Formulation of Multidimensional Frequency Characteristics of Second-Order Nonlinear IIR Filter

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***Abstract*—In this paper, the formulation of multidimensional frequency characteristics of a second-order nonlinear infinite impulse response (IIR) filter is presented. The second-order nonlinear IIR filter is based on a nonlinear differential equation for an electrodynamic loudspeaker and used to reduce nonlinear distortions of an electrodynamic loudspeaker. Hence, it is difficult to increase the filter order without considering the constraints of nonlinear differential equations to improve the capability of the filter to reduce nonlinear distortions. In this paper, the formulation of the frequency characteristics of the second-order nonlinear IIR filter is presented as a solution to this problem. This formulation is based on the principle of the second-order nonlinear IIR filter for reducing the nonlinear distortions. From this formulation, it is shown that the nonlinearities of the second-order nonlinear IIR filter can be represented by the multidimensional frequency characteristics, which is similar to Volterra filters with the feedback structure. The validity of the formulation is shown by comparing the frequency characteristics of the conventional nonlinear IIR filter structure with those of new formulated ones through some experiments.**

I. Introduction

Electrodynamic loudspeakers [1], [2] are widely used for audio systems, telephones, laptop PCs, and telecommunication meeting systems among others. In particular, microspeakers [3] are used in mobile phones, tablet-type devices, and portable audio devices. Essentially, electrodynamic loudspeakers have nonlinearities [4] and generate nonlinear distortions, which degrade sound quality. Moreover, they degrade the performance of acoustic echo cancellers, the noise reduction performance of active noise control systems, and so forth [5]. Therefore, it is necessary to reduce nonlinear distortions.

One of the methods of reducing the nonlinear distortions of an electrodynamic loudspeaker is to utilize a Volterra filter [6]. The Volterra filter can represent a nonlinear input–output relationship similarly to an electrodynamic loudspeaker [7], and nonlinear components of each order can be represented individually. There are some studies on nonlinear acoustic echo cancellers using a Volterra filter to reduce the nonlinear distortions of an electrodynamic loudspeaker [8]–[10]. Another method to reduce the nonlinear distortions of an electrodynamic loudspeaker is to utilize a nonlinear infinite impulse response (IIR) filter [11]–[18]. The nonlinear IIR filter is based on a nonlinear differential equation for an electrodynamic loudspeaker, and its filter coefficients are determined by the physical parameters of the loudspeaker. A second-order nonlinear IIR filter [11], [12] has the simplest structure, on which we focus on in this study. Nonlinear IIR filters have fewer filter coefficients, and their computational complexity is lower than that of Volterra filter-based systems [14], [15]. However, the improvement of the capability of these nonlinear IIR filters to reduce the nonlinear distortions can only be achieved by extending the nonlinear differential equation for an electrodynamic loudspeaker. That is, it is difficult to increase the filter order without considering the constraint of the nonlinear differential equation. Moreover, it is difficult to adopt adaptive algorithms to the nonlinear IIR filters owing to the complex filter structure and the difficulty to extend the filter order.

In this paper, the formulation of the multidimensional frequency characteristics of a second-order nonlinear IIR filter is presented. This formulation is based on the principle of the second-order nonlinear IIR filter for reducing nonlinear distortions. The principle of the nonlinear IIR filter can be represented by utilizing the Volterra filter. On the basis of this idea, it can be shown that the nonlinearities of the second-order nonlinear IIR filter can be represented by the multidimensional frequency characteristics similarly to the Volterra filter with the feedback structure. The validity of the formulated frequency characteristics is confirmed by comparing the frequency characteristics of the conventional nonlinear IIR filter structure with those of new formulated ones through some experiments.

II. Second-Order Nonlinear IIR Filter

A nonlinear IIR filter [11], [12], [15], [18] is derived from a nonlinear differential equation of an electrodynamic loudspeaker [4], [7]. In general, electrodynamic loudspeakers have nonlinearities caused by force factor, stiffness, and self-inductance [4], [7]. In this paper, the nonlinearities of force factor and stiffness are focused on for simplicity of the formulation. The nonlinear differential equation is as follows:

$$\begin{aligned} G_0 b(x) u(t) = {} & a(t) + k(x)\omega_0^2 x(t) \\ & + \left\{1 + \left(1 - \frac{Q_0}{Q_\mathrm{m}}\right)\left(b^2(x) - 1\right)\right\}\frac{\omega_0}{Q_0} v(t), \end{aligned} \tag{1}$$

where $u(t)$ is the input voltage [V], $x(t)$ is the displacement [m], $v(t) = dx(t)/dt$ is the velocity [m/s], $a(t) = d^2x(t)/dt^2$ is the acceleration [m/s^2] of the diaphragm, and t is the time

