Neuron Modulation: A Digital Modulation Based on Spiking Neuron Models

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***Abstract*—In this paper, a novel digital modulation technique based on spiking neuron models is proposed. Thanks to the extremely simple and low-power spiking neuron circuits, this modulation scheme allows a less complex receiver at the cost of a more complicated transmitter. Accordingly, it is a good candidate for downlink transmission. Since the transmitted signal is generated based on Shannon's interpolation formula, it is approximately bandlimited. Spectral analysis shows that the proposed scheme is as spectrally efficient as QAM wherein a root-raised-cosine filter with a small roll-off factor is employed. A comparison of the bit error rate performance of the proposed scheme and QAM is also described.**

I. Introduction

Nearly all neurons convey information with each other by spikes. Spiking neurons can efficiently transform a signal into a firing spike train. The simplest spiking model is integrate-and-fire (IF) neuron model. When the integration of a signal reaches a predefined firing threshold, it is reset to a base signal and a spike is generated. Thanks to the advantage of the simple and low-power hardware, IF neuron is a good substitute for the conventional analog-to-digital converter (ADC) in power-limited applications [1], [2]. The tradeoff is that the signal reconstruction from the spike train turns more complex.

In this paper, we propose a digital modulation, called *neuron modulation* (NM), based on the IF neuron model. The scheme of NM modulation is based on the signal reconstruction from the spike train while the scheme of NM demodulation is based on the IF neuron. As NM inherits the characteristic of the IF neuron model, it allows a less complex receiver and more complex transmitter. Another key feature of NM is that NM signal is approximately bandlimited without using any pulse shaping filter. As the radio spectrum has become very precious and increasingly crowded every day, the highest priority is usually good bandwidth efficiency with low bit error rate (BER). Spectral analysis shows that the proposed modulation is as spectrally efficient as QAM wherein a root-raised-cosine filter with a small roll-off factor is employed. Therefore, NM is a good candidate for downlink transmission.

II. Integrate-and-Fire Spiking Neuron and Signal Reconstruction

Shannon's celebrated theorem indicates that any signal $x(t)$ bandlimited to $[-B, B]$ can be reconstructed from its samples with Nyquist sampling rate $f_s = 2B$,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \text{sinc}\left(\frac{t - nT_s}{T_s}\right), \tag{1}$$

where $T_s = 1/f_s$. Let the sampling points be given by $\{s_j\}_{j\in\mathbb{Z}}$ rather than the uniform ones $\{nT_s\}_{n\in\mathbb{Z}}$. Following Strohmer [3], we can recover $x(t)$ by

$$x(t) = \sum_{j\in\mathbb{Z}} w_j \,\text{sinc}\left(\frac{t - s_j}{T_s}\right), \quad \text{where} \quad Rw = b \tag{2}$$

with the entries of R and b being $R_{i,j} = \text{sinc}((s_i - s_j)/T_s)$ and $b_i = x(s_i)$, respectively. It is apparent that $w_j = x(jT_s)$ as $s_j = jT_s$. Formula (2) cannot be actually implemented on a hardware due to the infinite-dimensional system of linear equations $Rw = b$. In practice, we truncate $x(t)$ to be time-limited and approximately bandlimited, and the reconstruction is modified as [3], [4],

$$x(t) = \sum_{j=1}^{L} w_j \,\text{sinc}\left(\frac{t - s_j}{T_s}\right), \quad \text{where} \quad \mathbf{Rw} = \mathbf{b}. \tag{3}$$

Based on the non-uniform sampling, Wei and Harris [1] introduce a method for perfect signal reconstruction from integrate-and-fire (IF) neuron model, the simplest spiking neuron model. In this model, the non-uniform sample points are generated by $s_i = (t_{i-1} + t_i)/2$ with the the firing times $\{t_i\}_{i=0}^{L}$, also called a spike train, satisfying

$$\int_{t_{i-1}}^{t_i} x(t)dt = \theta. \tag{4}$$

The mechanism that transforms a continuous signal into a spike train can be easily realized by a extremely simple and low-power hardware. Signal reconstruction from the spike train $\{t_i\}_{i=0}^{L}$ can be realized by (3) and (4),

