On Grating Lobe Elimination Of Difference Frequency In Parametric Loudspeaker

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***Abstract*—The parametric loudspeaker differs from other audio systems in its ability to create highly focused sound beam in air. This focused sound beam can be steered by adjusting a set of delays and weights of the ultrasonic transducer array (UTA), which is a component of parametric loudspeakers. In this paper, we use two ultrasonic primary signals to generate an audible signal at difference frequency. Grating lobes of primary signals occur when the spacing between ultrasonic transducers is larger than half the wavelength of primary signals, but grating lobes of the difference frequency are rarely observed. Therefore, in this paper, we examine this phenomenon and provide an analytical explanation.**

I. Introduction

Parametric loudspeakers transmit highly directional audible beams due to the virtual end-fire array formed by the nonlinearity of air. The principle of parametric loudspeakers, also known as parametric array in air, was first theoretically explained by Westervelt [1] in 1963 and then experimentally verified by Bennett and Blackstock [2] in 1975. Although adjustable sound beams are of great importance to realize 3-D sound effect [3], so far only few researchers have attempted to control the directivity of parametric loudspeakers. Pompei [4] worked out an optimum shape of transducers to suppress grating lobes of ultrasonic signals, which is potential to be applied in the parametric loudspeaker, but laborious to manufacture. Afterwards, Gan [5] and Tan [6] employed phased array techniques to the bifrequency primary signals and proposed a digital beamsteerer of difference frequency signals in the parametric array. They achieved to steer the difference frequency, but without the consideration of spatial aliasing.

When normal loudspeakers are arranged in a uniform linear array, the element spacing must be less than half the wavelength to ensure sufficient spatial sampling rate. One drawback of inadequate spatial sampling is the appearance of spurious mainlobes that are called grating lobes [7]. Because of the high primary frequency, spatial aliasing is extremely easy to observe in parametric loudspeakers due to the existence of more than one grating lobes. However, most grating lobes of primary frequencies are not inherited by the difference frequency. Eliminations of grating lobes occur in parametric loudspeakers at the difference frequency. In this paper, we examine and analyze the grating lobe elimination in a simplified beamsteerer of the UTA based on the product directivity principle. Furthermore, guidelines for designing steerable parametric loudspeakers are proposed according to the simulation results.

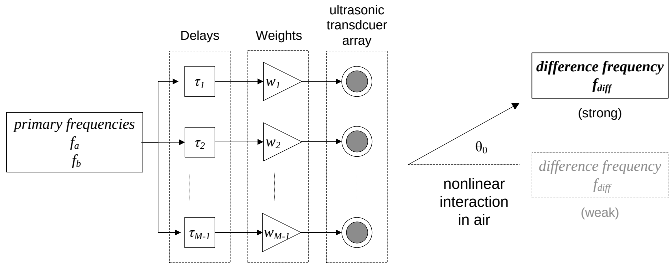

Fig. 1 Beamsteering structure of the difference frequency in parametric loudspeaker

II. Theory

To analyze the directivity of parametric loudspeakers, we consider the UTA as a group of M weighted and equally-spaced ultrasonic transducers, which is a component of parametric loudspeakers. Delay-and-sum beamsteering structure of the difference frequency is implemented in the parametric loudspeaker (see Fig. 1), where the UTA is steered in the same direction and shares the same group of weights for two primary frequencies. Gaussian directivity of ultrasonic transducers is also assumed. In this case, the directivity of the difference frequency can be predicted by the product of the directivities of primary frequencies [8].

Based on the quasilinear theory [5], the directivity of the difference frequency in the far-field is given by

$$D_{diff}(\theta) = D(k_a, \theta) D(k_b, \theta), \tag{1}$$

where θ is the incidence angle; k_a and k_b are the wavenumbers of primary frequencies f_a and f_b, respectively. We always assume that $f_a < f_b$ without lose of generality. The far-field beampattern of the steered UTA, $D(k,\theta)$, is given by

$$D(k,\theta) = \left| \sum_{m=0}^{M-1} w_m \exp\{ jmdk(\sin\theta - \sin\theta_0) \} \right|, \tag{2}$$

where w_m are the weights of ultrasonic transducers for m = 0,1,…, M−1; d is the spacing of ultrasonic transducers; k is the wavenumber of the primary frequency transmitted by the UTA; and θ_0 is the steering angle of the UTA.

