MIMO Detection Performance for Realistic Rician Fading of Estimated Statistics

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***Abstract*—The multiple-input multiple-output (MIMO) technique can greatly improve coverage and throughput in wireless networks. However, varying channel parameters create unfavorable conditions that make the transmission less reliable. Unfortunately, many previous MIMO evaluations suffer from limited practical relevance due overly-simplified assumption about the channel fading model and parameters. Therefore, it is important to match reality and thus acquire a more accurate understanding of MIMOs expected performance and challenges. In this paper, we investigate MIMO performance when the transmit and receive antennas experience correlation and when the channel gain matrix and its statistics (i.e., statistical channel state information — SCSI) are estimated. Furthermore, we investigate the effect of realistic random and correlated azimuth spread (AS) and K-factor. Numerical results for typical urban scenario demonstrate the performance difference between the unrealistic and realistic assumptions about channel knowledge and fading model.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems is a major breakthrough in providing faster and reliable wireless communication links. Space–time wireless technology that uses multiple antennas along with appropriate signaling and receiver techniques offers a powerful tool for improving wireless performance [1]. However, these gains may not be always fully achievable in practice due to channel condition [2], i.e., Rayleigh fading vs. Rician fading, independent fading parameter vs. correlated fading parameter, and transceiver impairments (instantaneous channel state information (ICSI) estimation error) [3].

Although many previous works have assumed perfect instantaneous and statistical channel information (ICSI, SCSI), in practice, the channel and its statistics have to be estimated. A common approach to MIMO channel estimation is to insert pilot symbols at the transmitter and to estimate the MIMO channel based on pilot samples at the receiver. Research has shown that ICSI estimation accuracy affects space-time decoding performance [4] [5] [6]. Different ICSI estimation methods offer different tradeoffs between performance and knowledge of statistical channel state information (SCSI) [7]. Accurate and efficient channel estimation is crucial for MIMO detection.

Furthermore, MIMO performance analyses have typically assumed Rayleigh fading channel, although Rician fading, which allows for specular or line-of-sight (LOS) propagation, has more often actually been measured — see for example the thorough WINNER II channel models [8].

In this paper, for spatially correlated Rician and Rayleigh fading, we employ estimated ICSI and SCSI for the low-complexity zero forcing (ZF) detection approach [9]. ICSI is estimated based on the minimum mean square error (MMSE) criterion. Since the MMSE ICSI estimate requires SCSI knowledge, the least square (LS) ICSI estimate is employed to estimate the channel correlation matrix and noise variance. For comparison purposes, our numerical results also show the ZF average symbol error rate (SER) for perfect ICSI and SCSI.

The rest of this paper is organized as follows. SectionsII and III introduce the structure of Rician fading MIMO channel model and its spatial correlation, respectively. SectionIV analyzes the ICSI and SCSI estimation. SectionV introduces the ZF detection method. Finally, SectionVI shows numerical results.

Notation

Scalars, vectors, and matrices are represented in lowercase italics, boldface lowercase, and boldface uppercase, respectively, e.g., x, $\mathbf{x}$, and $\mathbf{X}$; $\psi \sim \mathcal{N}(0,1)$ indicates that scalar ψ is a real-valued random variable of Gaussian distribution with zero-mean and unit variance; subscripts $\cdot_{\mathrm{d}}$ and $\cdot_{\mathrm{r}}$ identify, respectively, the deterministic (mean) and random components of a scalar or vector; index $\cdot_{\mathrm{n}}$ indicates a normalized variable; $i = 1 : N$ stands for the enumeration $i = 1, 2, \ldots N$; the superscripts $\cdot^T$ and $\cdot^H$ stand for transpose and Hermitian (complexconjugate) transpose; $[\cdot]_i$ and $[\cdot]_{i,j}$ indicate the ith and i,jth element of a vector and a matrix, respectively; $\mathbb{E}\{\cdot\}$ denotes statistical average.

