Maximum Likelihood Method for RFID Tag Set Cardinality Estimation using Multiple Independent Reader Sessions

Chuyen T. NGUYEN*, Kazunori HAYASHI*, Megumi KANEKO*, Petar POPOVSKI† and Hideaki SAKAI*
* Graduate School of Informatics, Kyoto University, Kyoto, Japan
E-mail: ntchuyen@sys.kyoto-u.ac.jp, kazunori@i.kyoto-u.ac.jp, meg@i.kyoto-u.ac.jp, hsakai@i.kyoto-u.ac.jp
† Department of Electronic Systems, Aalborg University, Aalborg, Denmark
E-mail: petarp@es.aau.dk

***Abstract*—In this paper, Radio Frequency IDentification (RFID) tag set cardinality estimation problem is considered under the model of multiple independent reader sessions with unreliable radio communication links in which transmission errors might occur. After the R-th reader session, the number of tags detected in j ($j = 1, 2, ..., R$) reader sessions is updated, which we call observed evidence. Then, in order to maximize the likelihood function of the number of tags and the transmission error probability given the observed evidences, we propose three different estimation methods depending on how to treat the discrete nature of tag set cardinality. Computer simulation results demonstrate the effectiveness of the proposed method.**

I. Introduction

One of the fundamental tasks in RFID systems is to estimate the total number of tags fast and reliably [1]. For example, in inventory management applications, the total number of products in a warehouse should be known. Also, the information of the tag set cardinality can be utilized in frame-slotted ALOHA based protocols for efficient tag identification [2]-[4]. In particular, Vogt method [2] minimizes the distance between the actual and theoretical reading results as represented by vectors (E, S, C) and $(\bar{E}, \bar{S}, \bar{C})$, where the elements of the vectors correspond to the number of empty, singleton and collision slots in the frame, respectively. Another simple estimation method is presented in [3] by assuming that the frame size is chosen to be equal to the tag set cardinality. Besides, [4] has proposed a method using Maximum a Posteriori (MAP) approach.

On the other hand, apart from the collision problem, transmission errors due to multi-path fading, obstacles in the radio path, blind spot phenomenon [5] or materials to which the tags are affixed [6], can be another limiting factor of the accuracy of the estimated tag set cardinality in RFID systems. Assuming transmission errors due to the channel, R. Jacobsen et al. [7] propose a practical method named *Remove Element Greater than Mean* (REGM) in which multiple independent reader sessions are employed. In particular, at the R-th reader session, appropriate weights of the observed evidences, which are defined as the number of tags detected in j ($j = 1, 2, ..., R$) reader sessions, are determined in order to estimate the transmission error probability and the total number of tags. Note that, the assumption of the independent reader sessions will be valid, for example, in time-selective fading environments, which could be seen in practical applications with moving target tags typically on conveyor belt systems.

In this paper, Maximum Likelihood (ML) approach is utilized to deal with the tag set cardinality estimation problem using the multiple independent reader sessions model in [7]. In particular, in order to maximize the likelihood function of the transmission error probability p and number of tags N given the current observed evidences, three different methods, namely, *Exhaustive ML (EML)*, *ML with Sample Mean for N (MLSM)*, and *ML with search Stopping Criterion (MLSC)* are proposed depending on the ways of treating the discrete nature of N. In EML method, the likelihood function is evaluated for all possible values of N, thus, it can achieve the best performance among all the methods but with high computational complexity due to the exhaustive search. In order to reduce the complexity, MLSM method is proposed as the simplest one. In this case, p is firstly estimated using a rough initial estimate of N, and then the estimate of N is updated by the sample mean of the observed evidences using the estimated p. In MLSC method, by assuming the continuous relaxation for N, the behavior of the likelihood function with respect to N is analyzed and a stop-searching criterion is given. The performance of the proposed methods is evaluated and compared with that of the conventional REGM method via computer simulations.

II. System Model and Conventional Approach

A. System Model

The considered RFID system consists of a reader and N unknown tags in the communication range where multiple independent reader sessions are performed. A reader session is defined as a reading round in which the reader broadcasts a message to all tags and receives responses from them. The transmission error probability p, which is the probability that a tag is not read in each reader session, is assumed to be unknown a priori and identical for all the tags. After R reading rounds, the number of tags observed in $R+1-j$ ($j = 1, .., R$) reader sessions k_j is updated, which is denoted as an observed

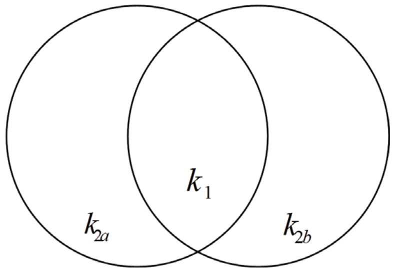

Fig. 1. Venn diagram of observed evidences ($R = 2$)

evidence. Our problem is to estimate the transmission error probability p and the tag set cardinality N using the observed evidences at each reader session.