Furthermore, the product directivity can be defined as

Proceedings of the Second APSIPA Annual Summit and Conference, pages 257–261,
Biopolis, Singapore, 14-17 December 2010.

$$D_{diff}\left(\beta,\gamma,\delta\right)=\left|\sum_{m=0}^{M-1}w_m\exp\left(j2\pi m\beta\delta\right)\right|\cdot\left|\sum_{m=0}^{M-1}w_m\exp\left(j2\pi m\beta\gamma\delta\right)\right|, \quad (3)$$

where β is the ratio of the spacing to the wavelength of the lower primary frequency f_a, given by $\beta = d / \lambda_a$; γ is the ratio of the higher primary frequency to the lower primary frequency, given by $\gamma = f_b / f_a$; and δ is the normalized angle, given by $\delta = \sin\theta - \sin\theta_0$.

III. GRATING LOBE ELIMINATION OF THE DIFFERENCE FREQUENCY

According to the assumption that the UTA is steered in the same direction and shares the same group of weights for two primary frequencies, a simplified beamsteering structure of the difference frequency in the UTA is shown in Fig. 1. Only two sine tones are delayed, amplified and transmitted as primary signals in the uniform linear UTA. The steering angle and beampattern of the difference frequency, as well as primary frequencies, are controlled by adjusting the delays and the weights.

Based on this simplified beamsteerer, simulations are carrier out for a UTA with 25 ultrasonic transducers. The spacing d is 16 mm, which is 1.86 times the wavelength of the lower primary frequency (f_a = 40 kHz). The higher primary frequency is selected as 42 kHz, 52 kHz, 60 kHz, resulting in γ = 1.05,1.3,1.5, respectively. The steering angle θ_0 is specified to 35˚ (see Fig. 1), and wm are chosen as Chebyshev weights. The simulated beam patterns for three sideband frequencies are plotted in Fig. 2-4. Grating lobes (1,2,3,…) in the figures are indexed from right to left, and the mainlobe is labeled as 0.

In Fig. 2, the first grating lobe of the lower primary frequency (f_a = 40 kHz) appears in the direction close to the first grating lobe of the higher primary frequency (f_b = 42 kHz). Two grating lobes that are closely spaced result in a sidelobe of the difference frequency (f_{diff} = 2 kHz) at about 3˚ that is lower than the mainlobe but higher than the highest sidelobe of primary frequencies. This sidelobe is considered as a partial eliminated grating lobe of primary frequencies.

In Fig. 3, grating lobes of the lower primary frequency (f_a = 40 kHz) coincide with sidelobes of the higher primary frequency (f_b = 52 kHz) that results in sidelobes of the difference frequency (f_{diff} = 12 kHz). Comparing to Fig. 2, heights of all the sidelobes in the difference frequency are not higher than the highest sidelobe of primary frequencies. This case is called the grating lobe elimination.

In Fig. 4, the second grating lobe of the lower primary frequency (f_a = 40 kHz) coincides with the third grating lobe of the higher primary frequency (f_b = 60 kHz). In this case, γ equals 1.5, and spatial aliasing of the difference frequency (f_{diff} = 20 kHz) occurs at about -30˚.

Grating lobes appear in the beampattern of primary frequencies when the spacing of ultrasonic transducers is larger than half the wavelength of primary frequencies in the UTA. This is commonly known as the Nyquist criterion in signal processing theory [7]. The above simulation results reveal that Nyquist criterion is no longer applied to the difference frequency generated by the parametric loudspeaker.

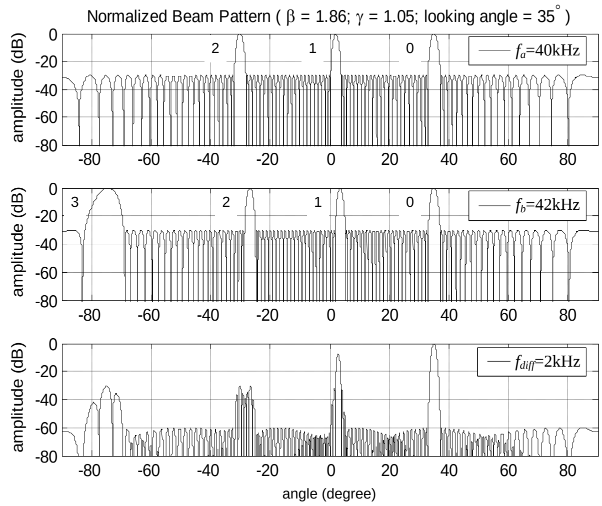

Fig. 2 Beampattern of primary frequencies (40 kHz, 42 kHz) and difference frequency (2 kHz) with partial grating lobe elimination

