Pseudo-IIR Adaptive Array Based on Spatial State-Space Filtering

Sho Iwazaki*, Koichi Ichige* and Hiroyuki Arai*

* Department of Electrical and Computer Engineering, Yokohama National University, Yokohama 240-8501, Japan.

Abstract—This paper presents a novel pseudo-IIR construction of adaptive array based on spatial state-space filtering. The output of adaptive array is basically given as a weighted sum of multiple inputs received by array sensors, which can be regarded as an FIR filtering in spatial domain. The proposed approach employs state-space representation of system modeling, and it enables an extended pseudo-IIR construction of adaptive arrays. Performance of the proposed approach is evaluated through some computer simulation in severe environment where the conventional FIR adaptive array does not work effectively.

I. INTRODUCTION

Adaptive array plays a significant role in mobile communication in extracting desired signal components while suppressing interferences [1]–[3]. The output of adaptive array is basically given as a weighted sum of multiple inputs received by array sensors, which can be regarded as an FIR filtering in spatial domain. MMSE-based optimization like LMS or RLS can be easily applied to update adaptive weights. The problem here is often said as the optimization performance like convergence speed or computational load, however we rather pay attention to the limited performance due to spatial FIR filtering. Enhancing spatial signal processing in adaptive array will lead better spatial beamforming performance and BER characteristics in mobile communication.

We recall that it is well-known for signal processing engineers that IIR filters can realize better frequency characteristics than FIR filters at lower order. Indeed IIR filtering has already been applied to temporal signal processing in TDL (Tapped Delay Line)-based adaptive arrays for wideband beamforming [4], [5]. The performance of temporal IIR filtering is superior than FIR filtering, however there is some restrictions or constraints in updating adaptive coefficients to guarantee the stability of IIR filtering as mentioned in [4]. Furthermore, TDL-based approach is not always effective for narrowband beamforming.

Here we consider to apply IIR filtering to spatial signal processing in adaptive array. One of the properties of spatial IIR filtering is that we can always guarantee the stability of IIR filtering by forward-backward IIR filtering [6], since the signals received by array sensors are all available at the same time. The problem is that the temporal IIR filtering construction cannot be directly applied to spatial adaptive array because there does not exist spatial delay elements. Therefore we have to consider an approach without using spatial delay elements. Based on the above discussion, we do not realize IIR filtering but do realize pseudo-IIR filtering by truncating the number of array elements and introducing state-space system modeling.

This paper studies a pseudo-IIR construction of adaptive array based on spatial state-space filtering. The proposed approach employs state-space representation of system modeling, and it enables an extended pseudo-IIR construction of adaptive arrays. We explicitly formulate the proposed pseudo-IIR adaptive array, and present a way of updating adaptive weights based on MMSE criterion using LMS algorithm. Performance of the proposed approach is evaluated through some computer simulation in severe environment where the conventional FIR adaptive array does not work effectively.

II. SPATIAL SIGNAL PROCESSING

We aim at improving the performance of spatial signal processing which directly corresponds to the pattern of array antenna.

A. Signal Model

Assume that we have an $n$-elements Uniform Linear Array (ULA) with the corresponding weights as shown in Fig. 1. The incident L waves from the azimuth directions of $\theta_1, \ldots, \theta_L$ arrive at the ULA under an Additive White Gaussian Noise (AWGN) environment (input signals and noises are uncorrelated). For the input signal vector $\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$, the $p$-th input of the ULA is represented as

$$x_p(k) = \sum_{\ell=1}^{L} s_\ell(k) z_{\ell u}^{-(p-1)} + \nu_p(k),$$

(1)

where $s_\ell(k)$ and $z_{\ell u} = e^{-j2\pi \lambda(\ell - \ell u) d / \sin \theta}$ denote complex waveforms and spatial delays, respectively. In addition, $\nu_p(k)$ is the noise received at $p$-th antenna, $\lambda$ is the wavelength of carrier wave, $d$ is the interelement distance and $\theta$ is DOA of the incident wave.

B. MMSE Adaptive Array

Let $\mathbf{w}(k) = [w_1(k), w_2(k), \ldots, w_n(k)]^T$ denote the adaptive weight vectors as in Fig. 1. The array output $y(k)$ is given as the inner product of those vectors, i.e., $y(k) = \mathbf{w}^H(k) \mathbf{x}(k)$. The error between the desired reference signal $r(k)$ and array output $y(k)$ is given as $e(k) = r(k) - y(k)$, and its expected power is given as

$$E = E \left[ |r(k)|^2 \right] - \mathbf{w}^T r_{\mathbf{x}r}^* - \mathbf{w}^H r_{\mathbf{x}\mathbf{w}} + \mathbf{w}^H \mathbf{R}_{\mathbf{xw}} \mathbf{w},$$

(2)
where $E[\cdot]$ is the expectation, $R_{ww} = E[x(t)x^H(t)]$ is the correlation matrix of input sequence, and $r_{xx}$ is the correlation vector of reference sequence and input vector defined by $r_{xx} = E[x(t)r^*(t)]$. The weight vector $w$ is recursively updated based on LMS algorithm by

$$w(k+1) = w(k) + \mu E [x(k)e^*(k)],$$

where $\mu$ is the step size in LMS algorithm.