Figure 1 shows a Venn diagram of the observed evidences for the case with two reader sessions, where k_1 is the number of tags that have been read in both reader sessions, and $k_{2\mathrm{a}}(k_{2\mathrm{b}})$ is the tag set cardinality that is read only in the first (second) reader session. In this case, k_1 and $k_2 = k_{2\mathrm{a}}+k_{2\mathrm{b}}$ are the observed evidences, which will be used for the estimation of N and p.

B. Conventional Approach-REGM Method [7]

For the case of two reader sessions ($R = 2$), REGM method utilizes the relations of

$$E[k_1] = Np_1 = N(1-p)^2, \quad (1)$$
$$E[k_2] = Np_2 = 2N(1-p)p, \quad (2)$$

where p_1 and p_2 are the probabilities of detecting a tag in both reader sessions and only one reader session respectively. Let $\hat{p}$ and $\hat{N}$ denote the estimates of p and N respectively obtained by replacing $E[k_j]$ with the observed evidence k_j $(j = 1, 2)$. Then, with the replacement, (1) and (2) become a set of linear equations of $\hat{p}$ and $\hat{N}$, which can be easily solved as

$$\hat{p} = \frac{k_2}{2k_1 + k_2}, \quad (3)$$

and

$$\hat{N} = \frac{k_1 + k_2}{1 - \hat{p}^2}. \quad (4)$$

The problem is also extended to the model with R (> 2) independent reader sessions in which the corresponding observed evidences are $k_1, k_2, ..., k_R$. In this case, N can be estimated for given $\hat{p}$ as

$$\hat{N} = \frac{\sum_{j=1}^{R} k_j}{1 - \hat{p}^R}, \quad (5)$$

while REGM estimates $\hat{p}$ by using a generalized form of (3) as

$$\frac{\sum_{j=1}^{R} \phi_{\mathrm{n}}(j) k_j}{\sum_{j=1}^{R} \phi_{\mathrm{d}}(j) k_j} = \frac{\sum_{j=1}^{R} \phi_{\mathrm{n}}(j) \binom{R}{R-(j-1)} (1-\hat{p})^{R-(j-1)} \hat{p}^{j-1}}{\sum_{j=1}^{R} \phi_{\mathrm{d}}(j) \binom{R}{R-(j-1)} (1-\hat{p})^{R-(j-1)} \hat{p}^{j-1}}. \quad (6)$$

Two window functions $\phi_{\mathrm{n}}(j)$ and $\phi_{\mathrm{d}}(j)$ determine the observed evidences used to compute $\hat{p}$, which are chosen as

$$\phi_{\mathrm{n}}(j) = \begin{cases} 1 & \text{if } w_j \neq 0, \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

and

$$\phi_{\mathrm{d}}(j) = \begin{cases} 1 & \text{if } w_j \neq 0 \text{ and } w_j < m_{\mathbf{w}}, \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where $\mathbf{w} = [w_1, w_2, ..., w_R]^T = \left[\frac{k_1}{\binom{R}{R}}, \frac{k_2}{\binom{R}{R-1}}, ..., \frac{k_R}{\binom{R}{1}} \right]^T$ and $m_{\mathbf{w}}$ is the sample mean of the nonzero elements in $\mathbf{w}$. The basic strategy here is to utilize the observed evidences with smaller normalized values except for zero elements, although no explicit justification is given in [7]. While it has been shown that the REGM can outperform the estimator based on [8], there might be a room to improve the estimation performance using the multiple reader sessions model, because (6) and the window functions are chosen in a rather heuristic manner.

III. Proposed Method

Here, we propose three different estimation methods using ML approach for the model of multiple independent reader sessions.

