Abstract—In multiple-input multiple-output (MIMO) systems, eigenbeam-space division multiplexing (E-SDM) can achieve high performance using channel state information that is fed back to the transmitter. Orthogonal frequency division multiplexing (OFDM) systems can efficiently operate in wideband fading channels using a simple 1-tap equalizer on each sub-carrier. Here, MIMO channels are generated with increasing correlation through Cholesky factorization of the channel covariance matrix. To maintain a given BER and hence compensate for the effects of correlation, an increase in the transmit power is required (e.g. 0.4 dB for \( \rho_{\text{tx}} = 0.25 \) at \( 10^{-4} \) BER for a reduced complexity pseudo-E-SDM system), but this increases significantly as \( \rho \) increases. In this paper, a description of the E-SDM system implementation on an FPGA-DSP platform is also detailed.

I. INTRODUCTION

It is well known that remarkable capacity gains can be obtained from multiple-input multiple-output (MIMO) systems. This capacity increases with the minimum number of transmit and receive elements. The gains can be achieved through deploying either spatially separated or polarized antennas that exploit the rich scattering and hence decorrelated fading. The capacity is maximized when eigen-beamforming, based on the channel subspace decomposition, is applied [1].

In eigenbeamforming, the channel eigenvectors obtained from the singular value decomposition (SVD) of the channel matrix, are used as the transmit and receive beamformer weights [2]. The complexity of eigenbeamforming systems for orthogonal frequency division multiplexing (OFDM) is high because the SVD must be calculated at each sub-carrier. The pseudo-eigenbeam-space division multiplexing (PE-SDM) technique however is based on a channel approximation that requires just one SVD calculation per time-slot [3].

The maximum MIMO channel capacity is achieved when the channels are independent. In practice however, there usually exists some degree of correlation. The effects of the correlation for narrowband MIMO systems were considered analytically in [4] and with actual channel data in [5]. Simulation results for an OFDM system employing space-time block codes, where the correlation is inherent due to restricting the range of the scatterers angles of arrival, was shown in [6].

The performance of practical beamforming systems with feedback are sensitive to a number of parameters including the accuracy of channel estimates, the feedback delay and the fixed precision bit-width. To determine the achievable performances in a realistic system a number of hardware testbeds have been developed. Examples of E-SDM testbeds are in [7] and [8]. In a previous paper, we described a real-time MIMO-OFDM PE-SDM testbed and compared its performance with E-SDM for uncorrelated channels [9]. In the work described in this paper, we have expanded the hardware design to enable the effects of spatial correlation to be analyzed for any given value of element correlation \( \rho \), and show results on the effects of correlation on error rate performance and capacity.

II. EIGENBEAMFORMING

A. E-SDM

The transmit data stream is divided into \( K \) substreams \((K \leq \min(N_{\text{tx}},N_{\text{rx}}))\), where \( N_{\text{tx}} \) and \( N_{\text{rx}} \) are the number of transmit and receive antennas respectively. The transmit signals are shaped by a weight matrix \( W_{\text{tx}} \) to form orthogonal beams and also to control power allocation. The receive signals are detected by a weight matrix \( W_{\text{rx}} \).

The optimal \( W_{\text{tx}} \) and \( W_{\text{rx}} \) weights are given by [2]

\[
W_{\text{tx}} = U\sqrt{P},
\]

\[
W_{\text{rx}} = U^H H^H,
\]

where \( P = \text{diag}(P_1,\ldots,P_K) \) is the transmit power matrix, \( H^H \) is the Hermitian transpose of the frequency domain channel \( H \), and \( U \in \mathbb{C}^{N_{\text{rx}} \times K} \) is obtained from the eigenvalue decomposition (EVD) as

\[
H^H H = U \Lambda U^H
\]

where \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_K) \).

The transmit weight perfectly matches the instantaneous MIMO channel response, are then given by

\[
y(t) = \Lambda\sqrt{P}s(t) + W_{\text{rx}} n(t),
\]

where the vector \( s(t) \in \mathbb{C}^K \) consists of signals \( s_1(t), \ldots, s_K(t) \) transmitted through the \( K \) subchannels, and \( n(t) \in \mathbb{C}^{N_{\text{rx}}} \) is additive white Gaussian noise (AWGN). The signal-to-noise power ratio (SNR) of the \( k \)th substream is given by

\[
\lambda_k P_k P_s / \sigma^2, \quad \text{where } P_s = E[|s_1(t)|^2] = \cdots = E[|s_K(t)|^2].
\]

The channel capacity and BER performance can be improved by adaptively assigning the data rate and transmitting power depending on the substream SNR [2].
However, the eigenvalues are output in descending magnitude order rather than in substream order and hence eigenvalues originating from the same substream may not necessarily be continuous in frequency. In addition, there is a random phase rotation associated with each SVD computation. Together, this greater frequency selectivity leads to an increase in the effective channel delay spread, which is given by the convolution of the beamformed impulse response and channel.