The output $y(k)$ of the adaptive array in Fig. 1 is written as

$$y(k) = \sum_{p=1}^{n} w_p^*(k)x_p(k)$$

$$= \sum_{p=1}^{n} w_p^*(k) \sum_{t=1}^{L} s_t(k) z_{st}^{-(p-1)} + \sum_{p=1}^{n} w_p^*(k) \nu_p(k)$$

$$= \sum_{t=1}^{L} H_t(z)s_t(k) + \tilde{\nu}(k),$$

where

$$H_t(z) = \sum_{p=1}^{n} w_p^*(k)z_{st}^{-(p-1)},$$

$$\tilde{\nu}(k) = \sum_{p=1}^{n} w_p^*(k) \nu_p(k).$$

We see that the output of adaptive array in Fig. 1 corresponds to the weighted sum of the FIR filter outputs as in (4).

III. PROPOSED APPROACH

FIR filters generally have a problem that higher order is required for better filter characteristics. That means, large number of elements is required for better antenna patterns in array signal processing. In this section, we propose a novel adaptive array structure that corresponds to pseudo-IIR filtering.

A. Applying State-space System Representation to Adaptive Array

We apply the state-space system model to the spatial signal processing in adaptive array. Let $H_\mu(z)$ denote the transfer function of state-space system $(A, b, c, d)$ defined as

$$H_\mu(z) = c(zI - A)^{-1}b + d.$$  \hfill (7)

Note that $I$ denotes the identity matrix whose size is equal to that of $A$. In addition, the state and output equations of the state-space system $(A, b, c, d)$ is also given as

$$\zeta(n+1) = A\zeta(n) + bx(n),$$

$$y(n) = c\zeta(n) + dx(n),$$

where $\zeta$ expresses the state variable vector. Besides, the term $(zI - A)^{-1}$ in (7) can be rewritten as

$$(zI - A)^{-1} = \sum_{p=1}^{\infty} z^{-p}A^{p-1}$$

$$= z^{-1}I + z^{-2}A + z^{-3}A^2 + \cdots,$$  \hfill (10)

provided that the matrix $A$ holds stability, that is, all the eigenvalues of $A$ are within the unit circle. Replacing $H_t(z)$ in (4) by $H_\mu(z)$ in (7), the array output $y(k)$ is rewritten as

$$y(k) = x_1(k) d(k) + x_2(k) e(k) b(k)$$

$$+ x_3(k) e(k) A(k) b(k)$$

$$+ x_4(k) e(k) A^2(k) b(k) + \cdots.$$ \hfill (11)

In (11), the array input vector is originally an infinite vector $x = [x_1, x_2, \cdots]^T$ but in fact it is truncated to a finite vector. The construction of the proposed system is shown in 2.

Besides, the IIR filter whose transfer function is given by

$$H_{IIR}(z) = \sum_{p=1}^{q} w_p^* z^{-p} 1 + \sum_{p=1}^{q} \nu_p^* z^{-p},$$

is realized by considering controllable canonical form of the state-space system $(A, b, c, d)$, i.e.,

$$A = \begin{bmatrix}
0 & 1 & 0 \\
0 & \ddots & \ddots \\
0 & \ddots & 1 \\
-v_0^* & -v_{q-1}^* & \cdots & -v_1^*
\end{bmatrix},$$

$$b = [0, \cdots, 0, 1]^T,$$

$$c = [w_0^*, w_{q-1}^*, \cdots, w_1]^T,$$

$$d = \gamma^*.$$ \hfill (16)

where $q$ is the size of the state-space which corresponds to the maximum size of $H_{IIR}(z)$, in this time, treated as $q = n$.  \hfill (14)
B. Optimization of State-space System Based on LMS Method

In this subsection, we consider LMS-based optimization for the state-space system \((\mathbf{A}^*, \mathbf{b}^*, \mathbf{c}^*, \mathbf{d}^*)\) represented by the following state and output equations:

\[
\zeta (n + 1) = \mathbf{A}^* \zeta (n) + \mathbf{b}^* x (n),
\]
\[
\hat{y} (n) = \mathbf{c}^* \zeta (n) + \mathbf{d}^* x (n).
\]

The objective function \(\tilde{J}\) to be minimized is given by

\[
\tilde{J} = E \left[ | e (t)|^2 \right] = E \left[ r (t) - \hat{y} (t)|^2 \right].
\]

