Minimization of PAPR in MIMO-OFDM Systems by Tone Reservation Techniques and Pilot Tones

Emmanuel MANASSEH*, Shuichi OHNO† and Yong JIN ‡
Dept. of Artificial Complex Systems Engineering, Hiroshima University
1-4-1 Higashi-Hiroshima, Japan
E-mail: *manasejc@ieee.org, †ohno@hiroshima-u.ac.jp, ‡jinyong2009@hiroshima-u.ac.jp

***Abstract*—In this paper a method that mixes tone reservation (TR) techniques and phase information of the pilot tones to efficiently reduce the peak to average power ratio (PAPR) of the multiple input multiple output (MIMO) orthogonal frequency division multiplexing (OFDM) signals is presented. First, we utilize convex optimization techniques to design dummy symbols to unused or reserved set of subcarriers to reduce the PAPR of the transmitted data in an OFDM symbol. Then, to the pilot tones dedicated for channel estimation, we design phase information to further reduce the PAPR. To design phase information of the pilot tones in MIMO-OFDM systems, we resort to the previously proposed cross entropy (CE) optimization techniques used for pilot phase design in single input single output (SISO) OFDM systems. Simulation results show that, by mixing the dummy symbols and pilot phase information the PAPR of the MIMO-OFDM signals can be effectively reduced.**

I. Introduction

Data throughput enhancement of the multiple input multiple output (MIMO) together with the robustness of the orthogonal frequency division multiplexing (OFDM) against frequency selective fading channels are regarded as the promising basis for the future high data rate communication systems [1], [2].

However, OFDM signals exhibit high peak to average power ratio (PAPR), causing MIMO-OFDM signals transmitted on different antennas to exhibit a prohibitively PAPR [2]. High PAPR is a critical issue in any multi-carrier system using OFDM. It can result in low power efficiency and large performance degradation of a system, due to the non-linearity of high-power amplifier (HPA) [3]–[5].

To mitigate PAPR problem, several techniques such as clipping and filtering, coding, selected mapping, interleaving and tone reservation have been proposed in the literature, (see [2]–[7] and the references therein). The objective of these methods has been to produce low complexity algorithms and schemes to reduce the high dynamic range to a reasonable level without (or with the minimum amount of) bandwidth loss [6].

Tone reservation (TR) technique is one of the promising approach for PAPR reduction in OFDM systems [3], [8], [9], where the transmitter mitigates the PAPR problem by sending dummy symbols (i.e., symbols not conveying information) in some reserved subcarriers [5]. In TR based schemes, there is no specific PAPR reduction information that needs to be communicated to the receiver. However, one problem with TR techniques lies on the computationally efficient determination of dummy symbols that effectively minimizes the PAPR [8].

The main differences in the TR techniques are based on the selection of the (convex) cost function, the possible constraints set and the algorithms used to obtain an optimal solution. In [5], TR technique that uses adaptive projected subgradient method to obtain dummy symbols to minimize PAPR of each symbol is proposed, while in [6], a subgradient optimization-based framework for iterative PAPR reduction is proposed. The approach in [6] minimizes the peak magnitude of the OFDM symbol vector by using tone values of the reserved subcarriers which are iteratively updated through a subgradient search. The algorithms have very simple update rules and low computational complexities. However, the number of updates required for a satisfactory peak level tends to be high.

In this paper a method that mixes TR technique and phase information of the pilot tones to efficiently reduce the PAPR of the MIMO-OFDM signals is presented. First, transmitters that reduce the PAPR by inserting dummy symbols in unused subcarriers in the active band is proposed. The optimal power distribution to the dummy symbols is determined by the solution of a convex optimization problem. Then, we utilize cross entropy (CE) optimization techniques proposed in [10] to design phase information to the pilot tones primarily dedicated for channel estimation [1], [11] to further reduce the PAPR.

To corroborate the effectiveness of our proposed method, the PAPR performance is evaluated using the complementary cumulative distribution function (CCDF). Simulation results show that, by mixing TR technique and phase information of the pilot tones, the PAPR of the MIMO-OFDM signals can be significantly reduced.

