



# A Flexible ICA-Based Method for AEC Without Requiring Double-Talk Detection

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*Abstract*—In this paper, we propose the use of the flexible independent component analysis (ICA) algorithm for the acoustic echo cancellation (AEC). The flexible ICA algorithm has a parametric score function which is controlled by the Gaussianity of the source signal (speech in our case). The probability density function of the speech signal is not always the super Gaussian, hence the inflexible score function used in the existing ICAbased AEC system is not always suitable. On the other hand, the proposed method is able to always provide the suitable score function depending on the present Gaussianity of the speech signal. The experimental results indicate that the proposed method significantly outperforms the existing ICA-based AEC algorithm and the variable step-size (VSS)-NLMS algorithm.

## I. INTRODUCTION

The face-to-face video calls feature of the latest handheld communication devices has changed our communication style. Instead of putting our phone next to our ear, we would set the loudspeaker on. However, the echo generated by reflection from the walls disrupts our conversation, therefore acoustic echo cancellers are necessary. The basic acoustic echo canceller (AEC) system originated from the network echo cancellation system where echo is caused by the unbalanced impedance matching between the two-wire loop and the fourwire loop in the telephone network [1]. In this case, adaptive filters of 32 ms in length are usually sufficient to give an adequate echo cancellation. On the other hand, the acoustic echo path is extremely long (can be in order of 125 ms) [1], and it may change at any time due to any movement of the phone user (for example, walking), therefore a fast-adapting AEC algorithm is required.

The simple normalized least mean square (NLMS) algorithm is commonly used to update the adaptive filter coefficients in the echo-cancellation system. In the presence of double-talk, the NLMS algorithm may become unstable and diverge [1]. To handle this problem, the filter adaptation should be halt when the double-talk is detected, and hence the doubletalk detector (DTD) is needed. There are several DTD algorithms for AEC: Geigel algorithm, cross-correlation method, and coherence method (see [1] and references therein). Nevertheless, there is some delay in the decision of a DTD; during this small delay, a few undetected large amplitude samples can perturb the echo path estimate considerably [2]. In order to avoid this perturbation of echo path model, several DTDfree AEC methods have recently been reported [2]-[5]. There are mainly two types of DTD-free AEC methods: one method uses a variable step-size (VSS) [2], [3], and the other method is based on the independent component analysis (ICA) [4], [5]. The first method is a generalization of the NLMS algorithm which estimates the power of the near-end signal, and hence its performance depends on the estimation accuracy. However, its performance is limited because it is using only up to second-order statistics (correlation-based method). On the other hand, the second method is based on independence (stronger statistics than correlation). It employs a semi-blind structure of the time-domain (TD) ICA (feedback network). However, the basic TD-ICA system has a slow convergence problem, therefore this method also has the same problem.

In this paper, we attempt to improve the existing ICA-based AEC system by employing flexible ICA [6]. The probability density function of the speech signal is not always the super Gaussian [7]. Therefore, the inflexible score function (i.e.,  $tanh(\cdot)$ ) is not always suitable. On the other hand, the flexible ICA has a parametric score function which is controlled by the Gaussianity of the source signal. In the original formulation of the flexible ICA algorithm, the score function is derived from the generalized Gaussian distribution (GGD) function. In [8], we have proposed generalized Cauchy distribution (GCD)-based score-function for flexible ICA. The GCD has the property of having heavy tails which makes it suitable for modelling a temporal signal such as speech. In the simulations, we evaluate both the GGD-based and the GCD-based flexible ICA algorithms and compare their performance with the original ICA-based AEC algorithm [4]. [5] and the VSS-NLMS algorithm [2], [3].

The rest of the paper is organized as follows. Section 2 describes the basic AEC model and gives an overview of several AEC methods. In Section 3, the flexible ICA-based AEC methods are proposed. The simulation results are presented in Section 4 followed by the concluding remarks presented in Section 5.