[s]. Also,

$$G_0 = \frac{A_0 Bl_0}{R_{\mathrm{dc}} m_0}, \qquad \omega_0 = \sqrt{\frac{K_0}{m_0}},$$
$$Q_0 = \frac{\sqrt{m_0 K_0}}{R_{\mathrm{m}} + Bl_0^2/R_{\mathrm{dc}}}, \quad Q_{\mathrm{m}} = \frac{\sqrt{m_0 K_0}}{R_{\mathrm{m}}},$$

where ω_0 is the lowest resonance angular frequency [rad/s], Q_0 is the quality factor at ω_0, and Q_{m} is the mechanical quality factor at ω_0. Moreover, m_0, K_0, and R_{m} are the mechanical mass [kg], stiffness [N/m], and resistance [Ns/m], respectively, R_{dc} is the electrical resistance [Ω], Bl_0 is the force factor [Wb/m], and A_0 is the analogue gain.

$$b(x) = 1 + b_1 x + b_2 x^2 \tag{2}$$

and

$$k(x) = 1 + k_1 x + k_2 x^2 \tag{3}$$

are respectively the nonlinearities of the force factor and stiffness [4], where b_1 and b_2 are the nonlinear parameters for the force factor and k_1 and k_2 are those for the stiffness. In [4], [7], the nonlinearities of the force factor and stiffness can be represented by the second-order polynomials of the displacement x. If the nonlinear distortions of the electrodynamic loudspeaker are completely reduced, the force factor and stiffness become constants, i.e., $Bl(x) = Bl_0$ and $K(x) = K_0$, respectively. Under this condition, the displacement, velocity, and acceleration are respectively given as follows.

$$x_{\mathrm{L}}(t) = \mathcal{L}^{-1}\left[G_0 \Big/ \left(s^2 + \frac{\omega_0}{Q_0}s + \omega_0^2\right)\right] * u(t), \tag{4}$$
$$v_{\mathrm{L}}(t) = \mathcal{L}^{-1}\left[G_0 s \Big/ \left(s^2 + \frac{\omega_0}{Q_0}s + \omega_0^2\right)\right] * u(t), \tag{5}$$
$$a_{\mathrm{L}}(t) = \mathcal{L}^{-1}\left[G_0 s^2 \Big/ \left(s^2 + \frac{\omega_0}{Q_0}s + \omega_0^2\right)\right] * u(t), \tag{6}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform, $s\ (= j\omega)$ is the complex variable in the Laplace domain, and $*$ denotes the convolution operator. Equations (4)–(6) represent the motions of the diaphragm without the nonlinearity.

If the nonlinear distortions of the electrodynamic loudspeaker are completely reduced, the sound propagated by the loudspeaker does not include the nonlinear distortions. Under this condition, the displacement, velocity, and acceleration do not also include the nonlinear distortions because of the proportional relationships of sound with displacement, velocity, and acceleration. This is the ideal condition and can be represented by replacing $x(t)$, $v(t)$, and $a(t)$ in (1) with (4)–(6), respectively, as follows.

$$G_0 b(x_{\mathrm{L}}) u_{\mathrm{L}}(t) = a_{\mathrm{L}}(t) + k(x_{\mathrm{L}})\omega_0^2 x_{\mathrm{L}}(t) + \left\{1 + \left(1 - \frac{Q_0}{Q_{\mathrm{m}}}\right)\left(b^2(x_{\mathrm{L}}) - 1\right)\right\}\frac{\omega_0}{Q_0} v_{\mathrm{L}}(t), \tag{7}$$

where $u_{\mathrm{L}}(t)$ is the compensation signal that satisfies the ideal conditions and used to reduce the nonlinear distortions. This signal in the discrete time domain can be obtained by using the bilinear transform to $x_{\mathrm{L}}(t)$, $v_{\mathrm{L}}(t)$, and $a_{\mathrm{L}}(t)$ and substituting them into (1) in the discrete time domain. For simplicity, $b(x)$ and $k(x)$ are directly used to generate $u_{\mathrm{L}}(t)$. A block diagram of the second-order nonlinear IIR filter is shown in Fig. 1, and the simplified block diagram in Fig. 1 is shown in Fig. 2. The filter coefficients are

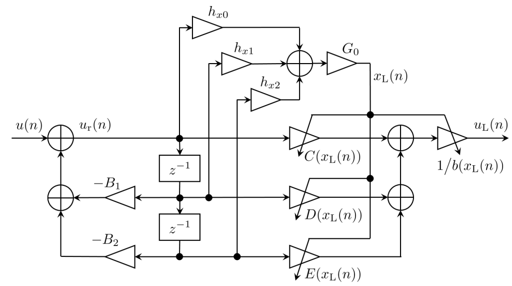


Fig. 1. Block diagram of second-order nonlinear IIR filter [11], [12].