**B. Pseudo E-SDM**

In order to achieve the frequency continuity, PE-SDM uses a concept of channel averaging. The auto-correlation of the estimated channel is written as

\[
R_{\text{tx}}(\tau) = \frac{1}{Q} \sum_{i=0}^{T_{\text{a}}} H_I(t) H^H_I(t + \tau),
\]

where \( Q \) is the FFT size, \( H_I(t) \) is the time domain channel impulse response and \( \tau \) is the delay lag [3]. The sum of the diagonal elements of \( H_I(t) H^H_I(t) \), \( \text{tr}[R_{\text{tx}}(0)] \), represents the total energy of the channel. \( R_{\text{tx}}(0) \) is equal to the sum of \( H_I(f) H^H_I(f) \) over all frequencies, where \( H_I(f) \) is the frequency domain channel. The SVD of \( R_{\text{tx}}(0) \) is given by

\[
R_{\text{tx}}(0) = V_{\text{tx}} A_{\text{tx}} V^H_{\text{tx}},
\]

where \( V_{\text{tx}} \) is a unitary matrix and \( A_{\text{tx}} \) contains descending ordered eigenvalues of \( R_{\text{tx}}(0) \). The transmit weight is finally obtained from the Gram-Schmidt (GS) orthonormalization after restoring the frequency response as

\[
W_{\text{tx}} = \text{GS}[H^H_I(f)V_{\text{tx},K}],
\]

### III. Spatial Correlation

The MIMO channel covariance matrix \( C \), is given by the Kronecker product \( R_{\text{MS}} \otimes R_{\text{BS}} \) as

\[
\begin{bmatrix}
\rho_{11} & \rho_{12} & \ldots & \rho_{1N} \\
\rho_{21} & \rho_{22} & \ldots & \rho_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{1N} & \rho_{2N} & \ldots & \rho_{NN}
\end{bmatrix}
\otimes
\begin{bmatrix}
\rho_{11} & \rho_{12} & \ldots & \rho_{1M} \\
\rho_{21} & \rho_{22} & \ldots & \rho_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{1M} & \rho_{2M} & \ldots & \rho_{MM}
\end{bmatrix}
\]

where \( R_{\text{MS}} \) is the \( N \times N \) covariance matrix seen at the mobile transmitter, \( R_{\text{BS}} \) is the \( M \times M \) covariance matrix seen at the base-station receiver, and \( \rho_{nm} \) is the correlation between antenna elements \( n \) and \( m \).

The Cholesky factorization of \( C \), such that \( C = LL^T \), is then represented by the lower diagonal matrix \( L \). The stacked output vector of correlated samples \( Z \), is obtained through multiplying \( L \) with the stacked vector of independent identically distributed (I.I.D.) input samples \( X \), as given by

\[
\begin{bmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
\vdots \\
Z_m
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
\alpha_{21} & \alpha_{22} & 0 & \ldots & 0 \\
\alpha_{31} & \alpha_{32} & \alpha_{33} & \ldots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha_{m1} & \alpha_{m2} & \alpha_{m3} & \ldots & \alpha_{mm}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
\vdots \\
X_m
\end{bmatrix}
\]

### IV. Performance Analysis

The performances of the SDM algorithms were simulated with a fixed total of eight coded bits assigned per subcarrier. The per substream allocated bit patterns were therefore \([4,4],[2,2,2,2],[6,2] \) or \([4,2,2] \) and correspond to IEEE 802.11n modulation coding scheme (MCS) values 11, 25, 34 and 39 respectively. The coding rate was 1/2, and hence all four schemes provide a 52 Mbps service. The TX resource adaption was determined by selecting the modulation on each substream that gave the minimum BER based on the Chernoff lower bound [12]. At each TX SNR point 100,000 packets were transmitted and an average BER calculated. The normalized TX power is the power normalized by the value yielding an average \( E_s/N_0 \) of 0 dB in the single antenna case in the same fading channel. The fading channels were correlated according to the process described in Section III, with correlation coefficients \( \rho_{nm} \). Varying between 0.0 and 1.0. The spatial coloring was repeated independently for each delay path.

The E-SDM family BER performances as a function of increasing coefficient \( \rho \) are shown in Fig. 1. The performance loss for E-SDM and PE-SDM systems when \( \rho \) was below 0.25 is small (specifically PE-SDM requires 0.4 dB more TX power at 10^{-3} BER). The BER for PE-SDM in correlated channels with varying frequency selectivity is shown in Fig. 2. At the BER of 1.0 \times 10^{-3}, an extra 7.4 dB total TX power is required in the 16-tap ChanA when \( \rho \) is 1.0 compared to 0.0.

1) Correlated Channel Capacity: The MIMO channel capacity assuming equal power on each transmit antenna, \( C \) is calculated using

\[
C = E \left\{ \sum_{k=1}^{K} \log_2 \left( 1 + \frac{\gamma_{AV}}{n} \lambda_k \right) \right\},
\]

where \( K \) is the rank or number of independent substreams, \( n \) is the number of transmitters, \( \gamma_{AV} \) is the average receive SNR and \( \lambda_k \) is the ith eigenvalue.