II. MIMO-OFDM Signals and PAPR Analysis

We consider MIMO-OFDM wireless system with N_t transmit antennas and N subcarriers for each OFDM symbol. For one OFDM symbol duration, we denote the transmitted OFDM symbol from the ith transmit antenna as

$$\boldsymbol{X}_i = [X_{i,0}, X_{i,1}, \ldots, X_{i,N-1}]^T. \tag{1}$$

At the transmitter, each $\boldsymbol{X}_i$ undergoes serial-to-parallel (S/P) followed by $\mathcal{L}N$-points inverse discrete Fourier transform (IDFT) to produce the $\mathcal{L}$-times oversampled-time domain baseband signals expressed as

$$\boldsymbol{x}_i = \boldsymbol{F}\boldsymbol{X}_i \tag{2}$$

where $\boldsymbol{F}$ is an $\mathcal{L}N \times N$ DFT matrix with

$$F_{t,k} = \frac{1}{\sqrt{\mathcal{L}N}} e^{j\frac{2\pi kt}{\mathcal{L}N}}, \quad t \in [0, \mathcal{L}N-1], \quad k \in [0, N-1]. \quad (3)$$

Note that, $\mathcal{L}$ denotes the oversampling factor sufficient to make the signal $\boldsymbol{x_i}$ as close as possible to the continuous signal [3].

The PAPR of the transmitted signal in (2) is defined as

$$PAPR_i = \frac{||\boldsymbol{x}_i||_\infty^2}{E\{|\boldsymbol{x}_i|^2\}}, \quad (4)$$

where $E\{|\boldsymbol{x}_i|^2\}$ is the average power of the signal and $E\{\cdot\}$ denotes expectation operation. Note that, $||\boldsymbol{x}_i||_\infty$ is the infinity norm of the time domain signals. The above definition clarifies that the PAPR is the maximum instantaneous power normalized by the average power among all possible signal patterns.

To avoid non-linear distortion in the power amplifiers and in turn the generation of undesired out-of-band radiation, the PAPR of all N_t transmit signals should be simultaneously as small as possible [12]. Since the performance is governed by the worst-case PAPR, we define $PAPR_{MIMO}$ as the maximum of all PAPR related to all N_t MIMO paths [2], [12]. Thus,

$$PAPR_{MIMO} = \max_{i=1,\ldots,N_t} PAPR_i. \quad (5)$$

Note that, PAPR is a random variable, hence the suitable description is the complementary cumulative distribution function (CCDF), which gives the probability Υ_o of exceeding a specified threshold γ, i.e.,

$$\Upsilon_o = Pr[PAPR > \gamma]. \quad (6)$$

III. PAPR Reduction using TR Techniques and Pilot Tones

For an OFDM symbol that consists of unused subcarriers, pilot and data subcarriers, we can utilize phase information of the pilot tones together with a certain number of unused subcarriers to reduce the PAPR of the OFDM symbol. The tone reservation (TR) technique makes use of some reserved or unused subcarriers and insert dummy symbols that can simultaneously minimize the peak levels of the OFDM signal $||\boldsymbol{x}_i||_\infty$. In line with the TR technique, careful design of the phase information to the pilot tones can substantially minimize the peak levels of the time domain OFDM signal.

In this section we propose a method that combines TR techniques and phase information of the pilot tones to significantly reduce the PAPR of the time domain OFDM signal $\boldsymbol{x}_i$. We resort to the pilot tones designed in [1], [11] for channel estimation in MIMO-OFDM systems with null edge subcarriers.

For a discrete set $\mathcal{I}$, we denote $|\mathcal{I}|$ as the cardinality of $\mathcal{I}$. Let $\mathcal{K}_s$ be the set of active subcarriers. As in [11], the number of pilot tones in each OFDM symbol is denoted as N_p. For an OFDM symbol transmitted from the ith transmit antenna, pilot and data symbols are located at subcarrier sets denoted as $\mathcal{K}_{p_i}$ and $\mathcal{K}_{d_i}$, respectively. To avoid interference between pilot tones from different antennas, $\mathcal{K}_{p_i}$ for $i = 1, 2, \ldots, N_t$ are set to be disjoint, hence

$$\mathcal{K}_{p_i} \cap \mathcal{K}_{p_n} = \emptyset \text{ for } i \neq n, \quad (7)$$

and we define a set of pilot tones for all transmit antennas as

$$\mathcal{K}_p = \left(\mathcal{K}_{p_1} \cup \mathcal{K}_{p_2} \ldots \cup \mathcal{K}_{p_{N_t}}\right). \quad (8)$$