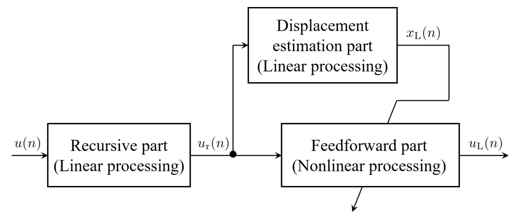


Fig. 2. Simplified block diagram of second-order nonlinear IIR filter [15].

$$C(x_{\mathrm{L}}(n)) = 1 + \beta(x_{\mathrm{L}}(n)) + \gamma(x_{\mathrm{L}}(n)),$$
$$D(x_{\mathrm{L}}(n)) = B_1 + 2\gamma(x_{\mathrm{L}}(n)),$$
$$E(x_{\mathrm{L}}(n)) = B_2 - \beta(x_{\mathrm{L}}(n)) + \gamma(x_{\mathrm{L}}(n)),$$
$$h_{x0} = h_{x2} = \frac{h_{x1}}{2} = \frac{1}{4f_{\mathrm{s}}^2\alpha}, \ \alpha = 1 + \frac{\omega_0}{2Q_0 f_{\mathrm{s}}} + \frac{\omega_0^2}{4f_{\mathrm{s}}^2},$$
$$B_1 = \left(-2 + \frac{\omega_0^2}{2f_{\mathrm{s}}^2}\right)\frac{1}{\alpha}, \ B_2 = \left(1 - \frac{\omega_0}{2Q_0 f_{\mathrm{s}}} + \frac{\omega_0^2}{4f_{\mathrm{s}}^2}\right)\frac{1}{\alpha},$$
$$\beta(x_{\mathrm{L}}(n)) = \beta_0\left\{b^2(x_{\mathrm{L}}(n)) - 1\right\}, \ \beta_0 = \frac{\omega_0}{2Q_0 f_{\mathrm{s}}\alpha}\left(1 - \frac{Q_0}{Q_{\mathrm{m}}}\right),$$
$$\gamma(x_{\mathrm{L}}(n)) = \gamma_0\left\{k\left(x_{\mathrm{L}}(n)\right) - 1\right\}, \ \gamma_0 = \frac{\omega_0^2}{4f_{\mathrm{s}}^2\alpha},$$

where f_{s} represents the sampling frequency [Hz]. The signals are

$$u_{\mathrm{L}}(n) = \frac{1}{b(x_{\mathrm{L}}(n))} u'_{\mathrm{L}}(n), \tag{8}$$
$$u'_{\mathrm{L}}(n) = \{C(x_{\mathrm{L}}(n))u_{\mathrm{r}}(n) + D(x_{\mathrm{L}}(n))u_{\mathrm{r}}(n-1) + E(x_{\mathrm{L}}(n))u_{\mathrm{r}}(n-2)\}, \tag{9}$$
$$x_{\mathrm{L}}(n) = G_0\{h_{x0}u_{\mathrm{r}}(n) + h_{x1}u_{\mathrm{r}}(n-1) + h_{x2}u_{\mathrm{r}}(n-2)\}, \tag{10}$$
$$u_{\mathrm{r}}(n) = u(n) - B_1 u_{\mathrm{r}}(n-1) - B_2 u_{\mathrm{r}}(n-2). \tag{11}$$

Here, the filter order depends on the order of the nonlinear differential equation (7) [1].

[1]In [14], [15], a third-order nonlinear differential equation with nonlinearities of the force factor, stiffness, and self-inductance is used and a third-order nonlinear IIR filter is obtained.

III. Formulation of Multidimensional Frequency Characteristics of Second-Order Nonlinear IIR Filter

In this section, the formulation of the multidimensional frequency characteristics of the second-order nonlinear IIR filter is presented. To formulate the frequency characteristics, each nonlinear component should be divided. Then, the frequency characteristics without the feedback structure are formulated on the basis of the principle of the second-order nonlinear IIR filter. Finally, the frequency characteristics with the feedback components are formulated.