A. Likelihood Function

The probability of detecting a tag in $R+1-j$ reader sessions is given by

$$p_j = \binom{R}{R-(j-1)} (1-p)^{R-(j-1)} p^{(j-1)}. \quad (9)$$

Therefore, if random variables representing the observed evidences obtained after R reader sessions are denoted by $K_1, K_2, ..., K_R$, they will follow the multinomial distribution. Thus, the likelihood function of N and p can be represented as

$$\begin{aligned} P(k_1, ..., k_R | N, p) &= P(K_1 = k_1, ..., K_R = k_R | N, p) \\ &= \frac{N!}{k_1! k_2! \cdots k_{R+1}!} \prod_{j=1}^{R+1} p_j^{k_j}, \end{aligned} \quad (10)$$

where $p_{R+1} = 1 - p_1 - \cdots - p_R$ and $k_{R+1} = N - k_1 - \cdots - k_R$. Our problem is thereby equivalent to finding values of N, p, which maximize the log likelihood function as

$$(\hat{N}, \hat{p}) = \arg \max_{N \in \mathbb{N}, p \in [0,1]} \ln P(k_1, k_2, ..., k_R | N, p). \quad (11)$$

Setting the derivative of the log likelihood function with respect to p to be equal to zero, we obtain

$$\hat{p} = \frac{RN - \sum_{j=1}^{R} (R+1-j) k_j}{RN}. \quad (12)$$

On the other hand, due to the discrete nature of N, we cannot obtain an estimate $\hat{N}$ by using the same approach with derivation. Thus, in the sequel, we propose three different approaches depending on how to deal with the discrete nature of N.

B. Exhaustive ML (EML)

An exhaustive search algorithm will be employed in this method. In particular, by substituting $\hat{p}$ of (12) into p of the log likelihood function, we obtain

$$\begin{aligned}
&\ln P(k_1,...,k_R|N) \\
&= \ln N! - \ln\left(N-\sum_{j=1}^{R}k_j\right)! - \ln k_1!k_2!\cdots k_R! \\
&+ \sum_{j=1}^{R+1}k_j\ln\binom{R}{R-(j-1)} + \sum_{j=1}^{R}(R+1-j)k_j \\
&\times \ln\frac{\sum_{j=1}^{R}(R+1-j)k_j}{RN} + \left(RN-\sum_{j=1}^{R}(R+1-j)k_j\right) \\
&\times \ln\frac{RN-\sum_{j=1}^{R}(R+1-j)k_j}{RN}. \qquad (13)
\end{aligned}$$

Then, for each discrete value of N, the log likelihood function (13) is evaluated. In the search algorithm, the total number of tags observed in the reader sessions

$$\hat{N}_0 = \sum_{j=1}^{R}k_j, \qquad (14)$$

can be used as the minimum possible value of N. The optimal value $\hat{N}$ is the one that maximizes the likelihood function. Hence, EML method can achieve the best performance among the three methods, which will be further discussed via simulation results. However, this method requires a high computational complexity due to the exhaustive search.

C. ML with Sample Mean of N (MLSM)

To reduce the computational complexity, MLSM method is proposed in this section. In this method, $\hat{p}$ is firstly estimated by using (12) where $\hat{N}_0$ is used as a rough estimate of N. Then, $\hat{N}$ will be updated by the sample mean of the observed evidences using (5). MLSM method can obtain the estimates of $\hat{p}$ and $\hat{N}$ from the observed evidences with very low computational cost. Although there will be some performance degradation compared as EML, we can expect good performance for a large number of reader sessions R, since $\hat{N}_0$ will be close to the actual total number of tags.

D. ML with search Stopping Criterion (MLSC)

EML method requires a high computational complexity because in the exhaustive search algorithm, all possible values of N have to be evaluated. To overcome this inconvenience, in this section, we derive a stopping criterion of the search algorithm by analyzing the log likelihood function. In particular, by using the continuous relaxation, we can consider the derivative of (13) with respect to N as

$$\begin{aligned}
f_{\mathrm{P}}(N) &= \Psi(N+1) - \Psi\left(N-\sum_{j=1}^{R}k_j+1\right) \\
&+ R\ln\frac{RN-\sum_{j=1}^{R}(R+1-j)k_j}{RN} = f_{1\mathrm{P}}(N)+f_{2\mathrm{P}}(N),
\end{aligned} \qquad (15)$$

where

$$f_{1\mathrm{P}}(N) = \Psi(N+1) - \Psi(N-\hat{N}_0+1), \qquad (16)$$

$$f_{2\mathrm{P}}(N) = R\ln\left(1-\sum_{j=1}^{R}\frac{R+1-j}{RN}k_j\right), \qquad (17)$$

and $\Psi(x)$ is the digamma function [9] i.e., $\Psi(x) = \frac{d}{dx}\ln\Gamma(x)$ and $\Gamma(x) = (x-1)!$. By analysis, we obtain