$$\theta = \sum_{j=1}^{L} w_j \int_{t_{i-1}}^{t_i} \text{sinc}\left(\frac{t - s_j}{T_s}\right) dt = \sum_{j=1}^{L} w_j c_{ij}, \tag{5}$$

for $1 \le i \le L$. Accordingly, we can calculate the weighting vector $\mathbf{w} = [w_1, w_2, \cdots, w_L]^T$ from $\theta\mathbf{1} = \mathbf{Cw}$ with the (i, j)-th component of $\mathbf{C}$ being c_{ij}. Unfortunately, $\mathbf{C}$ is usually ill-conditioned [1], and thus we apply the pseudo-inverse to the estimation of $\mathbf{w}$,

$$\widehat{\mathbf{w}} = \mathbf{C}^{+}\theta\mathbf{1}. \tag{6}$$

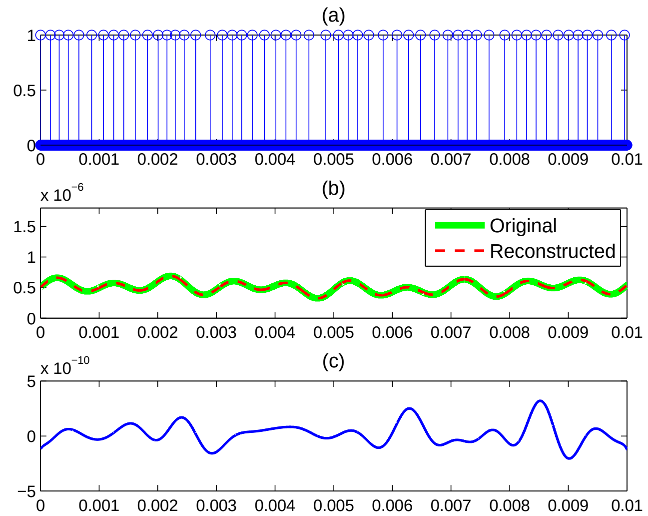

Fig. 1. Reconstruction from IF neuron: (a) the spike train, (b) the original and reconstructed signals, and (c) the reconstruction error.

and then $x(t)$ is reconstructed from $\widehat{\mathbf{w}}$ and $\{s_j\}_{j=1}^{L}$.

In a word, the uniform sampling and the IF neuron are special cases of the non-uniform sampling. The former prefixes the sampling points as $s_i = iT_s$ and uses only $x(s_i)$'s for reconstruction while the latter only requires well-designed sampling points s_i's. We give an experiment of the reconstruction from IF neuron shown in Fig. 1. In this experiment, $x(t) \in [-1000Hz, 1000Hz]$ implies the that the Nyquist frequency is $f_s = 1/T_s = 2000$, but Wei and Harris [1] recommend applying $\max_i |t_{i+1} - t_i| \leq T_s$ to the reconstruction from the IF neuron.

III. Digital Modulation Based on Integrate-and-Fire Spiking Neuron Model

In this section, we present a digital modulation based on the IF spiking neuron model, called *neuron modulation* (NM) for short. Simply speaking, NM modulator encodes the input binary stream to an artificial spike train and then generates a bandlimited signal by the reconstruction process in Section II, while NM demodulator uses the IF neuron to convert the received signal to a spike train and then obtain the decoded binary data stream. The main criteria of most digital modulation techniques are bandwidth efficient, bit error rate (BER), power consumption, and cost efficiency. Accordingly, there are some additional operating parameters, mechanisms and design rules crucial to NM, as discussed in the following.

A. Guard Interval

Consider an NM scheme with symbol rate $f_d = 1/T_d$ and M bits per symbol. A simple and straightforward temporal encoding strategy is dividing each time slot T_d into M subslots and transforming the data stream into a spike train,