II. Channel Model

For Rician fading, the channel matrix is:

$$\begin{aligned}\mathbf{H} &= \mathbf{H}_{\text{d}} + \mathbf{H}_{\text{r}} \\ &= \sqrt{\frac{K}{K+1}}\mathbf{H}_{\text{d,n}} + \sqrt{\frac{1}{K+1}}\mathbf{H}_{\text{r,n}},\end{aligned} \tag{1}$$

where $\mathbf{H}_{\text{d}}$ and $\mathbf{H}_{\text{r}}$ are the deterministic and random components of the channel matrix, respectively. $\mathbf{H}_{\text{d,n}}$ and $\mathbf{H}_{\text{r,n}}$ are their normalized value, with $||\mathbf{H}_{\text{d,n}}||^2 = N_{\text{T}}N_{\text{R}}$, and $\mathbb{E}\{|\,[\mathbf{H}_{\text{r,n}}]_{i,j}\,|^2\} = 1$, $\forall i = 1 : N_{\text{R}}, j = 1 : N_{\text{T}}$. Note that the Rician K-factor is the power ratio of the deterministic and random components of the channel, as

$$K = \frac{|[\mathbf{H}_{i,j}]|^2}{\mathbb{E}\{|[\mathbf{H}_{i,j}]|^2\}}. \tag{2}$$

Then, for correlated antennas, the Kronecker channel model is often employed, i.e., [1]:

$$\mathbf{H}_{\text{r,n}} = \mathbf{R}_{\text{Rx}}^{1/2}\ \mathbf{H}_w\ \mathbf{R}_{\text{Tx}}^{1/2}, \tag{3}$$

where $\mathbf{H}_w$ is a $N_{\text{R}} \times N_{\text{T}}$ is a matrix with zero-mean, i.i.d. Gaussian entries, and $\mathbf{R}_{\text{Rx}}$ is a $N_{\text{R}} \times N_{\text{R}}$ and $\mathbf{R}_{\text{Tx}}$ is a $N_{\text{T}} \times N_{\text{T}}$ describe receive and transmit correlation, respectively. In our current evaluation, we disregard receive correlation. Finally, we define $\mathbf{R}_{\text{H}} = \frac{1}{K+1}tr(\mathbf{R}_{\text{Rx}})\mathbf{R}_{\text{Tx}}$.

III. AS and K-Factor Statistics

WINNER II measurements for a wide range of scenarios have indicated that channel fading can be modeled as Rician with lognormally distributed and correlated AS and K-factor [8]. The numerical results shown later in this paper are for scenario C2 from [8]. For this scenario the AS and K statistical models are described in Table I, where $\chi, \psi \sim \mathcal{N}(0,1)$, and ρ is the correlation coefficient of χ and ψ. Depending on the scenario, this correlation can be negative, zero, or positive. For the numerical results shown in this paper, it has been imposed with:

$$\psi = \rho\chi + \sqrt{1-\rho^2}\omega, \tag{4}$$

where $\omega \sim \mathcal{N}(0,1)$ is independent of χ.

Our method to produce one side (transmitter or receiver) lognormally distributed and correlated AS and K-factor samples is:

Step 1 – Generate independent random, zero-mean, unit-variance χ and ω samples. These variables are independent.

TABLE I
AS and K statistics for scenario C2 (Typical urban macrocell)

	Tx	Rx
AS[°]	$10^{1.00+0.25\chi}$	$10^{1.7+0.19\chi}$
K	$10^{0.1(7+3\psi)}$	$10^{0.1(7+3\psi)}$
ρ	+0.1	-0.2

Step 2 – Generate samples of the zero-mean, unit-variance random variable ψ with (4)

Step 3 – Generate AS and K samples with equations given in Table I.

IV. Channel Estimation

A. ICSI Estimation

The received signal vector for the P pilot samples can be arranged into the $N_{\text{R}} \times P$ matrix [1]:

$$\mathbf{Y}_{\text{p}} = \mathbf{H}\mathbf{X}_{\text{p}} + \mathbf{N}, \tag{5}$$

where $\mathbf{X}$ is the transmitted pilot symbol matrix and $\mathbf{N}$ is the white noise matrix, whose elements are independent and identically distributed (i.i.d.) and have variance N_0.