- $\lim_{N\to\infty}\frac{-f_{1\mathrm{P}}(N)}{f_{2\mathrm{P}}(N)} < 1$, and hence $f_{\mathrm{P}}(N)$ has a negative value as $N\to\infty$.
- $\frac{df_{\mathrm{P}}(N)}{dN} < 0$ if $N \le N^*$, where

$$N^* = \frac{(\hat{N}_0-1)(R-1)(\hat{N}_0+\sum_{j=1}^{R}\frac{1-j}{R}k_j)}{R\hat{N}_0-\sum_{j=1}^{R}jk_j+1}. \qquad (18)$$

- $\frac{df_{\mathrm{P}}(N)}{dN} \ge 0$ if $N \ge N^{**}$, where

$$N^{**} = \frac{\hat{N}_0(R-1)(\hat{N}_0+\sum_{j=1}^{R}\frac{1-j}{R}k_j)}{R\hat{N}_0-\sum_{j=1}^{R}jk_j} \qquad (19)$$

Therefore, $f_{\mathrm{P}}(N)$ is monotonically decreasing for $N < N^*$ and is monotonically increasing for $N > N^{**}$. It is also easily verified that $N^{**} > N^* > 0$ and $N^{**} > \hat{N}_0$ while $N^* \approx N^{**}$ for sufficiently large $\hat{N}_0$.

In summary, assuming sufficiently large $\hat{N}_0$, we can conclude that $f_{\mathrm{P}}(N)$ has a unique minimum at $N^* \approx N^{**}$ in the region $N > 0$ and has a negative value as $N \to \infty$. This means that, if $f_{\mathrm{P}}(\hat{N}_0) > 0$, the equation $f_{\mathrm{P}}(N) = 0$ has a unique solution N^{***} where $N^{***} > \hat{N}_0$. In other words, the log likelihood function (13) is monotonically increasing with respect to $\hat{N}_0 \le N < N^{***}$ and then is monotonically decreasing with respect to $N > N^{***}$ after reaching to the maximum value at $N = N^{***}$. On the other hand, if $f_{\mathrm{P}}(\hat{N}_0) \le 0$, $f_{\mathrm{P}}(N)$ is negative in all the range $N \ge \hat{N}_0$ and hence, the log likelihood function is monotonically decreasing with respect to $N \ge \hat{N}_0$. It implies that the log likelihood function obtains the maximum value at $N = \hat{N}_0$. Thus, we evaluate the value of the log likelihood function for each discrete N in an increasing order starting from $N = \hat{N}_0$, and, if we observe the decrease of the likelihood, then N in the previous step is selected as the estimated value $\hat{N}$. We call this method MLSC.

IV. Simulation Results

In this section, we will show the performance of the proposed methods under different system parameters via computer simulations. The performance of the methods is also compared

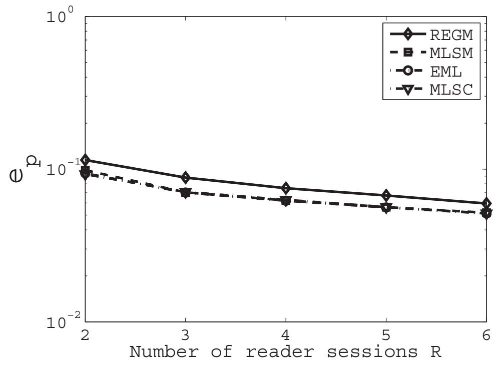

Fig. 2. RMSE of p ($p = 0.2,\ N = 10$)

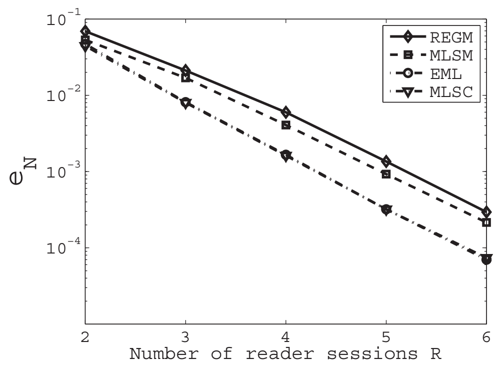

Fig. 3. Average normalized error of N ($p = 0.2,\ N = 10$)

with that of REGM method [7]. The simulation results are obtained by Monte Carlo method with $S = 10000$ runs.

