M)/2$.
5) The benchmark problems are specified by a class of low pass filters.

1) is the details of the indicator shown in section 3.

2) denotes that the stop band attenuation increases as the numerator order increases. On the other hand, the pass band approximation error decreases and the feasible region in the design is narrower as the denominator order increases.

In 3), the stop band attenuation can be estimated by the transition bandwidth and the pass bandwidth as shown in [11]. Thus, when the benchmark problems are set, either the above parameter must be given. In this paper, the pass bandwidth and the transition band width are given using the method in [1] so that the stop band attenuation can obtain $20[\mathrm{dB}]$, $30[\mathrm{dB}]$, $40[\mathrm{dB}]$ of low, middle and high order respectively. As for the attenuation of the low, the middle and the high order filters, the estimated values of the mean attenuation are calculated to $24.6[\mathrm{dB}]$, $35.1[\mathrm{dB}]$, $54.8[\mathrm{dB}]$ from the estimation formula of the attenuation shown in [11] respectively. However, considering the excess ripple, the set attenuation are less than the above estimation values actually.

4) intents the comparison with the perfect linear phase FIR filter. So, the comparison of the arbitrary group delay IIR filter with the same entire filter order is not considered in the benchmark problems. Therefore, as for the above comparison, the proposed benchmark problems are not perfectly. However, this issue can be resolved by the extended benchmark problems which the condition of the arbitrary group delay is added for the proposed benchmark problems, As for this task, it will be the future work.

5) intents that the arbitrary frequency selection IIR filter is got by spectral transformations [13] for the designed low pass filter. In addition, the high pass filter transformed by replacing $z \leftarrow -z$ from the low pass filter is guaranteed the optimality of the designed low pass filter. However, the properties of the band pass and the band elimination filter transformed by replacing $z \leftarrow z^2$ from the low pass filter is twice times of the filter order and the pass band group delay of the prototype filter. Thus, these filter types can be not guaranteed the optimal prototype filter. Therefore, the proposed benchmark problems are not enough to verify the design of the optimal band pass and band elimination filters. Considering the above conditions 1) to 5), the benchmark problems of frequency selection IIR filters based on the design examples from IEEE Xplore is

TABLE I
FREQUENCY SELECTION IIR FILTERS BENCHMARK PROBLEMS

Low order					Middle order					High order				
Ex.	N	M	ω_p	ω_s	Ex.	N	M	ω_p	ω_s	Ex.	N	M	ω_p	ω_s
1	8	2	0.2π	0.29π	13	16	4	0.2π	0.33π	25	24	6	0.2π	0.32π
2	8	2	0.4π	0.55π	14	16	4	0.4π	0.52π	26	24	6	0.4π	0.52π
3	8	2	0.6π	0.86π	15	16	4	0.6π	0.72π	27	24	6	0.6π	0.70π
4	6	4	0.2π	0.30π	16	12	8	0.2π	0.31π	28	18	12	0.2π	0.32π
5	6	4	0.4π	0.50π	17	12	8	0.4π	0.50π	29	18	12	0.4π	0.51π
6	6	4	0.6π	0.69π	18	12	8	0.6π	0.69π	30	18	12	0.6π	0.70π
7	5	5	0.2π	0.31π	19	10	10	0.2π	0.36π	31	15	15	0.2π	0.40π
8	5	5	0.4π	0.52π	20	10	10	0.4π	0.58π	32	15	15	0.4π	0.61π
9	5	5	0.6π	0.69π	21	10	10	0.6π	0.78π	33	15	15	0.6π	0.79π
10	4	6	0.2π	0.32π	22	8	12	0.2π	0.43π	34	12	18	0.2π	0.52π
11	4	6	0.4π	0.55π	23	8	12	0.4π	0.69π	35	12	18	0.4π	0.78π
12	4	6	0.6π	0.71π	24	8	12	0.6π	0.82π	36	12	18	0.6π	0.87π

shown in Table 1. In Table 1, the desired pass band group delay is 5, 10, 15 in each of the low, middle and high order design example.

V. CONCLUSION

In this paper, benchmark problems of the frequency selection IIR filter design problem are proposed to standardize to set the design example. The design difficulty was categorized into the three-stage by the investigated results based on 597 desired response in 224 papers of the IEEE Xplore and is reflected to the benchmark problems. However, the proposed benchmark problem is limited to the desired group delay to $(N + M)/2$ and is restricted to the band pass and the band elimination non-optimal filter transformed from the designed low pass filter by frequency transformation. Thus, in the future, those conditions will be imposed to the IIR filter design and be reflected to the benchmark problems.

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