A. Division of Compensation Signal into Each Nonlinear Component

To formulate the frequency characteristics of the second-order nonlinear IIR filter, the compensation signal $u_{\mathrm{L}}(n)$ is divided into linear and nonlinear components with the following form: [2] .

$$u_{\mathrm{L}}(n) = u(n) + \sum_{i=2}^{I_{\max}} u_{\mathrm{L},i}(n), \tag{12}$$

where $u_{\mathrm{L},i}(n)$ represents the ith-order nonlinear component of $u_{\mathrm{L}}(n)$ and $I_{\max}$ is the maximum order of the nonlinear component. $I_{\max}$ is determined by the polynomial order of $b(x)$. For example, $I_{\max} = 11$ when $b(x)$ is a second-order polynomial. It is easily shown that the linear component of $u_{\mathrm{L}}(n)$ is the same as the input signal $u(n)$ by expanding (11) with the polynomial approximation of b(x) shown as

$$1/b(x_{\mathrm{L}}(n)) \approx 1 + b_1' x_{\mathrm{L}}(n) + b_2' x_{\mathrm{L}}^2(n) + \cdots. \tag{13}$$

Here, it is easily seen that (12) is similar to the Volterra filter [6], [7] as

$$u_{\mathrm{L}}(n) = \sum_{i=1}^{I_{\max}} \mathbf{H}_{\mathrm{F},i}\left[u_{\mathrm{r}}(n)\right], \tag{14}$$

$$\mathbf{H}_{\mathrm{F},i}\left[u_{\mathrm{r}}(n)\right] = \sum_{p_1=0}^{N-1}\sum_{p_2=0}^{N-1}\cdots\sum_{p_i=0}^{N-1} h_{\mathrm{F},i}(p_1, p_2, \cdots, p_i) \, u_{\mathrm{r}}(n-p_1)u_{\mathrm{r}}(n-p_2)\cdots u_{\mathrm{r}}(n-p_i), \tag{15}$$

where $h_{\mathrm{F},i}(p_1, p_2, \cdots, p_i)$ is the ith-order Volterra kernel and N is the filter length. By using (14) and the Volterra model of the electrodynamic loudspeaker, we can obtain the compensation model of the nonlinear IIR filter, as shown in Fig. 3.

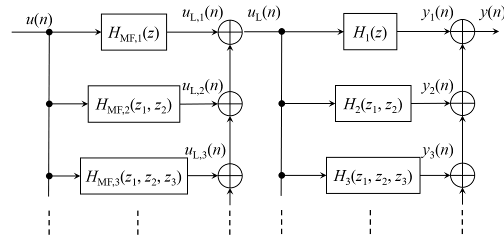


Fig. 3. Compensation model of second-order nonlinear IIR filter.

By using (8) and (13), we can rewrite the compensation signal $u_{\mathrm{L}}(n)$ as

$$u_{\mathrm{L}}(n) = \left\{1 + b_1' x_{\mathrm{L}}(n) + b_2' x_{\mathrm{L}}^2(n) + \cdots\right\} u_{\mathrm{L}}'(n), \tag{16}$$

$$\begin{aligned} u_{\mathrm{L}}'(n) &= u(n) + \sum_{i=2}^{5} u_{\mathrm{L},i}'(n), \\ u_{\mathrm{L},2}'(n) &= \left(2\beta_0 b_1 + \gamma_0 k_1\right) x_{\mathrm{L}}(n) u_{\mathrm{r}}(n) \\ &\quad + 2\gamma_0 k_1 x_{\mathrm{L}}(n) u_{\mathrm{r}}(n-1) \\ &\quad + \left(-2\beta_0 b_1 + \gamma_0 k_1\right) x_{\mathrm{L}}(n) u_{\mathrm{r}}(n-2), \end{aligned} \tag{17}$$

$$\begin{aligned} u_{\mathrm{L},3}'(n) &= \left\{\beta_0\left(b_1^2 + 2b_2\right) + \gamma_0 k_2\right\} x_{\mathrm{L}}^2(n) u_{\mathrm{r}}(n) \\ &\quad + 2\gamma_0 k_2 x_{\mathrm{L}}^2(n) u_{\mathrm{r}}(n-1) \\ &\quad + \left\{-\beta_0\left(b_1^2 + 2b_2\right) + \gamma_0 k_2\right\} x_{\mathrm{L}}^2(n) u_{\mathrm{r}}(n-2). \end{aligned} \tag{18}$$

Then, the components of (12) are represented as

$$u_{\mathrm{L},2}(n) = u_{\mathrm{L},2}'(n) + b_1' x_{\mathrm{L}}(n) u(n), \tag{19}$$

$$u_{\mathrm{L},3}(n) = u_{\mathrm{L},3}'(n) + b_1' x_{\mathrm{L}}(n) u_{\mathrm{L},2}'(n) + b_2' x_{\mathrm{L}}^2(n) u(n). \tag{20}$$

Here, the divided components above the fourth order are omitted owing to page limitation.