In Fig. 2, we show the root mean-square-error (RMSE) performance of the estimated error probability obtained by the proposed methods and the conventional REGM method with $p = 0.2$ and $N = 10$. The RMSE is defined as

$$e_p = \sqrt{\frac{1}{S}\sum_{i=1}^{S}(\hat{p}_i - p)^2}, \tag{20}$$

where $\hat{p}_i$ is the estimate of p at the i-th simulation run. We can observe that the estimation accuracy is improved by all the proposed methods compared to the REGM method. This is because the window functions used in REGM method are determined in a rather heuristic manner, while our methods utilize ML approach. The comparison is also presented in terms of the estimation of the cardinality in Fig. 3 with $p = 0.2$ and $N = 10$, where the average normalized error is given by

$$e_N = \frac{1}{S}\sum_{i=1}^{S}\frac{|\hat{N}_i - N|}{N}, \tag{21}$$

where $\hat{N}_i$ is the estimate of N at the i-th simulation run. From the figure, we can see that MLSM and conventional REGM have almost the same performance, while MLSM achieves better estimation accuracy of p. The reason is that they share the same estimation method of (5) for $\hat{N}$, and the improvement of $\hat{p}$ does not have a large impact on $\hat{N}$. However, MLSM method requires much smaller number of computations than that of REGM method, where $\hat{p}$ is computed by numerical least squares method. On the other hand, EML method, which requires high computational complexity due to the exhaustive search, always presents the best performance among the methods, while MLSC method achieves the same performance as EML method. This is because, unlike MLSM or REGM methods, EML and MLSC utilize ML approach for the estimation of N as well. The performance gain gets greater as the number of reader sessions is increased because all the information of the observed evidences can be utilized in the methods.

V. Conclusion

In this paper, we have studied practical and efficient estimation methods of the tag set cardinality and transmission error probability in RFID systems using the maximum likelihood approach. For multiple independent reader sessions model, three different methods, namely, *EML*, *MLSM*, and *MLSC* have been proposed in order to maximize the likelihood function by taking different approaches in the way of treating the discrete nature of N. Computer simulation results showed that the estimates of p and N of the proposed methods are more accurate than those of the conventional REGM method. In particular, MLSM method achieved a slightly better estimate of N than REGM method for small number of tags N, while MLSM method outperformed REGM method in terms of the accuracy of $\hat{p}$ regardless of N. MLSC method achieved the same performance as EML method, providing the performance bound for the ML approach. In the future work, we intend to study the problem of the tag set cardinality estimation in the presence of both collisions and transmission errors.

References

[1] M. Bolíc, D. Simplot-Ryl and I. Stojmenovíc, RFID Systems: Research Trends and Challenges, Wiley & Sons, 2010.

[2] H. Vogt, "Efficient Object Identification with Passive RFID Tags," Proc. Int. Conf. Pervasive Computing, pp. 98-113, 2002.

[3] F. C. Schoute, "Dynamic Frame Length ALOHA," IEEE Transactions on Communications, COM-31(4):565-568, Apr., 1983.

[4] W-T Chen, "An Accurate Tag Estimate Method for Improving the Performance of an RFID Anticollision Algorithm Based on Dynamic Frame Length ALOHA," IEEE Transaction on Automation Science and Engineering, Vol. 6, No. 1, Jan., 2009.

[5] L. W. F. Chaves, E. Buchmann, and K. Böhm, "Tagmark: Reliable Estimations of RFID Tags for Business Processes," KDD '08: Proceeding of the 14th ACM SIGKDD international conference on knowledge discovery and data mining, New York, 2008.

[6] "RFID for Logistic Applications - Test Results," in EPCglobal Inc., 2005.

[7] R. Jacobsen, K. F. Nielsen, P. Popovski and T. Larsen, "Reliable Identification of RFID Tags Using Multiple Independent Reader Sessions," IEEE International Conference on RFID, 2009.

[8] Z. E. Schnabel, "The Estimation of Total Fish Population of a Lake," The American Mathematical Monthly, vol. 45, no. 6, pp. 348-352, 1938.

[9] G. D. Anderson and S.-L. Qiu, "A Monotoneity Property of the Gamma Function," PROCEEDINGS OF THE AMERICAN MATHEMATICAL SOCIETY, vol. 125, no. 11, pp. 3355-3362, Nov. 1997.

[10] E. Süli and D. Mayers, An Introduction to Numerical Analysis, Cambridge University Press, 2003.

[11] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables, New York, 1972.

[12] S. J. Patterson, An Introduction to the Theory of the Riemann Zeta-Function, Cambridge University Press, 1988.