Note that, there are no pilot tones at $\mathcal{K}_{d_i}$, [1], [11] that is,

$$\mathcal{K}_{d_i} \subsetneqq \mathcal{K}_s \setminus \mathcal{K}_p \quad (9)$$

where $\setminus$ denotes set difference. Eq. (9) implies that all transmit antennas utilize the same set of subcarriers for carrying data. Let us denote the positions of pilots symbols as

$$\mathcal{K}_{p_i} = \{i_1 \ldots, i_{N_p}\}. \quad (10)$$

For each antenna transmitting an OFDM symbol with $\mathcal{K}_s$ active subcarriers, only K_{p_i} and $\mathcal{K}_{d_i}$ are used for pilot and data transmission, while subcarriers corresponding to the pilot tones of the other transmit antennas are not used. Thus to transmit data symbols, it is necessary to satisfy the condition $|\mathcal{K}_s| - N_p N_t > 0$.

A. Tone Reservation PAPR Reduction Method

Since for each transmit antenna link there are $N_r = |K_p \setminus K_{p_i}|$ number unused subcarriers within the active subcarrier band, then by sending dummy symbols (i.e., symbols not conveying information) in these subcarriers, the PAPR of the OFDM symbol can be reduced. We denote a set of unused subcarriers in the ith transmitter as

$$\mathcal{K}_{r_i} \subsetneqq \mathcal{K}_p \setminus \mathcal{K}_{p_i}. \quad (11)$$

Suppose that, we define $\boldsymbol{F_d}$, $\boldsymbol{F_{p_i}}$ and $\boldsymbol{F_{r_i}}$ as the DFT submatrix of $\boldsymbol{F}$ corresponding to $\mathcal{K}_d$, $\mathcal{K}_{p_i}$ and $\mathcal{K}_{r_i}$ subcarriers respectively. Then, we can decompose the expression in (2) as

$$\boldsymbol{x}_i = \boldsymbol{F}_d \boldsymbol{X}_{d_i} + \boldsymbol{F}_{p_i} \boldsymbol{X}_{p_i} + \boldsymbol{F}_{r_i} \boldsymbol{X}_{r_i} \quad (12)$$

where $\boldsymbol{X}_{d_i}$, $\boldsymbol{X}_{p_i}$ and $\boldsymbol{X}_{r_i}$ are the vectors containing data, pilot and dummy symbols respectively.

To reduce the PAPR we need to minimize $||\boldsymbol{x}_i||_\infty$, which is equivalent to the peak of the signal $\boldsymbol{x}_i$. The peak minimization problem can be written as

$$\min_{\boldsymbol{X}_{r_i}, \boldsymbol{\theta}_i} ||\boldsymbol{x}_i||_\infty = \min_{\boldsymbol{X}_{r_i}, \boldsymbol{\theta}_i} ||\boldsymbol{F}_d \boldsymbol{X}_{d_i} + \boldsymbol{F}_{p_i} \boldsymbol{X}_{p_i}(\boldsymbol{\theta}_i) + \boldsymbol{F}_{r_i} \boldsymbol{X}_{r_i}||_\infty \quad (13)$$

where

$$\boldsymbol{X}_{p_i} = \boldsymbol{X}_{p_i}(\boldsymbol{\theta}_i) = \left[|X_{i,i_1}|e^{j\theta_{i,i_1}}, \ldots, |X_{i,i_{N_p}}|e^{j\theta_{i,i_{N_p}}}\right]^T \quad (14)$$

with $\boldsymbol{\theta}_i = [\theta_{i,i_1}, \ldots, \theta_{i,i_{N_p}}]$ is an N_p vector containing phase information of the pilot tones in the ith transmit antenna.

Optimization of equation (13) is a difficult problem. Thus, we split it into two parts. First we will consider the optimization of the dummy symbols $\boldsymbol{X}_{r_i}$ to reduce the peaks in the data signals $\boldsymbol{X}_{d_i}$ as best as possible, and later we will consider phase design to the pilot tones to further reduce the PAPR. The first objective function can be expressed as

$$\Gamma(\boldsymbol{X}_{r_i}) = \min_{\boldsymbol{X}_{r_i}} ||\boldsymbol{F}_d \boldsymbol{X}_{d_i} + \boldsymbol{F}_{r_i} \boldsymbol{X}_{r_i}||_\infty. \quad (15)$$

Only $\boldsymbol{X}_{r_i}$ is allowed to change, thus we need to introduce some constraints to describe the desired characteristics of the convex set $\boldsymbol{X}_{r_i}$ and the signal amplitude constraints to limit the PAPR to an acceptable level.