$$t_i = (i-1)T_d + \frac{l+1}{M}T_d, \quad \text{and} \quad t_0 = 0.$$

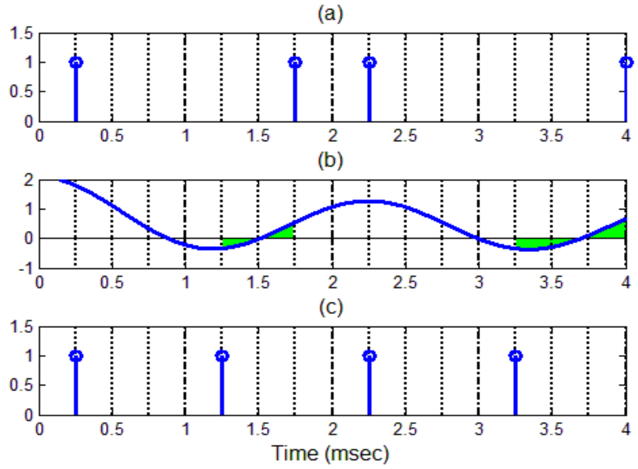

Fig. 2. Demodulation errors because $x(t) < 0$ for some t: (a) the spike train, (b) the signal x(t), and (c) the estimated spike train with input symbols $\{00, 11, 00, 10, \ldots\}$.

As $M = 4$ and gray coding is employed, $l = 0, 1, 2, 3$ are corresponding to symbols $00, 01, 11, 10$, respectively. However, this strategy may result in the undesirable demodulation errors because $x(t) < 0$ for some t. An example with $M = 4$ is shown in Fig. 2. The spike train is $\{t_1, t_2, t_3, t_4, \cdots\}$ but the estimated one is $\{t_1, t_2', t_3, t_4', \cdots\}$ because

$$\int_{t_2'}^{t_2} x(t)dt = 0, \quad \text{and} \quad \int_{t_4'}^{t_4} x(t)dt = 0.$$

To ensure $x(t) > 0$, a possible approach is reducing the difference of the values of Δt_i's, where $\Delta t_i = t_{i+1} - t_i$, and indirectly smoothing the variation of the signal amplitude. To make this possible, we sacrifice part of the slot interval as guard interval (GI) (see Fig. 3), and the spikes are $t_0 = 0$ and

$$t_i = (i-1)T_d + \gamma T_d + \frac{l}{M-1}(1-\gamma)T_d,$$

where GI ratio γ is defined as the ratio of GI to T_d. Fig. 4 is the same as the example of Fig. 2 except $\gamma = 0.7$. Large GI indeed decreases the probability of error induced by $x(t) < 0$ for some t, but it also increases the probability of error induced by the channel noise.

B. Membrane Potential Reset

Recall equation (4) where we show that as the membrane potential reaches a firing threshold $\theta + \theta_0$, it is reset to θ_0 and a spike is generated. Therefore, the estimated spike $\widehat{t}_i$ depends on $\widehat{t}_{i-1}$, $\widehat{t}_{i-1}$ depends on $\widehat{t}_{i-2}$, and so on. This phenomenon

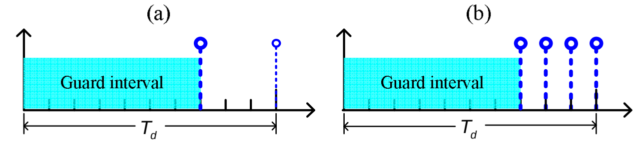

Fig. 3. Guard interval with (a) one bit per symbol and (b) two bits per symbol.

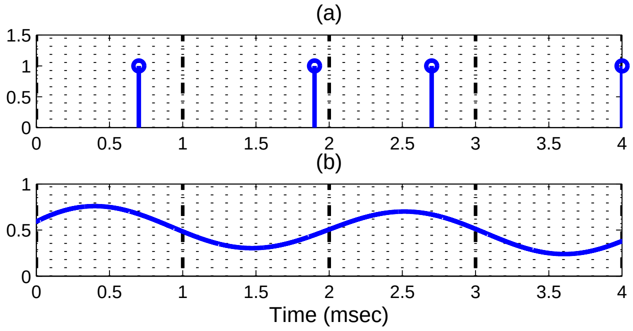

Fig. 4. Effect of using guard interval: (a) the spike train and (b) x(t) with input symbols $\{00, 11, 00, 10, \ldots\}$ and $\gamma = 0.7$.

induces serious error propagation as channel noise is large. To overcome this obstacle, the membrane potential is also reset as $t = iT_d$, and thus (4) and (5) are modified by

$$\int_{(i-1)T_d}^{t_i} x(t)dt = \theta, \quad i = 1, 2, \ldots, L. \tag{7}$$

This approach eliminates the error propagation effect since $\widehat{t_i}$'s are independent. The tradeoff is the requirement of a more accurate clock for the resetting time $t = iT_d$. This modified spiking neuron is referred to as IF-ADC.