#### II. AEC MODEL AND EXISTING METHODS

The basic system model for echo cancellation is shown in Fig. 1, where v(n) and x(n) denote the near-end signal (NES) and far-end signal (FES), respectively, and H(z) denotes transfer function of acoustic room impulse response (RIR). Assuming that H(z) is modeled as an FIR filter of length L,



Fig. 1. Basic system model for echo cancellation.

the echo signal y(n) is given as

$$y(n) = \mathbf{h}^T \mathbf{x}(n), \tag{1}$$

where  $\mathbf{h} = [h_0 \quad h_1 \quad \cdots \quad h_{L-1}]$  is the coefficient vector for H(z) and  $\mathbf{x}(n) = [x(n) \quad x(n-1) \quad \cdots \quad x(n-L+1)]^T$  is the input vector comprising L recent samples of recorded FES  $\mathbf{x}(n)$ . With  $\eta(n)$  denoting the ambient noise, the output of the near-end microphone is given as

$$d(n) = v(n) + y(n) + \eta(n),$$
 (2)

which acts as a desired response for the adaptive filter  $\hat{H}(z)$ , modelling echo path filter H(z). Taking FES x(n) as its input, the error signal for  $\hat{H}(z)$  is generated as

$$e(n) = d(n) - \hat{y}(n), \tag{3}$$

where

$$\hat{y}(n) = \hat{\mathbf{h}}^T(n)\mathbf{x}(n), \tag{4}$$

is the replica of far-end echo, and  $\hat{\mathbf{h}}(n) = [\hat{h}_0(n) \ \hat{h}_1(n) \ \cdots \ \hat{h}_{L-1}(n)]^T$  is the tap weight vector for  $\hat{H}(z)$ . The coefficients of  $\hat{H}(z)$  are adapted using NLMS algorithm as

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu(n)\mathbf{x}(n)e(n), \tag{5}$$

where

$$\mu(n) \equiv \mu_{\text{NLMS}}(n) = \frac{\tilde{\mu}}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta}$$
(6)

is the time-varying step-size for NLMS algorithm,  $\tilde{\mu}$  is a fixed step-size chosen experimentally, and  $\delta$  is a small positive constant to avoid division by zero.

The NLMS algorithm holds a single-talk scenario assumption, i.e., v(n) = 0. In the presence of NES, this assumption is not valid, therefore during a double-talk condition the NLMS algorithm diverges. In order to solve this problem, Paleologu, *et al.* [2], [3], proposed a variable step-size NLMS (VSS-NLMS) algorithm. The practical learning algorithm of the

VSS-NLMS is given as,

$$\mu_{\text{VSS}}(n) = \frac{\tilde{\mu}}{\delta + \mathbf{x}^T(n)\mathbf{x}(n)} \left| 1 - \frac{\sqrt{|\hat{\sigma}_d^2(n) - \hat{\sigma}_{\hat{y}}^2(n)|}}{\zeta + \hat{\sigma}_e^2(n)} \right|, \quad (7)$$

where  $\zeta$  is a small positive constant to prevent division by zero. The  $\hat{\sigma}_d^2(n)$ ,  $\hat{\sigma}_{\hat{y}}^2(n)$ , and  $\hat{\sigma}_e^2(n)$  are the power estimates of near-end microphone output d(n), echo replica  $\hat{y}(n)$ , and error signal e(n), respectively, and are computed as

$$\hat{\sigma}_d^2(n+1) = \lambda \hat{\sigma}_d^2(n) + (1-\lambda)d^2(n), \tag{8}$$

$$\hat{\sigma}_{\hat{y}}^2(n+1) = \lambda \hat{\sigma}_{\hat{y}}^2(n) + (1-\lambda)\hat{y}^2(n), \tag{9}$$

$$\hat{\sigma}_e^2(n+1) = \lambda \hat{\sigma}_e^2(n) + (1-\lambda)e^2(n).$$
 (10)

where  $\lambda$  is a weighting factor chosen as  $\lambda = 1 - 1/(KL)$ , with K > 1.

Another interesting approach is described in [4], [5] where the basic idea is to put system identification in the framework of ICA. In this approach, instead of (3), error signal is generated as

$$e(n) = a(n)d(n) - \hat{y}(n),$$
 (11)

where a(n) is a nonzero scalar quantity which in the usual adaptive filtering problem is set to 1. Then, instead of (5), a new natural gradient-based update rules are given as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_1 \left[\varphi(e(n))\mathbf{x}(n) + \left\{1 - \varphi(e(n))e(n)\right\} \hat{\mathbf{h}}(n)\right],$$
(12)

$$a(n+1) = a(n) + \mu_2[1 - \varphi(e(n))e(n)]a(n),$$
(13)

where  $\varphi(\cdot)$  is called the score function and is calculated as

$$\varphi(e(n)) = -\frac{\partial \log p(e(n))}{\partial e(n)},\tag{14}$$

and in [4], [5] the  $tanh(\cdot)$  function is used. Hereafter, natural gradient ICA-based AEC algorithm is referred as NG ICA-AEC.