Practically, one cannot select arbitrary values for $\boldsymbol{X}_{r_i}$, since they should obey the power spectral density (PSD) constraints imposed by the standards for the spectral compatibility reasons (see [3], [5], [6]). Therefore, the average power levels of the reserved tones are constrained by the PSD mask levels. Apart from the PSD constraints in frequency domain, we can add the peak power reduction requirement in time domain as well. Thus, we can write the peak minimization problem as the constrained optimization problem

$$\begin{aligned} \underset{\boldsymbol{X}_{r_i}}{\text{minimize}} \quad & \Gamma(\boldsymbol{X}_{r_i}) \\ \text{subject to} \quad & |\boldsymbol{X}_{r_i}| \leq \lambda, \\ & \Gamma(\boldsymbol{X}_{r_i}) \leq \rho ||\boldsymbol{F}_d \boldsymbol{X}_{d_i}||_\infty \end{aligned} \quad (16)$$

where λ is a PSD mask level constraint in frequency domain and ρ is a fraction value representing the desired peak level to be attained. The convex optimization problem in (16) can be efficiently solved by the using the cvx optimization package in [13]. It should be remarked that our two stages optimization may not converge to the optimal solution of the original problem at the cost of the reduction of computational complexity.

Note that, satisfying the PSD constraints in frequency domain and the desired peak magnitude constraints in time domain may become a contradictory requirements. To meet PSD mask and the desired PAPR reduction depends on the number of the dummy symbols as well as its placements. In strict condition where minimum number of dummy symbols are inserted, the peak level constraint can be relaxed, that is, we set $\rho = 1$.

Practically, the power allocated to subcarriers with pilot tones is higher than that of the data subcarriers, thus, we can mitigate interference introduced to the pilot tones by the dummy symbols by allocating lower power than that of the data subcarriers to the dummy symbols. There is a trade off between BER degradation due to interference caused by the dummy symbols and its effectiveness in PAPR reduction.

The proposed method can be used to design dummy symbols to a given set of null edge subcarriers to reduce PAPR and thereby mitigate the interference effects. However this will increase the spectrum or bandwidth of the OFDM symbol.

B. Pilot Tones based PAPR Reduction Method

For channel or carrier frequency offset (CFO) estimation, it is sufficient to design the placement and power of each pilot symbol, and there are no particular requirements for the phase information. Thus, we can design phases to a given set of pilot tones to enhance PAPR reduction. To ensure that pilot tones have low peaks we will design phase that lower the peak amplitudes of the pilot tones.

We consider a set of pilot tones designed for channel estimation of the received OFDM symbol from the ith transmit antenna and design plausible phase information to counteract the PAPR effects. The time domain representation of the pilot signals can be expressed as where,

$$\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i) = \boldsymbol{F}_{p_i} \boldsymbol{X}_{p_i}(\boldsymbol{\theta}_i). \quad (17)$$

The peak amplitude of the signal $\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i)$ is expressed as

$$\max |\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i)|. \quad (18)$$

Our objective is to design phases $\boldsymbol{\theta}_i$ of the pilot tones so that we can minimize the maximum of the absolute value of $\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i)$, i.e, to find the optimal $\boldsymbol{\theta}_i{}^*$ which given by

$$\boldsymbol{\theta}_i{}^* = \arg \min_{\boldsymbol{\theta}_i \in \boldsymbol{\phi}} \max |\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i)| \quad (19)$$

where $\boldsymbol{\phi}$ is a vector of length N_p whose elements are between 0 and 2π (or equivalently, $-\pi$ and π). Let γ_i^* be the minimum peak of the signal $|\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i)|$, then

$$\gamma_i^* = \min_{\boldsymbol{\theta}_i \in \boldsymbol{\phi}} \max |\boldsymbol{x}_{p_i}(\boldsymbol{\theta}_i)|. \quad (20)$$

Phase design to reduce PAPR is a non-convex and non-linear optimization problem [10]. The problem (20) is partially solved in [14], where equi-spaced and equi-powered pilot tones are considered, and exhaustive search is employed to obtain $\boldsymbol{\theta}_i$. However, the exhaustive search scheme becomes computationally prohibitive especially for pilot sets with a large number of subcarriers. Furthermore, the performance of the scheme depends on the search granularity. In [10], the problem is addressed by the cross entropy optimization techniques. Hence, we resort to the later approach to obtain the solution to this non-convex and non-linear optimization problem for multiple transmit antenna case.