C. The Reconstruction Procedure for NM

The reconstruction of $x(t)$ from the spike train is based on (3) and (6). The critical target in NM demodulator is minimizing the spike train estimating errors, $|t_i - \widehat{t_i}|$, rather than the reconstruction error of $x(t)$. Therefore, $x(t)$ must "perfectly" satisfy (7), that is, $\widehat{\mathbf{w}}$ in (6) must satisfy

$$\mathbf{C}\widehat{\mathbf{w}} = \mathbf{C}\mathbf{C}^{+}\theta\mathbf{1} = \theta\mathbf{1}; \tag{8}$$

otherwise, $x(t)$ may yield a wrong spike train despite $x(t) > 0$ and no channel noise concrescence. There is experimental evidence showing that $\mathbf{C}$ is not ill-conditioned if

$$T_s \leq \frac{1}{L-1}\sum_{i=1}^{L-1}|s_{i+1} - s_i| \approx \frac{1}{L}\sum_{i=0}^{L-1}|t_{i+1} - t_i| \approx T_d.$$

To achieve the best bandwidth efficiency, $T_s = T_d$ is utilized and $x(t)$ is approximately bandlimited to $[-\frac{1}{2T_d}, \frac{1}{2T_d}]$. Besides, to reduce the computational complexity, we can use uniform sampling points $s_j = (i-1)T_d + \frac{1}{2}T_d$. Therefore, $x(t)$ can be reconstruct by a conventional digital-to-analog converter (DAC) from $w_j = x((i-1)T_d + \frac{1}{2}T_d)$.

To estimate the L spikes, we require $x(t)$ for $0 \leq t \leq LT_d$. Therefore, the actual output NM signal is a cascade of signals of length LT_d. The complexity of matrix inverse in (6) and the actual bandwidth of the output NM signal depend on the value of L. Larger L provides more spectrally efficient transmission at the cost of higher complexity in modulator, but it does not affect the complexity of the demodulator.

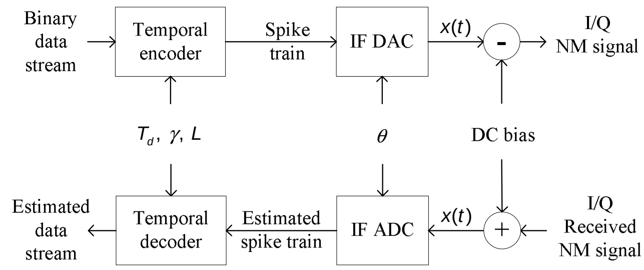

Fig. 5. Block diagrams of NM transmitter and receiver without I/Q Modulation.

D. DC Bias and I/Q Modulation

As $x(t)$ is forced to be positive, the waste of DC biasing power of $x(t)$ should be eliminated to improve the performance. The DC bias denoted by dc is given by

$$dc = \frac{1}{LT_d}\int_{t=0}^{t=LT_d} x(t)dt \approx \frac{1}{t_L}\int_{t=0}^{t=t_L} x(t)dt = \frac{L\theta}{t_L}.$$

As L and γ are large enough, $t_L \approx LT_d$ leads to $dc \approx \theta/T_d$.

Assume $x'(t) = x(t) - dc$. To improve the bandwidth efficiency, two physically independent $x'(t)$, say in-phase signal $x'_I(t)$ and quadrature signal $x'_Q(t)$, are mixed to occupy the same channel bandwidth. That is, the actual output NM signal is given by

$$x''(t) = x'_I(t)\cos(2\pi f_c t) + x'_Q(t)\sin(2\pi f_c t), \tag{9}$$

where f_c is the carrier frequency. This I/Q modulation can be realized by simple circuits.