Note that \(\tilde{J}\) in (19) is continuous for \(a_{ij}, b, c\) and \(d (\ell = 1, 2, \ldots, q, j = 1, 2, \ldots, q)\) and each gradient is derived by calculating the following partial differentials:

\[
\frac{\partial \tilde{J}}{\partial a_{ij}} = -2E \left[ \hat{e}^* (k) x (k) \left[ (zI - \mathbf{A}^*)^{-1} \right]_i \right],
\]
\[
\frac{\partial \tilde{J}}{\partial \mathbf{b}^*} = -2E \left[ \hat{e}^* (k) x (k) \left[ (zI - \mathbf{A}^*)^{-1} \right]^T \right],
\]
\[
\frac{\partial \tilde{J}}{\partial \mathbf{c}^*} = -2E \left[ \hat{e}^* (k) x (k) \left[ (zI - \mathbf{A}^*)^{-1} \right] \right],
\]
\[
\frac{\partial \tilde{J}}{\partial \mathbf{d}^*} = -2E \left[ \hat{e}^* (k) x (k) \right].
\]

In order to correspond \(\tilde{J}\) to each input of the adaptive array, we use the state equation (8) for (22). For (21), we consider the state equation of dual system:

\[
\beta (n + 1) = \mathbf{A}^H \beta (n) + \mathbf{c}^H x (n).
\]

where \(\beta\) is also the state variable vector. Additionally, to realize (20), we can use the following state equation via applying the state \(\beta\) of dual system (24) to the input of this system and employing the each state variable vector \(\alpha_i\), which is expressed as

\[
\alpha_i (n + 1) = \mathbf{A}^i \alpha_i (n) + \mathbf{b}^i \beta_i (n).
\]

From these facts, the recursive update formulae of the state-space system \((\mathbf{A}^*, \mathbf{b}^*, \mathbf{c}^*, \mathbf{d}^*)\) are provided by

\[
\alpha (k + 1) = \alpha (k) + 2 \delta \mathbf{e}^* (k) \mathbf{e} (k),
\]

\[
\beta (k + 1) = \beta (k) + 2 \delta \mathbf{e}^* (k) \zeta (k),
\]

\[
\gamma (k + 1) = \gamma (k) + 2 \delta \mathbf{e}^* (k) \hat{y} (k),
\]

\[
\delta (k + 1) = \delta (k) + 2 \delta \mathbf{e}^* (k) x (k).
\]

where \(\delta\) is the step size of LMS algorithm, \(\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_q]^T\). We optimize those weights based on LMS algorithm. Moreover, those weights are again updated via Line Search (LS) in the case that any of the poles of this system is not within the unit circle.

IV. SIMULATION

The proposed method is evaluated through some computer simulation. Specifications of simulations are shown in Table I. Severe simulation condition as in Table I is tested for evaluation in order to demonstrate that the proposed method effectively works where the conventional FIR array does not work well. Note that two different values of the step size (indicated as #1,#2) are employed so as to see that the converged BER characteristics do not depend on the value of the step size if enough number of iteration is assured. Naturally, as there are differences between conventional and proposed method, each of the step sizes has different value.

Figure 3 shows the comparison of beam patterns of FIR-and the proposed pseudo-IIR-adaptive array. We see from Fig. 3(a),(b) that the undesired ripples are well-suppressed by the proposed pseudo-IIR approach.

The Bit Error Ratio (BER) characteristics are also shown in Fig. 4. We see that the proposed method realizes better BER than the conventional FIR approach for both of the step sizes #1 and #2.

Furthermore, the convergence characteristics of the errors are depicted in Fig. 5. The proposed state space approaches well convergence point in the error than the conventional FIR-based methods. But divergences are occasionally observed.
because we employ only 10 points to LS in this time. Be aware that it remains as one of future studies to give an explicit upper limit of the step size for the proposed method.

One of suppressible reason about these behaviors described above are that the design of estimating model via state-space system become more proper and flexible than that of FIR, just a linear weighted combine.

However, computational load (the number of addition/multiplication operations) of the proposed method is $O(m^2)$ which is larger than the load for FIR-based method $O(n)$. Note that we do not claim computational efficiency at this moment, what we emphasize in this paper is a novel pseudo-IIR adaptive array structure and its good performance in severe environment. Reducing computational load also remains as one of future studies. Add lately that the proposed approach perform well in general case of course.

V. CONCLUDING REMARKS

This paper studied a pseudo-IIR construction of adaptive array based on spatial state-space filtering. We explicitly formulated the proposed pseudo-IIR adaptive array, and presented a way of updating adaptive weights based on MMSE criterion using LMS algorithm. Performance of the proposed approach was evaluated through some computer simulation in severe environment where the conventional FIR adaptive array does not works effectively, and we confirmed that the proposed method effectively works even in such a severe environment. Simpler computation procedure remains as one of the future problems.

REFERENCES