Fig. 3 shows the capacity variation as \( \rho \) increases from 0.0 to 1.0 for both 2x2 and 4x4 systems at 20 dB SNR. At the
4.3 bps/Hz as this capacity is reduced by 13.7 bps/Hz, 6.5 bps/Hz and ChanA = \{0, -1, ..., -15dB\} (16 taps); ChanD = \{0, -6, -12dB\} (3 taps); ChanE = \{0, -3, -6, ..., -18dB\} (6 taps).

10% CDF level, the maximum 4×4 capacity is 18.8 bps/Hz. This capacity is reduced by 13.7 bps/Hz, 6.5 bps/Hz and 4.3 bps/Hz as \(\rho\) varies from 1.0, 0.9 and 0.8 respectively.

V. HARDWARE IMPLEMENTATION

The channel emulator, TX and RX systems each consist of a Koden E-1071 signal processing platform. This comprises a base PC, DSP, signal conditioning and clock generation cards. The DSP card contains three Xilinx Virtex-4 FPGAs and four Analog Devices TigerSharc TS-201 DSPs. The signal conditioning card at the TX/RX contains eight DACs/ADCs and four Analog Devices TigerSharc TS-201 DSPs. The signal conditioning card at the TX/RX contains eight DACs/ADCs respectively and five supporting FPGAs (Fig. 4).

A. Transceiver Design

The principle blocks of the TX and RX are shown in Fig. 5. At the transmitter, the input data is scrambled and 1/2-rate convolutionally encoded. The parser multiplexes groups of bits onto separate substreams, and the interleaver distributes the bits across non-adjacent subcarriers. After puncturing, the data is symbol mapped and the spatial mapper implements the cyclic delay diversity by rotating the phases of the mapper outputs on different substreams. A 64-point IFFT converts the frequency domain data into a time domain signal. In practice, two stages are required in the transmission of one data packet. A non-beamformed sounding packet is sent in the first packet enabling the RX to make a MIMO channel estimate. In the subsequent packet, beamforming is applied.

At the RX after timing recovery and FFT, the channel state information (CSI) is estimated. The DSP then computes the beamform weights, and an optional zero-forcing (ZF) solution is computed on an FPGA. The Viterbi decoder computes with a trace-back length of 84 samples. The symbol-level operations in each block are computed within a 4 µs period and this enables a real-time processing of 260 Mbps.

B. Channel Emulator Design

The channel emulator generates the fading according to the channel model selected from: a) Gaussian I.I.D., b) modified Jakes and c) Zheng models (Fig. 6). The Gauss I.I.D. and AWGN samples are generated using independent PN shift registers for each path. Zheng proposed a statistical model in which the initial phases on the orthogonal branches are independent and randomly initialized [13]. The spatial coloring module takes the I.I.D. generated samples as inputs. Two approaches for implementing the Cholesky factorization in FPGA were considered. The first method involves storing the precomputed factors in memory. In the second approach, the factorization was achieved using a Xilinx AccelDSP core. The core reads the channel covariance matrix and stores the factors in an output memory in preparation for the spatial coloring process in (10).

C. Performance

The SDM and E-SDM performances were evaluated using ModelSim software. This simulates the exact hardware response using the actual VHDL (hardware description language) code that is used in the final hardware compile. The SDM results correspond to those from an uncorrelated AWGN channel (Fig. 7). For E-SDM systems, a packet consisting of \(N_T\) times 40 OFDM symbols was transmitted over 64 arbitrary Gaussian 2×2, 3×3 and 4×4 single-tap MIMO channels with...
\[ \rho = 0.0, 0.1, 0.2 \text{ and } 0.5. \] For the 4x4 system, an additional 0.8 dB (2.4 dB) transmit power was required when \( \rho = 0.1 \) (0.2) compared to when \( \rho = 0.0 \) in order to maintain a constant \( 10^{-4} \) bit error rate (BER). Similar results were obtained for the 2x2 and 3x3 systems.

VI. CONCLUSION

This paper has described both a software and hardware testbed for investigating the effects of spatial correlation on MIMO-OFDM beamforming systems. Cholesky factorization of the channel covariance matrix with increasing levels of element correlation were computed. In the multipath channels, a small increase of 0.4 dB transmit power is required when the correlation increases from 0.0 to 0.25 for PE-SDM systems to maintain a 10^{-5} BER. However when \( \rho = 0.99 \), an extra 7.4 dB TX power is required to maintain the BER of 10^{-3}, and the 4x4 capacity decreases by 72% at the 10% CDF level. The testbed can generate multipath channels and these will be evaluated for the PE-SDM system as part of future work.

ACKNOWLEDGMENT

This work was supported by the Strategic Information and Communications R&D Promotion Program from the Ministry of Internal Affairs and Communications, Japan and the Global Center of Excellence program.

REFERENCES