#### III. FLEXIBLE ICA-BASED AEC

In ICA, the choice of the score function  $\varphi(\cdot)$  plays a crucial role in determining the performance of the algorithm [9]. Due to this, we put our focus on finding the best score function to improve the ICA-based AEC system. We consider that the commonly used  $tanh(\cdot)$  function does not give a good enough solution to the problem. The function  $tanh(\cdot)$  is a common choice for super Gaussian source signals. however the probability density function of the speech signal is not always the super Gaussian [7]. Hence, in this paper, we propose the use of the flexible ICA to solve the AEC problem.

The idea of the flexible ICA is to provide a parametric score function which is controlled by the Gaussianity of the source signal. To measure the Gaussianity of the source signal, the following online kurtosis estimation is adopted.

$$\kappa(n) = \frac{M_4(n)}{\hat{M}_2^2(n)} - 3,$$
(15)

where  $\hat{M}_2$  and  $\hat{M}_4$  are the estimation of the second- and forth-order moments, respectively. These estimates can be recursively computed as

$$\hat{M}_2(n+1) = \lambda \hat{M}_2(n) + (1-\lambda)e^2(n)$$
(16)

$$\hat{M}_4(n+1) = \lambda \hat{M}_4(n) + (1-\lambda)e^4(n).$$
(17)

There are two parametric score-functions which are discussed in this paper. One is the generalized Gaussian distribution (GGD)-based score-function, which is provided in the original flexible ICA algorithm [6]. The other one is the generalized Cauchy distribution (GCD)-based score-function which we had already proposed [8].

# A. GGD-Based Score Function

The probability density function (PDF) for the generalized Gaussian distribution is given as [6]

$$p(e(n);\gamma(n)) = \frac{\gamma(n)}{2\hat{\sigma}_e(n)\Gamma(\frac{1}{\gamma(n)})} \exp\left(-\left|\frac{e(n)}{\hat{\sigma}_e(n)}\right|^{\gamma(n)}\right)$$
(18)

where  $\Gamma(\cdot)$  is Gamma function given by

$$\Gamma(x) = \int_0^\infty t^{x-1} \exp(-t) dt.$$
 (19)

Inserting (18) into the score-function formula (14), we obtain the GGD-based score-function formula [6]

$$\varphi(e(n)) = |e(n)|^{\gamma(n)-1} \operatorname{sgn}(e(n))$$
(20)

Note that, for  $\gamma(n) = 1$ , (20) becomes a sgn(·) function, and thus  $\varphi(e(n)) = \text{sgn}(e(n))$ , which can be derived from the Laplacian density model for sources. For  $\gamma(n) = 2$ , (20) becomes a linear function, and thus  $\varphi(e(n)) = e(n)$ , which can be derived from the Gaussian density model for sources. For  $\gamma(n) = 4$ , (20) becomes a cubic function which is known to be a good choice for sub Gaussian signals [6]. Depending upon kurtosis  $\kappa(n)$  estimation in (15), the shape parameter  $\gamma(n)$  is chosen as

$$\gamma(n) = \begin{cases} 1; & \kappa(n) \ge 0 \quad \text{(super Gaussian)} \\ 4; & \kappa(n) < 0 \quad \text{(sub Gaussian).} \end{cases}$$
(21)

Hereafter, GGD-based algorithm is referred as Flexible ICA-AEC1.