B. Formulation of Multidimensional Frequency Characteristics

The frequency characteristics of the second-order nonlinear IIR filter are obtained on the basis of its principle shown above and the Volterra filter (14). Here, the Volterra filter has multidimensional frequency characteristics that can be represented by multidimensional z transform as follows [19]:

$$H_{\mathrm{F},i}(z_1, z_2, \cdots, z_i) = \sum_{p_1=0}^{N-1}\sum_{p_2=0}^{N-1}\cdots\sum_{p_i=0}^{N-1} h_{\mathrm{F},i}(p_1, p_2, \cdots, p_i) \, z_1^{-p_1} z_2^{-p_2} \cdots z_i^{-p_i}, \tag{21}$$

where $z_i = e^{j\omega_i}$ is the complex variable in the z transform domain, and $\omega_i = 2\pi f_i$ is the angular frequency [rad/s].

First, from (10), (16), (19), and (20), second- and third-order components are respectively written similarly to the Volterra filter representation (15) as

$$u_{\mathrm{L},2}(n) = \sum_{p_1=0}^{2}\sum_{p_2=0}^{2} h_{\mathrm{F},2}(p_1, p_2) u_{\mathrm{r}}(n-p_1) u_{\mathrm{r}}(n-p_2), \tag{22}$$

$$u_{\mathrm{L},3}(n) = \sum_{p_1=0}^{2}\sum_{p_2=0}^{2}\sum_{p_3=0}^{2} h_{\mathrm{F},3}(p_1, p_2, p_3) u_{\mathrm{r}}(n-p_1) \, u_{\mathrm{r}}(n-p_2) u_{\mathrm{r}}(n-p_3), \tag{23}$$

[2]This division can be carried out by expanding (8) with polynomial approximation of $1/b(x(n))$ shown in (13). Then, it is shown that the first-order component is completely the same as the input signal.

where

$$h_{\mathrm{F},2}(p_1,p_2) = G_0 h_{xp_2}\left(h_{\mathrm{N2},p_1} + b_1' B_{p_1}\right), \tag{24}$$

$$h_{\mathrm{F},3}(p_1,p_2,p_3) = G_0^2 h_{xp_2} h_{xp_3}\left(h_{\mathrm{N3},p_1} + b_1' h_{\mathrm{N2},p_1} + b_2' B_{p_1}\right), \tag{25}$$

and

$$\begin{aligned}
&h_{\mathrm{N2},0} = 2\beta_0 b_1 + \gamma_0 k_1,\ h_{\mathrm{N2},1} = 2\gamma_0 k_1,\\
&h_{\mathrm{N2},2} = -2\beta_0 b_1 + \gamma_0 k_1,\\
&h_{\mathrm{N3},0} = \beta_0\left(b_1^2 + 2b_2\right) + \gamma_0 k_2,\ h_{\mathrm{N3},1} = 2\gamma_0 k_2,\\
&h_{\mathrm{N3},2} = -\beta_0\left(b_1^2 + 2b_2\right) + \gamma_0 k_2,\ B_0 = 1.
\end{aligned}$$

Here, to obtain (24) and (25), the following equation is used:

$$u(n) = u_{\mathrm{r}}(n) + B_1 u_{\mathrm{r}}(n-1) + B_2 u_{\mathrm{r}}(n-2). \tag{26}$$

As shown in (21), the frequency characteristics of the second- and third-order components in the z transform domain are respectively written as

$$H_{\mathrm{F},2}(z_1,z_2) = \sum_{p_1=0}^{2}\sum_{p_2=0}^{2} h_2(p_1,p_2) z_1^{-p_1} z_2^{-p_2}, \tag{27}$$

$$H_{\mathrm{F},3}(z_1,z_2,z_3) = \sum_{p_1=0}^{2}\sum_{p_2=0}^{2}\sum_{p_3=0}^{2} h_3(p_1,p_2,p_3) z_1^{-p_1} z_2^{-p_2} z_3^{-p_3}. \tag{28}$$

Here, (27) and (28) do not include the frequency characteristics owing to B_1 and B_2, that is, they cannot fully represent the frequency characteristics of the nonlinear IIR filter since they do not include the effect of the feedback path shown in Fig. 1.

