Mathematical Model of a Horse and the Rider during a Jump

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Abstract—The horse-human interaction during horse riding is discussed here. The movements of a horse and its rider at trot are modeled by spring-damper-mass (SDM) models, which extracted the optimality of riding techniques in terms of the biomechanics. However, the models did not reproduce the trajectories at jump well because a trot is oscillatory while a jump is not. To improve the reproducibility of the trajectories at jump, we modified the existing SDM model for the horse at jump and evaluated the reproducibility of the models using the trajectories of the horse and the rider at canter and at jump, which are collected by tracking them in videos. As a result, our model succeeded to reproduce the trajectories at jump well.

I. Introduction

Horse-riding is an interaction between a horse and its rider. One way to examine a rider's skill is to understand their movements. Biomechanical models of their movements based on spring-damper-mass (SDM) models revealed that a horse's movement during trot depends on the experience of the rider and an 'extreme' modern jockey technique minimizes the peak force of the rider and the work of the horse [1], [2], [3], [4].

Trots are an oscillatory movement characterized by the frequency, the amplitude, and the phase, while jumps are an aperiodic movement. Thus, jumps have been analyzed from various aspects such as the effects of the rider's position [5], the rider's proficiency on jump [6], and the effects of the angular momentum between the horse and the rider [7], [8]. However, biomechanical models at jump is not established yet and hence what mechanical properties are important at jump is not clear.

As the first step toward the criteria for jumps, we examined whether the two SDM models for trots in [9] are applicable to jumping and found that only one of the two can reproduce the trajectories of riders at jump [10]. However, we gave the trajectries of the horses therein from the observed data because the model failed to reproduce the trajectories of the horses.

This paper is the second step toward the criteria for jumps, that is, we propose a modified model to reproduce the trajectories of the horses. Our approach is to replace a sinusoidal function in the model to a square wave by taking into account the difference between a trot and a jump. As a result, our model succeeded to reproduce both trajectories of the riders and the horses.

II. Materials and methods

A. Data collection and processing

The same two videos were used as our previous work in [10], collected from the Internet video site (Youtube; Data 1, www.youtube.com/watch?v=GS8WGSPZAKU, 29.7 fps; Data 2, www.youtube.com/watch?v=Gtn2W8-QbjI, 25.0 fps), each of which includes a jump and a canter for the approach run. Note that the scales (pixel/m) of the videos were calibrated so that the horse withers height was 1.6 meters.

The anatomical points of the riders and the horses were also the same and were extracted by using DeepLabCut [11], which were the head, the shoulders, the hip, the elbows, the wrists, the fingertips, the knees, the ankles, the heels, and the toes of a rider and the head, the neck, the trunk, the shoulders, and the thighs of a horse. Using the anatomical points of a rider, we calculated the center of gravity (CoG) of each of the four body parts (the upper body, the upper legs, the lower legs, and the feet) according to [12] and the CoG of the rider as

$$z_{\mathrm{G}} = \sum_{i=1}^{n} \frac{z_{\mathrm{G},i} m_i}{m}, \tag{1}$$

where m is the mass, $z_{\mathrm{G},i}$ and m_i are the CoG position and the mass in each body parts, respectively, where the mass of the rider was set to 60 kg. In a similar way, we calculated the CoG of each of the five body parts (the head, the neck, the trunk, the shoulders, and the thighs) according to [13] and the CoG of the horse using (1), where the mass of the horse was set to 600 kg.

Note that the extracted points with low likelihood for estimation and those that appear wrong were manually corrected and that the CoGs calculated frame by frame were smoothened with Savitzky–Golay filter [14].

B. Spring-damper-mass model

Our SDMs for a rider and for a horse at jump are almost the same as the SDMs at trot proposed in [9], where the rider and the horse are regarded as mass points, m_r and m_h, connected with springs, k_r and k_h, and dampers, c_h and c_r, and only their vertical displacements, z_r and z_h, are discussed.