#### B. GCD-Based Score Function

Instead of the generalized Gaussian distribution given in (18), we proposed the use of a generalized Cauchy distribution to derive a new score-function [8]. The GCD has a PDF given by,

$$p(e(n), \hat{\sigma}_e, q(n), n) = g(n)\hat{\sigma}_e(\hat{\sigma}_e^{q(n)} + |e(n)|^{q(n)})^{-\frac{2}{q(n)}}$$
(22)

where  $\hat{\sigma}_e$  denotes the standard deviation of error signal, q(n) is the shape parameter, and g(n) is given as

$$g(n) = \frac{q(n)\Gamma\left(\frac{2}{q(n)}\right)}{2\left(\Gamma\left(\frac{1}{q(n)}\right)\right)^2},$$
(23)

TABLE I SUMMARY OF PROPOSED FLEXIBLE ICA-BASED ALGORITHMS FOR DTD-FREE AEC

Initialize: $a(n), K, \hat{M}_2(0), \hat{M}_4(0), \text{ and } \hat{\mathbf{h}}(0).$								
while $\{x(n), v(n)\}$ is available do								
1.	$\hat{y}(n) = \hat{\mathbf{h}}^T(n)\mathbf{x}(n),$							
2.	$e(n) = a(n)d(n) - \hat{y}(n),$							
3.	$\hat{M}_2(n+1) = \lambda \hat{M}_2(n) + (1-\lambda)e^2(n),$							
	$\hat{M}_4(n+1) = \lambda \hat{M}_4(n) + (1-\lambda)e^4(n),$							
	$\kappa(n)=rac{\hat{M}_4(n)}{\hat{M}_2^2(n)}$							
4.	$\gamma^*(n) = \begin{cases} 1; & \kappa(n) \ge 0  \text{(super Gaussian)} \\ 4; & \kappa(n) < 0  \text{(sub Gaussian).} \end{cases}$							
	$q^{+}(n) = \begin{cases} 1; & \kappa(n) \ge 0 \text{ (super Gaussian)} \\ 5; & \kappa(n) < 0 \text{ (sub Gaussian),} \end{cases}$							
5.	Compute score-function $\varphi(e(n))$							
	- (20) for Flexible ICA-AEC 1,							
	- (24) for Flexible ICA-AEC 2.							
6.	Update $\hat{H}(z)$ : $\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu \mathbf{x}(n)\varphi(e(n))$ .							
end while								

\* : shape parameter for GGD,

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+ : shape parameter for GCD.
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where  $\Gamma(\cdot)$  is the gamma function (given in (19)). Substituting (22) in (14), the GCD-based score-function is obtained as

$$\varphi(e(n)) = 2(\hat{\sigma}_e^{q(n)} + |e(n)|^{q(n)})^{-1} |e(n)|^{q(n)-1} \operatorname{sgn}(e(n))$$
(24)

Depending upon kurtosis  $\kappa(n)$  estimation in (15), the shape parameter q(n) is chosen as

$$q(n) = \begin{cases} 1; & \kappa(n) \ge 0 \quad \text{(super Gaussian)} \\ 5; & \kappa(n) < 0 \quad \text{(sub Gaussian)}, \end{cases}$$
(25)

where values assigned for this cases are decided empirically. Hereafter, GCD-based algorithm is referred as Flexible ICA-AEC2.

We found that the natural gradient given in (12) causes the convergence rate to be slow. We also set the scalar quantity a(n) to 1 (fixed) as in the usual adaptive filtering, therefore the learning rule for a(n) given in (13) can be eliminated., Based on these, we apply the following usual gradient ICA algorithm instead.

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu \varphi(e(n)) \mathbf{x}(n).$$
(26)

Hereafter, usual gradient ICA-based AEC algorithm is referred as UG ICA-AEC. As a reference, Table I gives summary of proposed flexible ICA-based methods for DTD-free AEC.

# IV. EXPERIMENTAL RESULTS AND DISCUSSION

Simulations are carried out in MATLAB. For the FES signal, two kinds of signals are used: speech signal and Gaussian signal of the same variance, and for the NES signal only speech signal is used. The length of each signal is 60 seconds (sampling frequency is 8 kHz) and the waveplots are given in Fig. 2. An echo path of length L = 512 (64 ms) is generated using MATLAB's room impulse response (RIR) generator [11] and is shown in Fig. 3. An independent