Next, the multidimensional frequency characteristics with the feedback part are formulated. From (19) and (20), the ith-order output signals consist of weighted products of $u_{\mathrm{r}}(n)$, for example, $h_{\mathrm{F},2}(p_1,p_2)u_{\mathrm{r}}(n-p_1)u_{\mathrm{r}}(n-p_2)$. Hence, ith-order filters include the characteristics of the feedback part, and total multidimensional frequency characteristics are represented by

$$H_{\mathrm{MF},2}(z_1,z_2) = \frac{H_{\mathrm{F},2}(z_1,z_2)}{H_{\mathrm{B},2}(z_1,z_2)}, \tag{29}$$

$$H_{\mathrm{MF},3}(z_1,z_2,z_3) = \frac{H_{\mathrm{F},3}(z_1,z_2,z_3)}{H_{\mathrm{B},3}(z_1,z_2,z_3)}, \tag{30}$$

where

$$H_{\mathrm{B},2}(z_1,z_2) = \sum_{p_1=0}^{2}\sum_{p_2=0}^{2} B_{p_1}B_{p_2} z_1^{-p_1} z_2^{-p_2}, \tag{31}$$

$$H_{\mathrm{B},3}(z_1,z_2,z_3) = \sum_{p_1=0}^{2}\sum_{p_2=0}^{2}\sum_{p_3=0}^{2} B_{p_1}B_{p_2}B_{p_3} z_1^{-p_1} z_2^{-p_2} z_3^{-p_3}. \tag{32}$$

(29) and (30) represent the second- and third-order multidimensional frequency characteristics of the second-order nonlinear IIR filter, respectively.

TABLE I
SPECIFICATIONS OF TARGET LOUDSPEAKER.

Diameter	6.5 cm
Rated input	6 W (4.9 V)
Nominal impedance	4 Ω
Enclosure type	Closed box
Enclosure volume	0.6 l

TABLE II
LINEAR AND NONLINEAR PARAMETERS OF TARGET LOUDSPEAKER.

ω_0	1052 rad/s
Q_0	1.78
Q_{m}	5.32
R_{dc}	4.13 Ω
R_{m}	0.62 Ns/m
m_0	3.14 $\times 10^{-3}$ kg
K_0	3480 N/m
Bl_0	2.26 Wb/m

b_1	−108 /m
b_2	−78800 /m^2
k_1	−38 /m
k_2	17100 /m^2
b_1'	112 /m
b_2'	93500 /m^2
b_3'	8.92$\times 10^6$ /m^3
b_4'	4.36$\times 10^9$ /m^4
b_5'	6.48$\times 10^{12}$ /m^5
b_6'	2.70$\times 10^{15}$ /m^6

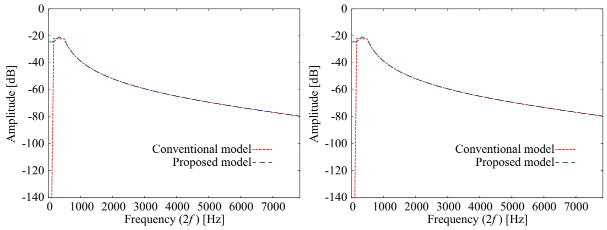
IV. EXPERIMENTAL RESULTS

We confirmed the validity of the formulated multidimensional frequency characteristics of the second-order nonlinear IIR filter by computer simulation and compensation experiment. In the simulation and experiment, the frequency characteristics and compensation results for the formulated characteristics should be similar to those for the conventional nonlinear IIR filter.

In the following simulation and experiment, we used a real electrodynamic loudspeaker and its parameters. Tables I and II show the specifications and the linear and nonlinear parameters of the target loudspeaker, respectively. Here, the parameters were measured using a Klippel measurement instrument [20]. The sampling frequency f_{s}, discrete Fourier transform (DFT) size, and analogue gain A_0 were set to 16000 Hz, 200, and 5.17, respectively. In the case of $A_0 = 5.17$, the RMS values of the input and output voltages were 0.87 and 4.5 Vrms, respectively. Hereafter, we call the formulated multidimensional characteristics (29) and (30) as the "proposed model" and the conventional nonlinear IIR filter (8) as the "conventional model."