IV. Simulation Results and Discussion

We present computer simulation results to demonstrate the effectiveness of the combined tones reservation (TR) technique and phase information of the pilot tones in PAPR reduction. We adopt the disjoint pilot tones presented in [1], [11], primarily used for channel estimations in MIMO-OFDM systems. An OFDM transmission frame with $N = 256$ (see in [15, p.429]) is considered. In a data-carrying symbol 200 subcarriers are used for data and pilot tones. Of the other 56 subcarriers, 28 are null in the lower frequency guard band, 27 are nulled in the upper frequency guard band and one is the null central DC subcarrier.

The PAPR complementary cumulative distribution functions (CCDF) of an uncoded OFDM signal are computed with an oversampling factor of $\mathcal{L} = 8$ and are obtained by considering 10^3 independent OFDM symbols. The data subcarriers are loaded with QPSK signals.

Fig. 1 depicts the CCDF curves of PAPR performance of an

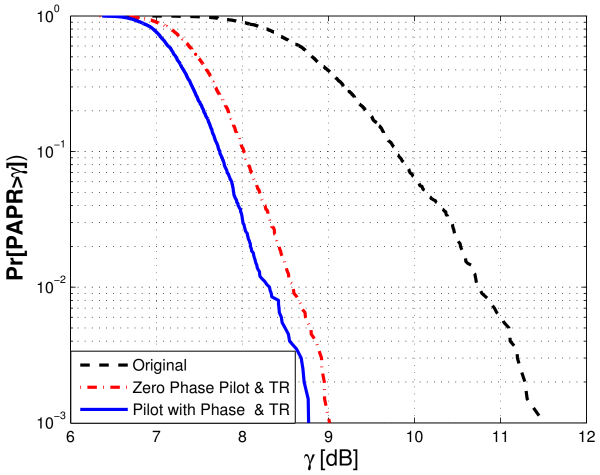

Fig. 1. CCDF performance of the TR and pilot phase information for $N_p = 8$ and $N_t = 4$.

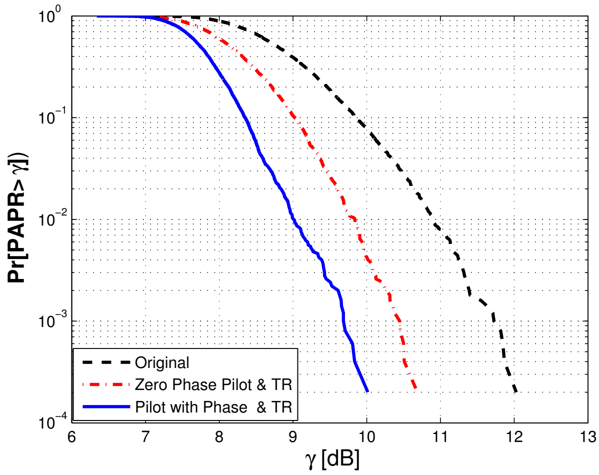

Fig. 2. CCDF performance of the TR and pilot phase information for $N_p = 16$ and $N_t = 2$.

OFDM system with $N_t = 4$ and $N_p = 8$ in each transmitter. The results show relatively lower PAPR for the OFDM symbols that utilize dummy symbols to reduce the PAPR over the data and pilot tones without any modifications. By combining phase information of the pilot tones with dummy symbols, the PAPR can further be reduced. From the figure it is clear that, there is slight improvement in PAPR reduction for the combined dummy symbols and phase information as compared to the dummy symbols only. This verifies the potential of our proposed combined technique in minimizing the PAPR.