SUMMARY OF SIMULATION PARAMETERS FOR VARIOUS ALGORITHMS STUDIED IN THIS PAPER							
	Gradient	Parameters					
NLMS	usual (5)	$\hat{\mathbf{h}}(0) = 0, \ \mu(n) = \mu_{\text{NLMS}}(n) \ (6), \ \tilde{\mu} = 1, \ \delta = 30\sigma_x^2$					
VSS-NLMS [3]	usual (5)	$\hat{\mathbf{h}}(0) = 0, K = 2, \lambda = 1 - 1/(KL), \mu(n) = \mu_{\text{VSS}}(n)$ (7), $\tilde{\mu} = 4 \times 10^{-1}, \delta = 30\sigma_x^2$ ,					
		$\hat{\sigma}_d^2(0) = 0, \ \hat{\sigma}_{\hat{y}}^2(0) = 0, \ \hat{\sigma}_e^2(0) = 0, \ \eta = 10^{-8}.$					
NG ICA-AEC [4]	natural (12)	$\hat{\mathbf{h}}(0) = 0, a(0) = 1, \varphi(e(n)) = \tanh(e(n)), \mu_1 = 10^{-2}, \mu_2 = 10^{-2}$					
UG ICA-AEC	usual (26)	$\hat{\mathbf{h}}(0) = 0,  a(n) = 1,  \varphi(e(n)) = \tanh(e(n)),  \mu = 10^{-2}$					
	usual (26)	$\hat{\mathbf{h}}(0) = 0,  a(n) = 1,  \varphi(e(n)) = \operatorname{sgn}(e(n)),  \mu = 10^{-3}$					
Flexible ICA-AEC1	usual (26)	$\hat{\mathbf{h}}(0) = 0, a(n) = 1, K = 2, \lambda = 1 - 1/(KL), \hat{M}_2(0) = 10^{-2}, \hat{M}_4(0) = 10^{-4}, \varphi(e(n))$ : GGD (20),					
		$\mu = 10^{-3}.$					
Flexible ICA-AEC2	usual (26)	$\hat{\mathbf{h}}(0) = 0, a(n) = 1, K = 2, \lambda = 1 - 1/(KL), \hat{M}_2(0) = 10^{-2}, \hat{M}_4(0) = 10^{-4}, \varphi(e(n))$ : GCD (24),					
		$\mu = 4 \times 10^{-5}$					

 TABLE II

 Summary of simulation parameters for various algorithms studied in this pape



Fig. 2. Waveplots for various signals used in computer simulations: [a] FES (speech), [b] FES (Gaussian), and [c] NES (speech)



Fig. 3. Impulse response of echo path H(z) used in simulations

Gaussian signal  $\eta(n)$  is added to the echo signal y(n) with 20 dB signal-to-noise ratio (SNR) for all the experiments. Table II details simulation parameters for various algorithms investigated in this paper.

After an AEC system has converged, the condition

$$\hat{\mathbf{h}} \approx a\mathbf{h}$$

should be fulfilled. Hence, as a performance measure, the

following normalized filter misalignment (NFM) is used.

$$\operatorname{NFM}(n) = 20 \log \left( \frac{\|a(n)\mathbf{h} - \hat{\mathbf{h}}(n)\|_2}{\|a(n)\mathbf{h}\|_2} \right) [d\mathbf{B}], \qquad (27)$$

where  $\|\cdot\|_2$  denotes the  $l_2$  (Euclidean) norm.

### A. Case 1

In this case, the influence of natural gradient and the adaptive scale parameter a(n) given in (12) and (13) is evaluated. Simulations are conducted using speech FES for the NG ICA-AEC and the UG ICA-AEC algorithms. The NFM(n) are calculated for both methods at each iteration and the results are shown in Fig. 4. The natural gradient (NG) method is relatively a more stable algorithm compared to the usual gradient (UG) method: its obtained NFM(n) shows less striking changes than what obtained by the usual gradient. However, it also has a drawback of having slower convergence than the usual gradient. Therefore, based on this result, the choice of using the usual gradient in our proposed flexible ICA-based AEC method is made.

The probability density function of the speech signal is not always super Gaussian [7]. To demonstrate this, simulations are conducted using speech FES and speech NES signals for both our proposed flexible ICA-AEC1 method and the UG ICA-AEC method employing  $sgn(\cdot)$  score function. From (20), it can be seen that when  $\gamma(n) = 1$  the GGD-based score function is equal to  $sgn(\cdot)$  score function. Hence, if a speech signal was always super Gaussian, flexible ICA-AEC1 method and the UG ICA-AEC method employing  $sgn(\cdot)$  score function should give exactly the same performance measured by the NFM(n). However, the evolution of NFM(n) obtained by the two methods are different as shown in Fig. 5, which implies that speech signal is not always super Gaussian. It can also be observed from the figure that for most of the time, the flexible ICA-AEC1 method gives smaller error than the UG ICA-AEC employing  $sgn(\cdot)$  score function. This is because the flexible ICA has an on-line Gaussianity measure and chooses