The MMSE estimator requires knowledge of N_0 and $\mathbf{R}_{\text{H}}$ and is given by [10]:

$$\widehat{\mathbf{H}}_{\text{MMSE}} = \mathbf{Y}_{\text{p}}(\mathbf{X}_{\text{p}}^H\mathbf{M}\mathbf{X}_{\text{p}} + N_{\text{R}}N_0\mathbf{I}_p)^{-1}\mathbf{X}_{\text{p}}^H\mathbf{M}, \tag{6}$$

where

$$\mathbf{M} = \mathbf{H}_{\text{d}}^H\mathbf{H}_{\text{d}} + \mathbf{R}_{\text{H}}. \tag{7}$$

The mean square error (MSE) of the MMSE estimate is given by [7]:

$$\begin{aligned}J_{\text{MMSE}} &= \mathbb{E}\{||\mathbf{H} - \widehat{\mathbf{H}}_{\text{MMSE}}||_F^2\} \qquad (8)\\ &= tr\{(\mathbf{R}_{\text{H}}^{-1} + \frac{1}{N_0N_{\text{R}}}\mathbf{X}\mathbf{X}^H)^{-1}\}. \qquad (9)\end{aligned}$$

B. SCSI Estimation

We exploit the LS ICSI estimate on $i = 1 : N_S$ consecutive slots in a frame (with same AS and K) to average over the fading. Then, the SCSI estimates are given by[10]:

$$\widehat{\mathbf{H}}_{\text{LS},i} = \mathbf{Y}_{\text{p,i}}\mathbf{X}_{\text{p,i}}^H(\mathbf{X}_{\text{p,i}}\mathbf{X}_{\text{p,i}}^H)^{-1} \tag{10}$$

$$\widehat{\mathbf{H}}_{\text{d,n}} = \frac{1}{N_S}\sum_{i=1}^{N_S}\widehat{\mathbf{H}}_{\text{LS},i}, \tag{11}$$

$$\widehat{\mathbf{R}}_{\text{H}} = \frac{\widehat{K}+1}{\sqrt{\alpha}N_S}\sum_{i=1}^{N_S}\left[\left(\widehat{\mathbf{H}}_{\text{LS},i}\right)^H\left(\widehat{\mathbf{H}}_{\text{LS},i}\right) - \left(\widehat{\mathbf{H}}_{\text{d,n},i}\right)^H\left(\widehat{\mathbf{H}}_{\text{d,n},i}\right)\right], \tag{12}$$

where

$$\widehat{K} = \frac{||\widehat{\mathbf{H}}_{\mathrm{d,n}}||_F^2}{\frac{1}{N_S}\sum_{i=1}^{N_S}||\widehat{\mathbf{H}}_{\mathrm{LS},i} - \widehat{\mathbf{H}}_{\mathrm{d,n}}||_F^2} \tag{13}$$

is the K-factor estimate, and

$$\alpha = \frac{\widehat{K}+1}{N_S}\sum_{i=1}^{N_S}(||\widehat{\mathbf{H}}_{\mathrm{LS},i}||_F^2 - ||\widehat{\mathbf{H}}_{\mathrm{d,n}}||_F^2). \tag{14}$$

Now, the MMSE ICSI estimate from (6) can be computed.

V. MIMO ZF Detection

Letting $\mathbf{x} = [x_1, x_2, \cdots, x_{N_\mathrm{T}}]^T$ denote the N_T -dimensional vector with the transmitted symbols (M-PSK modulation, $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_{N_\mathrm{T}}$), the N_R-dimensional vector with the received signals can be represented as [1]:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}, \tag{15}$$

For perfect ICSI, the ZF detection weight matrix $\mathbf{W}$ is given by the pseudo-inverse of channel matrix $\mathbf{H}$, i.e.,

$$\mathbf{W} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H. \tag{16}$$

Then, the ZF symbol vector estimate $\hat{\mathbf{X}}$ is [9]:

$$\begin{aligned}\widehat{\mathbf{X}} &= \mathbf{W}\mathbf{Y}\\ &= (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{Y}\\ &= \mathbf{X} + (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{N},\end{aligned} \tag{17}$$

i.e., ZF detection attempts to eliminate the interstream interference.

Given the estimate $\hat{\mathbf{H}}$ of $\mathbf{H}$, ZF detectionestimates the symbol transmitted through the kth antenna by mapping the kth element of the N_T-dimensional vector [11] [9] [12]

$$\widehat{\mathbf{W}}\,\mathbf{Y} = \left[\widehat{\mathbf{H}}^H\,\widehat{\mathbf{H}}\right]^{-1}\,\widehat{\mathbf{H}}^H\,\mathbf{X} + \left[\widehat{\mathbf{H}}^H\,\widehat{\mathbf{H}}\right]^{-1}\,\widehat{\mathbf{H}}^H\,\mathbf{N}. \tag{18}$$

into the closest modulation constellation symbol (i.e., slicing) [11]. Note that this approach employs the channel matrix estimate as if it were the true channel matrix [1].