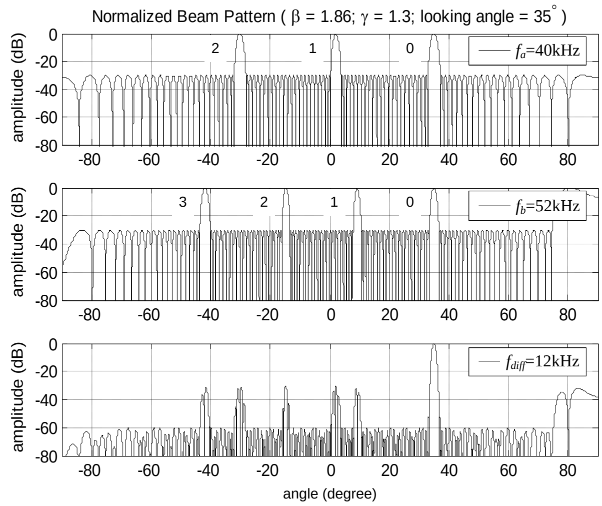

Fig. 3: Beampattern of primary frequencies (40 kHz, 52 kHz) and difference frequency (12 kHz) with grating lobe eliminated

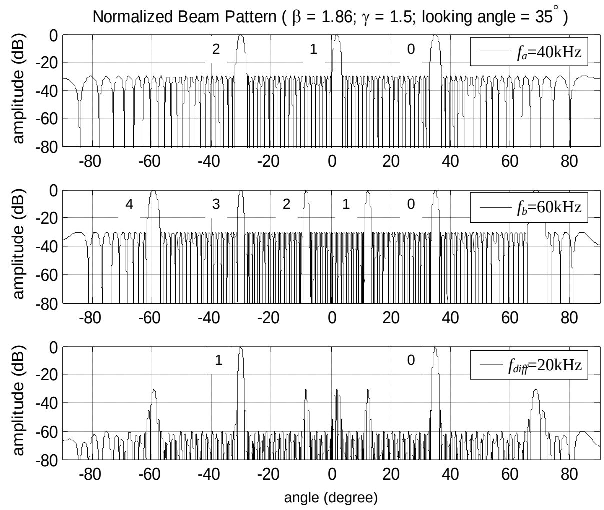

Fig. 4: Beampattern of primary frequencies (40 kHz, 60 kHz) and difference frequency (20 kHz) without grating lobe elimination

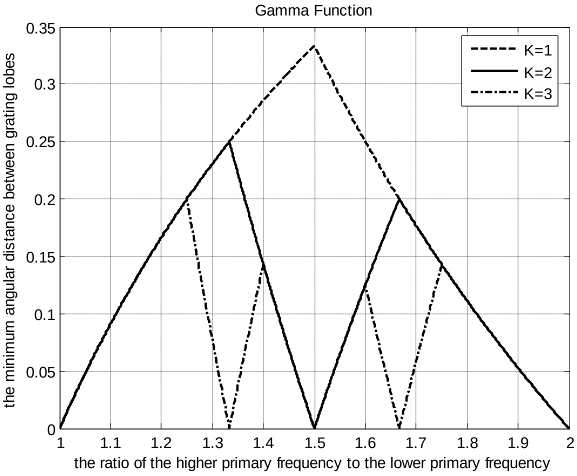

Fig. 5. Gamma function when K = 1,2,3

This is due to the fact that grating lobes of two primary signals do not coincide at the same direction all the times. It allows the grating lobes of primary frequencies to suppress each other at the difference frequency based on the product directivity principle. Therefore, the spacing limitation has been relaxed to enable sufficient spatial sampling rate at the difference frequency in parametric loudspeakers.

IV. PREREQUISITE TO THE GRATING LOBE ELIMINATION

The grating lobe elimination occurs in a specific pattern for the difference frequency, depending on the characteristics of parametric loudspeakers. In the following subsections, we derive the prerequisite to the grating lobe elimination and propose a guideline for designing a steerable parametric loudspeaker. In addition, we assume that the weights in the uniform linear UTA are symmetric with reference to the center transducer, and choose the steering angle $0° \leq \theta_0 \leq 90°$ without lost of generality.