Rider: The rider model has two springs, the saddle spring with a constant stiffness, $k_{r,s}$, and the active spring with a variable stiffness, $k_{r,l}$. The stiffness $k_{r,l}$ sinusoidally changes between $k_{r,l,base}$ and $k_{r,l,base} + k_{r,l,amp}$ with the angular frequency ω_r and the phase difference γ_r. The springs are not fixed with the horse but can be apart. Force contact factors, $\eta_{r,s}$ and $\eta_{r,l}$, represent the state of the two springs, respectively. Thus, the dynamics of the rider at time t is described as

$$
\begin{aligned}
m_r \ddot{z}_r &= -\eta_{r,c} c_r (\dot{z}_r - \dot{z}_h) \\
&\quad - \eta_{r,s} k_{r,s} \varepsilon_{r,s} - \eta_{r,l} k_{r,l} \varepsilon_{r,l} - m_r g, \qquad \text{(2a)} \\
k_{r,l} &= k_{r,l,base} \\
&\quad + k_{r,l,amp}(0.5 - 0.5 \sin(\gamma_r + \omega_r t)), \qquad \text{(2b)} \\
z_{r,\eta_l} &= z_{r,\eta_l,base} - z_{r,\eta_l,amp} \sin(\gamma_r + \omega_r t), \qquad \text{(2c)} \\
\varepsilon_{r,s} &= \frac{(z_r - z_h) - z_{r,\eta_s}}{z_{r,\eta_s}}, \qquad \text{(2d)} \\
\varepsilon_{r,l} &= \frac{(z_r - z_h) - z_{r,\eta_l}}{z_{r,\eta_l}}, \qquad \text{(2e)} \\
\eta_{r,s} &= \frac{1}{1 + \exp(a \varepsilon_{r,s})}, \qquad \text{(2f)} \\
\eta_{r,l} &= \frac{1}{1 + \exp(a \varepsilon_{r,l})}, \qquad \text{(2g)} \\
\eta_{r,c} &= \begin{cases} \eta_{r,s} & (\eta_{r,s} \geq \eta_{r,l}), \\ \eta_{r,l} & (\eta_{r,s} < \eta_{r,l}), \end{cases} \qquad \text{(2h)}
\end{aligned}
$$

where z is the vertical displacement, $\dot{z}$ is the vertical velocity, ε is the strain of the movement, η is the force contact factor, z_{r,η_s} and $z_{r,\eta_l,base}$ are the average of the difference between the heights of the rider and the horse, g is the constant gravitational acceleration, and a is a constant that determines the tendency of the contact's switch.

Horse: Our SDM for a horse at jump is based on the SDM at trot proposed in [9], which has a simple spring with a constant stiffness, k_h, and a force contact factor, η_h, similarly to the rider model. Our model is modified at the forcing function, FF_h, that excites the horse body. While the original in [9] was a sinusoid, ours is a square wave function (Fig. 1), since jump is an instantaneous movement.

Thus, the dynamics of the horse at time t is described as

$$
\begin{aligned}
m_h \ddot{z}_h &= -\eta_h c_h \dot{z}_h - \eta_{r,s} c_r (\dot{z}_h - \dot{z}_r) - \eta_h k_h \varepsilon_h \\
&\quad + \eta_{r,s} k_{r,s} \varepsilon_{r,s} + \eta_{r,l} k_{r,l} \varepsilon_{r,l} \\
&\quad - m_h g + \eta_h FF_h, \qquad \text{(3a)} \\
\varepsilon_h &= \frac{z_h - z_{h,\eta}}{z_{h,\eta}} \qquad \text{(3b)} \\
\eta_h &= \frac{1}{1 + \exp(a \varepsilon_h)}, \qquad \text{(3c)} \\
FF_h &= \begin{cases} A_{h1} & \text{if } t_{s1} \leq t \leq t_{e1}, \\ A_{h2} & \text{if } t_{s2} \leq t \leq t_{e2}, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{(3d)}
\end{aligned}
$$

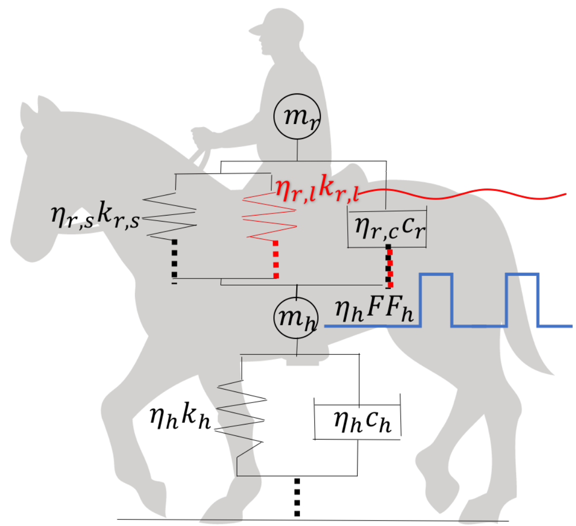


Fig. 1. Spring-Damper-Mass with an active spring for the rider and a square wave forcing function for the horse

where A_{h1} is the amplitude for the take-off from time t_{s1} to t_{e1} and A_{h2} is the amplitude for the landing from time t_{s1} to t_{e1}.