VI. Numerical Results

We first evaluate the normalized mean square estimation error (NMSE) for the estimation performance measurement, given by:

$$\mathrm{NMSE} = \frac{\mathbb{E}\{||\mathbf{H} - \widehat{\mathbf{H}}||_F^2\}}{||\mathbf{H}||_F^2}. \tag{19}$$

TABLE II
Simulation parameters

Scenario	C2
Tx antennas	4
Rx antennas	4
Pilot symbols	16
Data symbols	1000
$\mathbf{H}_\mathrm{d}$ rank	rank one
AS-K samples	1000
Modulation	QPSK

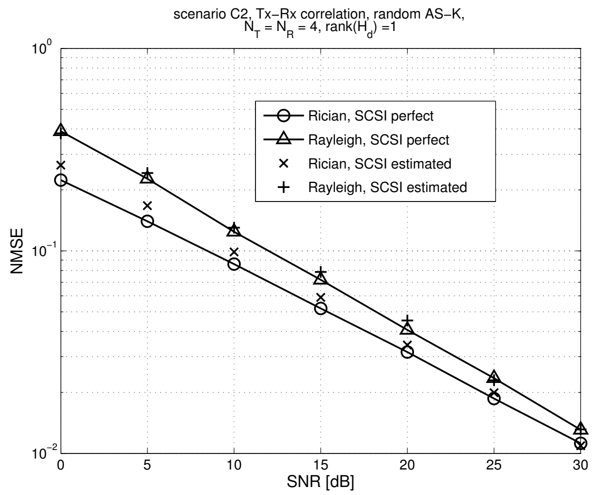

Fig. 1. NMSE for 4×4 MIMO MMSE channel estimation, for scenario C2 with random AS and K, for both Tx and Rx correlation.

Then, the SER is computed as a measure of detection performance. NMSE and SER are shown vs. signal-to-noise ratio (SNR $= \frac{Es}{N_0}$). Our simulation parameters are listed in Table II. The deterministic channel matrix component has rank one, which is practically more realistic [1].

Fig. 1 depicts estimation performance for random AS and K as in the WINNER II C2 scenario. Note that the MMSE ICSI estimate is more accurate for Rician fading than for Rayleigh fading. However, MMSE estimate accuracy degrades for estimated vs. perfect SCSI more for Rician fading than for Rayleigh fading. For example, for Rician fading, accuracy degrades by about 2 dB at NMSE level 10^{-1}. However, for SNR higher than about 20 dB, differences are negligible. Finally, compared with results for transmit-only correlation from [13], both transmit and receive correlation yields lower ICSI estimation error, for estimated SCSI.

Fig. 2 depicts symbol detection performance for per-

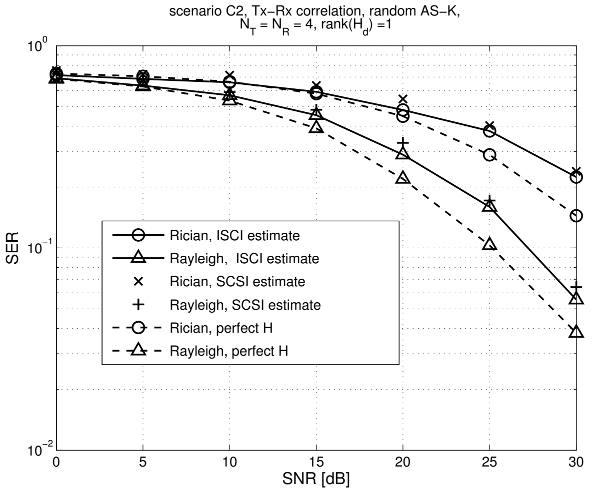

Fig. 2. Mean (over AS and K) SER performance for 4×4 MIMO ZF detection, for scenario C2 with random AS and K, for both Tx and Rx correlation.

fect ICSI and SCSI, estimated ICSI and perfect SCSI, and for estimated ICSI and SCSI. Interestingly, the Rician vs. Rayleigh relative performance is reversed for ZF detection than for MMSE channel estimation. Rician fading yields poorer SER performance than Rayleigh fading, due to the rank-one deterministic channel component. Thus, the LOS in Rician fading helps yield a better channel estimate but no better detection performance. On the other hand, ICSI estimation degrades SER performance significantly whereas SCSI estimation does not degrade performance much.