Fig. 4. Evolution of NFMs(n) obtained under Case 1 scenario by NG ICA-AEC [4], [5] and UG ICA-AEC algorithms versus the number of iterations



Fig. 5. Evolution of NFMs(n) obtained under Case 1 scenario by UG ICA-AEC with sgn $(\cdot)$  score-function and Flexible ICA-AEC1 algorithms versus the number of iterations.

a suitable score-function based on the measurement. Since the performance of gradient-based ICA algorithm depends on the score function selection based on source distribution, the flexible ICA-AEC with its adaptive score functions gives a better performance than the UG ICA-AEC.

#### B. Case 2

In this case, Gaussian FES is used (FES is speech) and the evolution of NFMs(n) obtained by NLMS, VSS-NLMS, NG ICA-AEC, Flexible ICA-AEC1 and Flexible ICA-AEC2 algorithms are shown in Fig. 6. The NLMS algorithm, despite of having the fastest convergence, becomes unstable and diverges in the presence of NES (double-talk condition). The NG ICA-AEC algorithm [4], [5], on the other hand, has the slowest convergence and in the steady-state condition, it has a fluctuating performance shown by drastic changes in its NFM(n) along the iteration axis. The proposed Flexible ICA-AEC1 and Flexible ICA-AEC2 methods have a faster convergence and are more stable than the NG ICA-AEC [4], [5] method. Nevertheless, the best performance is obtained by the VSS-NLMS algorithm [2], [3] as it gives the lowest



Fig. 6. Evolution of NFMs(n) obtained under Case 2 scenario by NLMS, VSS-NLMS [2], [3], NG ICA-AEC [4], [5], Flexible ICA-AEC1, and Flexible ICA-AEC2 algorithms versus the number of iterations



Fig. 7. Evolution of NFMs(n) obtained under Case 3 scenario by NLMS, VSS-NLMS [2], [3], NG ICA-AEC [4], [5], Flexible ICA-AEC1, and Flexible ICA-AEC2 algorithms versus the number of iterations

error and a convergence rate almost as fast as the NLMS algorithm. However, in the presence of NES, it shows drastic changes along the iteration axis as can be seen in Fig. 6. This superior performance of VSS-NLMS algorithm can be explained by the ICA point of view. According to (20), when  $\gamma(n) = 2$ , it follows that  $\varphi(e(n)) = e(n)$  and it is a linear function which can be derived from the Gaussian density model for sources. The VSS-NLMS algorithm, just like the other LMS-type algorithms, are using this score-function and hence are good for applications involving Gaussian signals. The VSS-NLMS even has a variable step-size which has a role of controlling its convergence speed, therefore it has fast convergence. However, in practical AEC scenarios, FES and NES are speech signals and are rarely Gaussian as studied in next experiment.

# C. Case 3

In this case, both FES and NES are speech and the evolution of NFMs(n) obtained by NLMS, VSS-NLMS, NG ICA-AEC, Flexible ICA-AEC1 and Flexible ICA-AEC2 algorithms are

	Step of Algorithm	Eq. Number	No. of Multipliers	No. of Adders	Another Function	Total Cost
NLMS	error calculation	(3) & (4)	L	L	-	3L + 2
	step size	(6)	L+1	1	-	multipliers
	score function	-	-	-	-	and $2L+1$
	filter update	(5)	L+1	L	-	adders
VSS-NLMS [3]	error calculation	(3) & (4)	L	L	-	3L + 16
	step size	(7)-(10)	L + 15	10	$\operatorname{sqrt}(\cdot)$	multipliers,
	score function	-	-	-	-	2L + 10 adders,
	filter update	(5)	L+1	L	-	and a $sqrt(n)$
NG ICA-AEC [4]	error calculation	(11) & (4)	L+1	L	-	4L + 5
	step size	constant	-	-	-	multipliers
	score function	tanh()	-	-	$\tanh(\cdot)$	and $3L+3$
	filter update	(12) & (13)	3L + 4	2L + 3	-	adders
Flexible ICA-AEC1	error calculation	(3) & (4)	L	L	-	2L + 10 or
	step size	constant	-	-	-	2L + 13
	score function	(15)-(17),(20), $\gamma(n) = 1$	9	5	-	multipliers
		(15)-(17),(20), $\gamma(n) = 4$	12	5	-	and $2L + 10$
	filter update	(26)	L+1	L	-	adders
Flexible ICA-AEC2	error calculation	(3) & (4)	L	L	-	2L + 12 or
	step size	constant	-	-	-	2L + 17
	score function	(15)-(17),(24), q(n) = 1	11	6	$\operatorname{sqrt}(\cdot)$	multipliers,
		(15)- $(17)$ , $(24)$ , $q(n) = 5$	16	6	$\operatorname{sqrt}(\cdot)$	2L+12 adders
	filter update	(26)	L+1	L	-	and a $sqrt(\cdot)$ .