C. Parameter Estimation

The parameters in the model were estimated from the observed displacements z_r and z_h, using Differential Evolution (DE) [15], [16], which is an evolutionary algorithm for optimization. The objective function for the optimization was the mean square error between the observed and modeled displacements of the horse's and rider's CoGs, in the same way as [17]. The search range of the parameters are presented in Table 1 that was determined from the values in [18], [19], [3], [20].

D. Evaluation

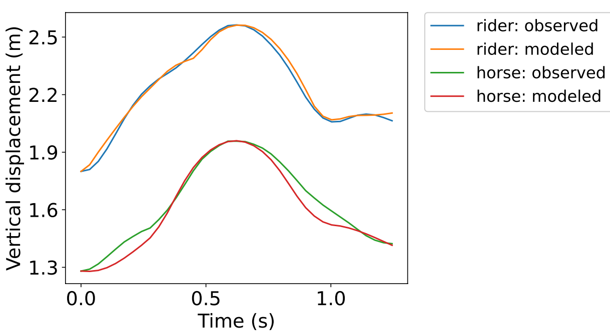
To see how much the models explain the observed trajectries of the rider at jump and canter, the differential equations of the models with the estimated parameters were solved using RK45 (scipy.integrate.solve_ivp [21]) and the reproduced trajectries were compared with the observed ones.

III. Results

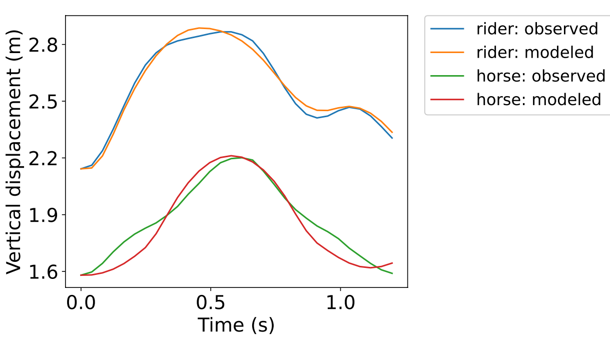
The SDM models for the horse and the rider successfully reproduced their observed displacements at jump (Fig. 2), where the coefficients of determination, R^2, are rider= 0.990, horse= 0.952 in Data 1 and rider=0.985, horse= 0.906 in Data 2.

TABLE I
Search range of the model parameters

Horse's parameters	Search range
damping coefficient c_h (kg s^{-1})	0 – 10000
spring stiffness k_h (kN m^{-1})	0 – 80
amplitude $A_{h\ takeoff,\ landing}$ (N)	0 – 12000
take-off time window t_{s1}, t_{e1} (s)	0 – half of the whole time
landing time window t_{s2}, t_{e2} (s)	half of the whole time – end of the time
Rider's parameters	**Search range**
damping coefficient c_r (kg s^{-1})	0 – 3000
saddle spring stiffness $k_{r,s}$(kN m^{-1})	0 – 80
active spring base stiffness $k_{r,l,base}$ (kN m^{-1})	0 – 40
active spring increase stiffness $k_{r,l,amp}$ (kN m^{-1})	0 – 40
phase difference γ_r	0 – 2π
leg's amplitude $z_{r,\eta_l,amp}$ (m)	0 – 0.3
time frequency f_r (Hz)	0 – 3



Data 1



Data 2

Fig. 2. Vertical displacements of the horse and the rider, observed (blue, green) and reproduced by our SDM models (orange and red).

IV. Discussion

To improve the reproducibility of the trajectories at jump by the SDM models for the horse and the rider, we modified the SDM model for the horse at jump by changing the force function and confirmed that the model reproduces the trajectories at jump. This is a surprising result since both a horse and the rider during a jump make pitch rotations of their bodies, which are not included in the SDM models. The simple SDM models will help understand the nature of the horse-rider interaction at jump as was done at trot in [3].

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