A. Gamma Function

The angular distances between grating lobes of primary frequencies helps us to quantify the level of grating lobe eliminations. For example, the grating lobe elimination of the difference frequency can be achieved when the angular distance between grating lobes of primary frequencies are far apart as in Fig. 3. The direction where grating lobes occur is found at $\delta_a = n_a / \beta$ for the lower primary frequency f_a and $\delta_b = n_b / \beta\gamma$ for the higher primary frequency f_b, where n_a and n_b are indices of grating lobes. The gamma function, describing the minimum angular distance between grating lobes of two primary frequencies, is defined as

$$\Gamma_K(\gamma) = \min_{\substack{1 \leq n_a \leq K \\ 1 \leq n_b \leq L}} \left(\frac{|n_a - n_b/\gamma|}{\beta} \right), \tag{4}$$

where K is the largest index of grating lobes of the lower primary frequency f_a; L is the largest index of grating lobe of the higher primary frequency f_b. Because $f_a < f_b$ is assumed, K must be not more than L, and $\gamma > 1$.Furthermore, to obtain the values of the gamma function, K needs to be assigned in advance. After that, L is estimated by the closest integer to (K + 1) times γ.

Nulls in the gamma function show the occurrences of grating lobes for the difference frequency (see Fig. 5). When two grating lobes of primary frequencies arise in the same direction, the value of the gamma function reduces to 0. This observation is verified in Fig. 4. When the second grating lobe of 40 kHz primary signal coincides with the third grating lobe of 60 kHz primary signal, a null in the gamma function exists at γ = 1.5. Therefore, nulls in the gamma function constrain the range of γ that must be chosen between two neighboring nulls to prevent grating lobes of the difference frequency. To ensure the ability of generating low frequency, the null where γ = 1 must be chosen as the lower bound of the valid range of γ. From (4), nulls in the gamma function are located where $\gamma = n_b / n_a$. Setting $n_a = K$ and $n_b = K + 1$ gives the upper bound of the valid range of γ, which represents the null that is closest to 1. Thus, the upper bond of γ is determined by $\gamma = 1 + 1/K$. Note that the location of nulls in gamma function is not related to the estimation of L. So the range of γ, given by $1 < \gamma < 1 + 1/K$, is valid whenever K is assigned.

Derived from the range of γ, the range of the difference frequency is $0 < f_{diff} < f_a / K$. If the parametric loudspeaker is designed to generate all the audible frequencies, K should be less than f_a / 20 kHz. In practice, fa is normally chosen to be around 40 kHz. Therefore, it is reasonable to assign K = 2.

B. Intersection Function

As discussed above, the grating lobes of the difference frequency appear where there are nulls in the gamma function. In addition, grating lobes are suppressed within the vicinity of nulls in the gamma function due to the product directivity principle (see Fig. 2 and 3). Thus, intersection function is proposed to describe the elimination of grating lobes of the difference frequency when the distance between grating lobes of primary frequencies is given by the gamma function. Intersection function is defined as

$$I_K(\gamma) = \max_{\delta} \left[\begin{array}{l} \left| \sum_{m=0}^{M-1} w_m \exp\left(j\pi m\beta\delta \right) \right| \cdot \\ \left| \sum_{m=0}^{M-1} w_m \exp\left[j\pi m\beta\gamma\left(\delta - \Gamma_K(\gamma) \right) \right] \right| \end{array} \right]. \tag{5}$$

Note that the intersection function is a function of γ, as well as K and the weights wm. When K = 2, $1 < \gamma < 1.5$, the intersection functions with four sets of weights (Chebyshev weights with various attenuations and equal weights) are plotted in Fig. 6 and Fig. 7 for the number of ultrasonic transducers M = 15 and M = 25, respectively.

C. Guidelines For Parametric Loudspeaker Design

Table 1 and Table 2 are extracted from Fig. 6 and Fig. 7, respectively, showing the range of γ where the grating lobe elimination happens and the difference frequency range when the carrier frequency (*i.e.* the lower primary frequency) is selected as 30 kHz or 40 kHz.

Comparing the four groups of weights in Tables 1 and 2, we notice that Chebyshev weights with 20 dB attenuation results in a comparable difference frequency range as equal weights. Chebyshev weights with 10 dB attenuation result in the broadest difference frequency range, while Chebyshev weights with 30 dB attenuation lead to the narrowest range of the difference frequency. The difference frequency range has

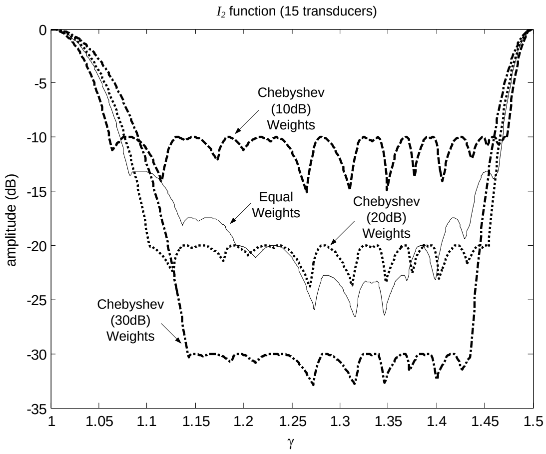

Fig. 5: Intersection function I2 with four groups of weights (15 transducers in the UTA)