VII. Conclusions

This paper evaluates the MIMO channel estimation and symbol detection performance for various assumptions about the channel fading and ICSI and SCSI knowledge. Transmit and receive antenna correlation has been considered. AS and K are modeled as lognormally distributed and correlated. It is discovered that for Rayleigh fading (which may be considered unrealistic), estimated SCSI yields nearly the same MMSE ICSI estimation accuracy as perfect SCSI. On the other hand, for more realistic, i.e., Rician, channel fading, SCSI estimation can significantly degrade the MMSE ICSI estimate accuracy. Finally, although ICSI estimation inaccuracy can significantly degrade ZF detection performance, we observed negligible performance degradation due to SCSI estimation. Since SCSI-based channel estimates are known to yield better detection performance than SCSI-independent estimates, and since SCSI can be obtained at low cost, we conclude that MIMO detection with realistic MMSE ICSI is feasible.

References

[1] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, UK: Cambridge University Press, 2005.

[2] C. Siriteanu and S. D. Blostein, “Performance of smart antenna arrays with maximal-ratio eigen-combining,” Queen’s University, Department of Electrical and Computer Engineering, Kingston, Ontario, Canada, Tech. Rep. 0401, Jan. 2004. [Online]. Available: http://ipcl.ee.queensu.ca

[3] C. Siriteanu, X. Shi, and Y. Miyanaga, “MIMO zero-forcing detection performance for correlated and estimated Rician fading with lognormal azimuth spread and K-factor,” in *IEEE International Conference on Communications, (ICC’11)*, accepted 2011.

[4] A. Assalini, E. Dall’Anese, and S. Pupolin, “On the robustness of MIMO LMMSE channel estimation,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3313 –3319, november 2010.

[5] F. Dietrich and W. Utschick, “Pilot-assisted channel estimation based on second-order statistics,” *IEEE Transactions on Signal Processing*, vol. 53, no. 3, pp. 1178 – 1193, march 2005.

[6] H. Nooralizadeh, S. Moghaddam, and H. Bakhshi, “Optimal training sequences in MIMO channel estimation with spatially correlated Rician flat fading,” in *Industrial Electronics Applications, 2009. ISIEA 2009. IEEE Symposium on*, vol. 1, 4-6 2009, pp. 227 –232.

[7] M. Biguesh and A. Gershman, “Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals,” *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 884 – 893, Mar. 2006.

[8] P. Kyosti, J. Meinila, L. Hentila, and *et al.*, “WINNER II Channel Models.” 2008.

[9] C. Wang, E. Au, R. Murch, W. H. Mow, R. Cheng, and V. Lau, “On the performance of the MIMO zero-forcing receiver in the presence of channel estimation error,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 3, pp. 805 –810, march 2007.

[10] G. Taricco and G. Coluccia, “Optimum receiver design for correlated rician fading MIMO channels with pilot-aided detection,” *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 7, pp. 1311 –1321, Sep. 2007.

[11] C. Wang, E. K. S. Au, R. D. Murch, W. H. Mow, R. S. Cheng, and V. Lau, “On the performance of the MIMO zero-forcing receiver in the presence of channel estimation error,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 3, pp. 805 –810, Mar. 2007.

[12] K. Nobandegani and P. Azmi, “Effects of inaccurate training-based minimum mean square error channel estimation on the performance of multiple input-multiple output vertical bell laboratories space-time zero-forcing receivers,” *Communications, IET*, vol. 4, no. 6, pp. 663 –674, 16 2010.

[13] X. Shi, C. Siriteanu, and Y. Miyanaga, “MIMO channel estimation performance for realistic processing and propagation conditions,” in *The 26th International Technical Conference on Circuits/Systems, Computers and Communications, (ITC-CSCC 2011)*, accepted 2011.