Next we consider an OFDM system with $N_t = 2$ and $N_p = 16$. Fig. 2 depicts the CCDF curves of the PAPR reduction for the combined dummy symbols and pilot phase information, dummy symbols and the original (data and pilot tones with no phase information) signals. For the pilot tones with phases, the PAPR (i.e., $PAPR_{MIMO}$) of the OFDM symbol is significantly minimized. This PAPR reduction is obtained without any spectrum loss since no any side information is required. This suggests that, our proposed method can be used together with the TR or other PAPR reduction techniques to significantly reduce the PAPR of the OFDM symbol.

When $N_p = 8$, there is no significant difference between the combined (TR and phase information pilot tones) and the TR techniques with zero phase pilot. This is because PAPR reduction is governed mainly by the dummy symbols when the number of pilot tones is relatively small. Likewise, position and number of dummy symbols may lead to a lower PAPR, however, in our simulation example, there is no choice as to which tones shall be used for PAPR reduction rather than data transmission. If the system is able to freely choose, the distribution of these dummy symbols over the system bandwidth, more PAPR reduction may be possible.

References

[1] Q. Huang, M. Ghogho, and S. Freear, “Pilot design for MIMO OFDM systems with virtual carriers,” *IEEE Transactions on Signal Processing*, vol. 57, no. 5, pp. 2024–2029, 2009.

[2] B. Rihawi, Y. Louet, and S. Zabre, “PAPR reduction scheme with SOCP for MIMO-OFDM,” in *Proc. Int. Conf. Wireless Communications, Networking and Mobile Computing WiCom 2007*, 2007, pp. 271–274.

[3] I. M. Mahafeno, Y. Louet, and J.-F. Helard, “Peak-to-average power ratio reduction using second order cone programming based tone reservation for terrestrial digital video broadcasting systems,” *IET Communications*, vol. 3, no. 7, pp. 1250–1261, 2009.

[4] K. C. Chen, K. A. Morris, and M. A. Beach, “Increasing the power efficiency of an IEEE 802.11a power amplifier,” in *Proc. VTC 2005-Spring Vehicular Technology Conference 2005 IEEE 61st*, vol. 2, May 30–Jun. 1, 2005, pp. 954–957.

[5] R. Cavalcante and I. Yamada, “A flexible peak-to-average power ratio reduction scheme for OFDM systems by the adaptive projected subgradient method,” *IEEE Transactions on Signal Processing*, vol. 57, no. 4, pp. 1456–1468, 2009.

[6] A. Erdogan, “A low complexity multicarrier PAR reduction approach based on subgradient optimization,” *Signal Processing*, vol. 86, no. 12, pp. 3890–3903, Dec. 2006.

[7] J. Tellado and J. M. Cioffi, “Efficient algorithms for reducing PAR in multicarrier systems,” in *Proc. IEEE International Symposium on Information Theory*, Aug. 16–21, 1998, p. 191.

[8] S. Janaaththanan, C. Kasparis, and B. G. Evans, “A gradient based algorithm for PAPR reduction of OFDM using tone reservation technique,” in *Proc. IEEE Vehicular Technology Conf. VTC Spring 2008*, 2008, pp. 2977–2980.

[9] D. Guel, J. Palicot, and Y. Loue, “Tone reservation technique based on geometric method for orthogonal frequency division multiplexing peak-to-average power ratio reduction,” *IET Communications*, vol. 4, no. 17, pp. 2065–2073, 2010.

[10] E. Manasseh, S. Ohno, and M. Nakamoto, “Design of low PAPR preamble and pilot symbol for channel estimation in OFDM systems,” *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 1, pp. 39–50, Jan 2011.

[11] ——, “Pilot symbol design for channel estimation in MIMO-OFDM systems with null subcarriers,” in *European Signal Processing Conference*, Aug. 2010, pp. 1612–1616.

[12] R. F. H. Fischer and M. Hoch, “Peak-to-average power ratio reduction in MIMO OFDM,” in *Proc. IEEE Int. Conf. Communications ICC ’07*, 2007, pp. 762–767.

[13] S. Boyd and L. Vandenberghe, “Convex optimization,” *Cambridge University Press*, 2004.

[14] S. Hosokawa, K. Teo, S. Ohno, and T. Hinamoto, “Pilot tone design with low peak-to-average power ratio in OFDM,” *IEICE Trans. Fundamentals*, vol. E88-A, no. 8, pp. 2117–2123, Aug. 2005.

[15] *IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface for Fixed Broadband Wireless Access Systems*, IEEE Std., 2004.