Table 1: Difference frequency range for 15 transducers in UTA

weights / attenuation	range of γ	diff. freq. range (30 kHz carrier)	diff. freq. range (40 kHz carrier)
Equal (13.02 dB)	1.087-1.448	2.61-13.44 kHz	3.48-17.92 kHz
Chebyshev (10 dB)	1.065-1.460	1.95-13.80 kHz	2.60-18.40 kHz
Chebyshev (20 dB)	1.114-1.434	3.42-13.02 kHz	4.56-17.36 kHz
Chebyshev (30 dB)	1.164-1.408	4.92-12.24 kHz	6.56-16.32 kHz

Table 2: Difference frequency range for 25 transducers in UTA

weights / attenuation	range of γ	diff. freq. range (30 kHz carrier)	diff. freq. range (40 kHz carrier)
Equal (13.02 dB)	1.052-1.468	1.56-14.04 kHz	2.08-18.72 kHz
Chebyshev (10 dB)	1.038-1.477	1.14-14.31 kHz	1.52-19.08 kHz
Chebyshev (20 dB)	1.065-1.461	1.95-13.83 kHz	2.60-18.44 kHz
Chebyshev (30 dB)	1.094-1.445	2.82-13.35 kHz	3.76-17.80 kHz

to be sacrificed to achieve more attenuation. Once the range of γ is determined, improving the carrier frequency widens the range of different frequency. However, the bound of γ is proportional to the bound of the difference frequency in parametric loudspeakers. Improving the carrier frequency increases both the lower and upper bounds of the difference frequency. After all, a better way to build the UTA is to increase the number of ultrasonic transducers, which widens the ranges of γ and the difference frequency, and preserves the ability of generating low frequency as well.

Importantly, the steering angle of the UTA, as well as β and γ, affects the largest index of grating lobes of primary frequencies. Similarly to the Nyquist criterion [7], the range of β is approximately given by

$$\beta \leq \frac{K+1}{1+\sin\varphi}, \qquad (6)$$

where φ is the maximum steering angle. For example, setting $K = 2$ and $\varphi = 90°$, β is found to achieve 1.5 at the most. The spacing of ultrasonic transducers is extended to three times of half the wavelength of the lower primary frequency. In the case of upper single sideband modulation, the carrier frequency which is also the lower primary frequency fa, can be chosen as 40 kHz. Thus, the maximum non-aliasing spacing (given by $\beta\cdot\lambda_a$) is 12.86 mm. If the steering range is restricted to an angle between 0° to 30°, the maximum value of β is 2. Therefore, the maximum non-aliasing spacing is corresponded to be 17.15 mm that allows the grating lobe elimination to be valid.

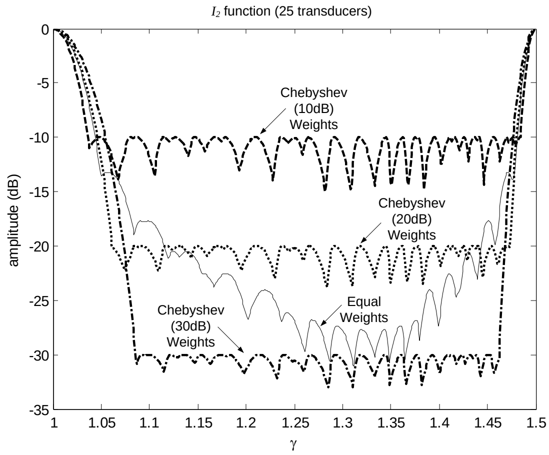

Fig. 6: Intersection function I2 with four groups of weights (25 transducers in the UTA)

V. CONCLUSION

The grating lobe elimination indicates that the non-aliasing spacing in the UTA of parametric loudspeakers is extended to $K+1$ times of half the wavelength of the lower primary frequency, where K is the largest index of grating lobes of the lower primary frequency and is normally stated as 2. A trade-off between the sidelobe attenuation and the valid range of the difference frequency becomes practically insolvable when the minimum sideband frequency is close to 0 Hz, unless a low carrier frequency and limited steering range is employed in the parametric loudspeaker. However, increasing the number of ultrasonic transducers in the UTA is recommended to design steerable parametric loudspeakers.

ACKNOWLEDGMENT

This work is supported by the Singapore National Research Foundation Interactive Digital Media R&D Program, under research grant NRF2007IDM-IDM002-086.

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