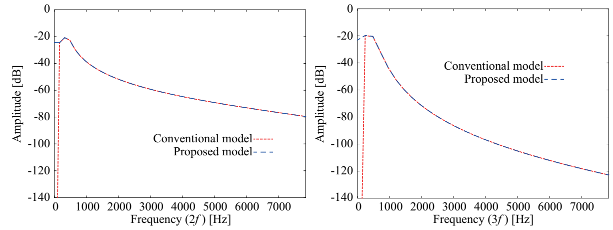
A. Computer Simulation

In the simulation, we compared the frequency characteristics of the proposed model (29) and (30) with those of the conventional model. The frequency characteristics of the conventional model were identified by the frequency response method [21]. Figure 4 shows the diagonal elements of the multidimensional characteristics obtained by the conventional and proposed models. From Fig. 4, the proposed model shows the same frequency characteristics as the conventional model except at low frequencies. This is because the frequency characteristics of the conventional model include higher-order intermodulation distortions owing to its identification principle



(a) Second-order frequency characteristic (b) Third-order frequency characteristic

Fig. 4. Diagonal elements of multidimensional frequency characteristics.



(a) Second-order frequency characteristic (b) Third-order frequency characteristic

Fig. 5. Diagonal elements of multidimensional frequency characteristics without higher-order components.

[21], that is, the harmonic components are

$$\begin{aligned}
U_{\mathrm{L}}(2\omega) = {} & H_2(\omega,\omega)U(\omega)U(\omega) \\
& + H_4(\omega,\omega,\omega,-\omega)U(\omega)U(\omega)U(\omega)U(-\omega) \\
& + H_6(\omega,\omega,\omega,-\omega,\omega,-\omega) \\
& \qquad U(\omega)U(\omega)U(\omega)U(-\omega)U(\omega)U(-\omega) \\
& + H_8(\omega,\omega,\omega,-\omega,\omega,-\omega,\omega,-\omega) \\
& U(\omega)U(\omega)U(\omega)U(-\omega)U(\omega)U(-\omega)U(\omega)U(-\omega) \\
& + H_{10}(\omega,\omega,\omega,-\omega,\omega,-\omega,\omega,-\omega,\omega,-\omega) \\
& \quad U(\omega)U(\omega)U(\omega)U(-\omega)U(\omega)U(-\omega) \\
& \qquad U(\omega)U(-\omega)U(\omega)U(-\omega),
\end{aligned} \tag{33}$$

$$\begin{aligned}
U_{\mathrm{L}}(3\omega) = {} & H_3(\omega,\omega,\omega)U(\omega)U(\omega)U(\omega) \\
& + H_5(\omega,\omega,\omega,\omega,-\omega)U(\omega)U(\omega)U(\omega)U(\omega)U(-\omega) \\
& + H_7(\omega,\omega,,\omega,\omega,-\omega,\omega,-\omega) \\
& \quad U(\omega)U(\omega)U(\omega)U(\omega)U(-\omega)U(\omega)U(-\omega) \\
& + H_9(\omega,\omega,\omega,\omega,-\omega,\omega,-\omega,\omega,-\omega) \\
& \ U(\omega)U(\omega)U(\omega)U(\omega) \\
& \ U(-\omega)U(\omega)U(-\omega)U(\omega)U(-\omega) \\
& + H_{11}(\omega,\omega,\omega,\omega,-\omega,\omega,-\omega,\omega,-\omega,\omega,-\omega) \\
& \quad U(\omega)U(\omega)U(\omega)U(\omega)U(-\omega)U(\omega)U(-\omega) \\
& \qquad U(\omega)U(-\omega)U(\omega)U(-\omega),
\end{aligned} \tag{34}$$

where $U(\omega)$ and $U_{\mathrm{L}}(\omega)$ are DFT of $u(n)$ and $u_{\mathrm{L}}(n)$ at frequency bin ω, respectively. From (33) and (34), the second- and third-order harmonic components include higher-order intermodulation components, for example, the $\omega+\omega+\omega-\omega$ component, respectively. On the other hand, (29) and (30) respectively consist of only second- and third-order components because of the derivation procedures of (29) and (30). Moreover, the components of the conventional model at 0 Hz cannot be identified because of its identification principle [21].