E. Modulation and Demodulation Schemes

With the knowledge of NM design rules mentioned above, we conclude our modulation and demodulation schemes. The block diagrams of the NM transmitter and receiver without I/Q Modulation are shown in Fig. 5. The temporal encoder and decoder are the forward and inverse transformations, respectively, between the input binary stream and the artificial spike train based on the encoding strategy mentioned in Section III-A. The IF-DAC is the mechanism that generates $x(t)$ with parameters mentioned in Sections III-B and III-C. The IF-ADC is a modified IF neuron circuit that estimates and then quantizes the spike train, as mentioned in Section III-B. The removal of DC bias and restoration of DC level can reduce the power consumption of the output NM signal.

IV. Bandwidth Efficiency, Complexity and Performance Analysis

In this section, we compare the bandwidth efficiency, complexity and BER between NM and QAM, which is one of the most popular digital modulation schemes. Here, we consider a QAM scheme with root-raised-cosine filters (RRC) employed in the transmitter and receiver. We define M-NM is the mixture of the I and Q NM signals each with $m/2$ bits per symbol where $M = 2^m$. As the symbol rate is $1/T_d$, the bit rate is $log_2 M/T_d$. As L is large enough, the bandwidth of the

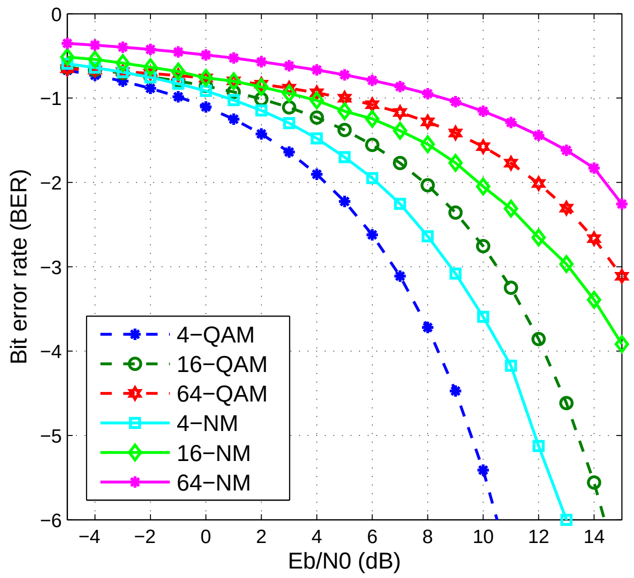

Fig. 6. BER versus E_b/N_0 for 4-QAM, 16-QAM, 64-QAM, 4-NM, 16-NM and 64-NM

NM signal is approximately $1/T_d$. Accordingly, the bandwidth efficiency of M-NM is $log_2 M$ bits/s/Hz which is the same as QAM if the roll-off factor of RRC is small enough.

Along the complexity of transmitters, we restrict our attention to the RRC in QAM and the IF-DAC in NM. Both RRC and IF-DAC contain a conventional DAC, yet IF-DAC is more complex due to its matrix inverse computation. To achieve higher bandwidth efficiency, both RRC and IF-DAC become more complex because the former requires smaller roll-off factor and more taps while the latter requires larger L (larger dimension of the matrix inverse). Along the complexity of receivers, we focus on the RRC in QAM and the IF-ADC in NM. RRC with small roll-off factor and IF-ADC both require a high accurate clock, but IF-ADC is less complex since its modified IF neuron circuit is very simple and low power consumption. Besides, NM receiver has the advantage that its complexity is independent of L.

Performance of QAM and NM is shown in Fig. 6. M-NM requires an E_b/N_0 that is about 2dB more than the corresponding values for M-QAM to realize the same BER.

V. Conclusion

In this paper, we have proposed a novel digital modulation, called neuron modulation (NM), based on the integrate-and-fire (IF) neuron model. NM has the property that less complex receiver is traded off for more complex transmitter. Based on the signal reconstruction from a spike train, the NM transmitter encodes the input binary stream to an artificial spike train and then generates a bandlimited NM signal. Based on the mechanism of the IF neuron, the NM receiver converts the received signal into a spike train and then obtain the decoded binary data stream. Besides, for improving the performance of NM, several design rules and mechanisms have addressed, including guard interval, membrane potential reset, DC bias, etc. We have shown that NM has very good bandwidth efficiency, say $log_2 M$ bits/s/Hz for M-NM. Also, simulation results has shown that M-NM requires an E_b/N_0 that is about 2dB more than the corresponding values for M-QAM to realize the same BER.

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