 TABLE III

 Summary of The Computational Complexity of Each AEC Methods Used in The Comparison

<sup>a</sup> In the table, L denotes the length of the input buffer,

<sup>b</sup> There are many methods to compute  $tanh(\cdot)$  and  $sqrt(\cdot)$  and the complexity depends on the method used.

shown in Fig. 7. Even on this case, the NLMS algorithm has the fastest convergence and diverges in the presence of NES (double-talk situation). The NG ICA-AEC method [4], [5] has the slowest convergence but it has minor fluctuations on its error performance. The VSS-NLMS [2], [3], on the other hand, has a better performance compared to the NG ICA-AEC method explained by having lower error rate and faster convergence. Nevertheless, the best performance are given by our proposed Flexible ICA-AEC1 and Flexible ICA-AEC2 methods. The proposed methods, have faster convergence and lower error rate compared to both the NG ICA-AEC and the VSS-NLMS algorithms. Moreover, the Flexible ICA-AEC2 is relatively more stable than the Flexible ICA-AEC1 as it gives less fluctuations on the NFM(n). This is because the GCD has a characteristic of having heavy-tails, thus it matches the distribution of impulsive signal (speech in our case) better than GGD. However, concerning the convergence rate, our proposed method is still slower than the NLMS algorithm (before the presence of NES) and hence we would like address this issue in our future work.

# D. Computational Complexity

Table III shows the summary of the computational complexity of each AEC methods used in the comparison. We can see from the table that the VSS-NLMS has the highest computational complexity among the five methods because it includes the computation of the step size of NLMS algorithm and three moving averages given in (8), (9), and (10). The second highest computational cost is given by the NG ICA-AEC algorithm because its natural gradient ICA-based filter update rule has a more complex equation. It also has an update rule for the scalar quantity a(n) and the score function formula  $tanh(\cdot)$  which computational complexity depends on the computational method used. The third highest computational complexity is given by the NLMS algorithm because at every iteration this algorithm calculates  $\mathbf{x}^T \mathbf{x} + \delta$  which requires L multipliers and L adders. The proposed Flexible ICA-AEC1 and Flexible ICA-AEC2 on the other hand has the lowest computational cost among the five algorithms. The score function formula of both methods are indeed complex but each of them only applies to the scalar quantity e(n). Moreover among the two proposed methods, the Flexible ICA-AEC2 method has a slightly higher computational cost than the Flexible ICA-AEC1 method for it has a more complex score function equation which requires more multipliers, adders and the  $sqrt(\cdot)$  function.

#### V. CONCLUDING REMARKS

In this paper, we have proposed Flexible ICA-based AEC algorithms. In simulations involving Gaussian far-end signal (FES), our proposed Flexible ICA-AEC1 and Flexible ICA-AEC2 methods outperform the NG ICA-AEC method [4], [5] but underperform the VSS-NLMS algorithm [2], [3]. This is because the VSS-NLMS and the other LMS-type algorithms are good for applications involving Gaussian signals (see Case 2 in Section IV). However, in practical AEC scenarios, where FES is speech signal and rarely Gaussian, our proposed methods outperform both the NG ICA-AEC and the VSS-NLMS algorithms. Moreover, the Flexible ICA2 is more stable than the Flexible ICA1 as it shows less fluctuations on its NFM(n) (see Case 3 in Section IV). In the future, we would like to improve the convergence rate of our proposed methods.

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