Next, we compared the frequency characteristics of the proposed model with those of the conventional model, which consists of only $u(n)$, $u_{\mathrm{L},2}(n)$, and $u_{\mathrm{L},3}(n)$, that is, the conventional model is represented by $u(n)$, (19), and (20). Figure 5 shows the multidimensional frequency characteristics under this condition. Here, the frequency characteristics of the proposed model are the same as those shown in Fig. 4. From Fig. 5, the frequency characteristics of the conventional and proposed models perfectly match with each other except at 0 Hz. Although other components except the diagonal elements cannot be shown, they also completely match with each other. Therefore, it can be said that the proposed model can represent the conventional model, and the multidimensional frequency characteristics are successfully formulated.

TABLE III
EXPERIMENTAL CONDITIONS.

Input signal	Multiple swept sinusoidal wave
Sampling frequency f_{s}	16000 Hz
Fixed frequency f_1	160 Hz
Swept frequency f_2	80 – 2400 Hz
Input voltage	4.68 Vrms

B. Compensation Experiment

In this experiment, we evaluated the amounts of reduction of nonlinear distortions of the loudspeaker used in the previous experiment. The compensation signals of the conventional model were generated using (8), and that of the proposed model was generated using (14), (29), and (30). Here, if the frequency characteristics are successfully formulated, the amounts of reduction obtained using (14) are almost the same as those obtained using (8).

Measurement conditions and the arrangement of the measurement devices are shown in Table III and Fig. 6, respectively. The experimental results for second-order nonlinear distortion are shown in Fig. 7. From Fig. 7, the compensation amounts of both models are very similar, although there are small differences between them due to the polynomial coefficients in (13). These results are similar to those shown in [18], indicating that our experiment was correctly conducted. As stated above, the second-order nonlinear IIR filter has limited its capability to reduce nonlinear distortions. Hence, it can be expected that the multidimensional frequency characteristics of the third-order nonlinear IIR filter may improve its capability to reduce nonlinear distortions. Moreover, it can also be expected that the frequency characteristics of the modified second-order nonlinear IIR filter may also improve its capability to reduce nonlinear distortions. From these results, it can be said that the multidimensional frequency characteristics of the second-order nonlinear IIR filter were successfully formulated.

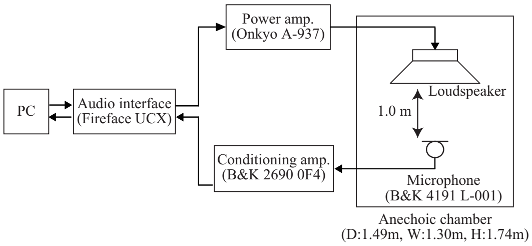
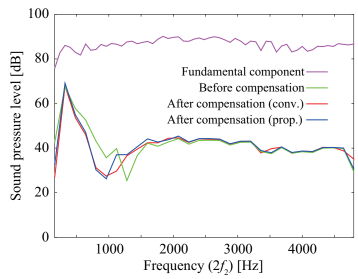
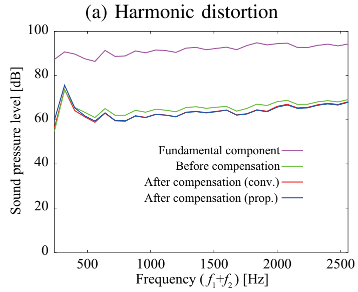


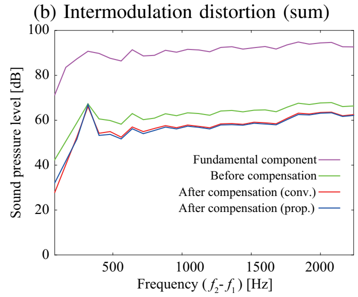
Fig. 6. Arrangement of measurement devices.



(a) Harmonic distortion



(b) Intermodulation distortion (sum)



(c) Intermodulation distortion (difference)

Fig. 7. Compensation results for second-order nonlinear distortion.

V. Conclusion

In this paper, the formulation of the multidimensional frequency characteristics of the second-order nonlinear IIR filter was presented. The frequency characteristics were formulated on the basis of the principles of the nonlinear IIR filter and Volterra filter. Some experimental results showed that the formulated frequency characteristics can represent the same nonlinear characteristics of the original second-order nonlinear IIR filter. In the future, we will study an adaptive algorithm for the second-order nonlinear IIR filter represented by the formulated multidimensional frequency characteristics and adopt it to nonlinear acoustic echo cancellations.

Acknowledgment

A part of this work is supproted by KDDI Foundation Research Grant Program 2019 and JSPS KAKENHI (JP21K17773).

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