Sub-block Encoder Design of High-rate QC-LDPC Code for Image Transmission

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***Abstract*— In this paper, quasi-cyclic low-density parity-check (QC-LDPC) over additive white Gaussian noise (AWGN) channel for image transmission system is presented. The code is used as forward error correction (FEC) technique to protect transmitted source data over the channel. The sub-block encoder is designed with various AWGNs at three high rates QC-LDPC, 0.7294, 0.8438 and 0.9000. The performance of the proposed sub-block encoder is evaluated by bit error rate (BER) improvement 13.5% and highest peak signal-to-noise ratio (PSNR) 33.7 at a low SNR.**

I. INTRODUCTION

Nowadays, high efficient data transmission code is needed for wireless data communication applications, such as image transmission. Channel coding is necessary for improving the transmission. This paper focuses on a channel coding using any desired high rates and any block lengths of quasi-cyclic low-density parity-check (QC-LDPC) codes. It is a certain error correction code technique to prevent the image data from noises and for more transmission efficiency.

Forward error correction (FEC) techniques are one of many available tools made for achieving consistent data transmission, which is used in many applications such as DVB2 (Digital Video Broadcast 2), magnetic recorder system, optical storage system, etc. Low-density parity-check (LDPC) codes was introduced by Gallager [1] in 1962 and it was rediscovered by Mackay and Neal [2] as one of many kinds of linear block codes that have been studied vastly in this decade. The main advantages of the codes are that it provides a performance that closes to the limited capacity for many different channels and it is linear complexity algorithms for data encoding. There are two types of LDPC codes, regular LDPC code and irregular LDPC code. In previous works, many researchers had studied in encoding complexity problems [3-6]. The regular type of LDPC codes is called quasi-cyclic LPDC code. It is a quasi-cyclic structure which is taken a simpler encoding scheme by dividing the parity-check matrix into sub matrices scheme. It is suitable for high-rate applications [7]. Myung [8] proposed an encoding based on a generation matrix instead of the parity-check matrix. It is still based on breaking the parity-check matrix into sub matrices and then translating them into the generation matrix.

In this paper, a FEC technique, called quasi-cyclic low-density parity-check codes, is designed for image transmission in order to reduce bit error rate and to keep a quality of the received image at three high rates. The new channel coding is designed using LDPC approach having a lower complexity comparatively than the others, while it is still effective. The quasi-cyclic LDPC is then applied to the image transmission system in order to improve its performance.

This paper is organized as follows. In Section 2, LDPC code algorithm is introduced. In Section 3, we present the methodology. The simulation results are in Section 4. Finally, the conclusion is made in Section 5.

II. LOW-DENSITY PARITY-CHECK (LDPC) CODES

A. *Array-based LDPC code*

An array-based LDPC code or QC-LDPC code [7] is a type 3 of non-systematic LDPC codes with quasi-cycle property.

The parity-check matrix $\mathbf{H}$ as shown in equation (2) can be constructed from the permutation matrix $\mathbf{P}$ with a dimension size $L \times L$, defined as in equation (1). The matrix $\mathbf{H}$ has a dimension $(n-k) \times n$

$$\mathbf{P}_{L\times L} = \begin{bmatrix} 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & & 0 \end{bmatrix} \quad (1)$$

where k is the message length and
n is the code length.

The matrix $\mathbf{P}$ is a shifted identity matrix $\mathbf{I}$ one time. The matrix $\mathbf{H}$ can be built from

$$\mathbf{H} = \begin{bmatrix} I & I & I & \dots & I \\ I & P & P^2 & \dots & P^{k-1} \\ I & P^2 & P^4 & \dots & P^{2(k-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I & P^{j-1} & P^{2(j-1)} & \dots & P^{(j-1)(k-1)} \end{bmatrix}. \quad (2)$$

Basically, the matrix $\mathbf{P}$ expands in rows by j times and in columns by k times, respectively. The permutation matrix can be generated by shifting a matrix $\mathbf{I}$ in order that the matrix $\mathbf{H}$ for an encoding process has lower complicated linearly as shown in equation (3).

$$\mathbf{H} = \begin{bmatrix} I & I & I & \dots & I & \dots & I \\ 0 & I & P & \dots & P^{(j-2)} & \dots & P^{(k-2)} \\ 0 & 0 & P & \dots & P^{2(j-3)} & \dots & P^{2(j-3)} \\ : & & & & : & & \\ 0 & \dots & 0 & I & P^{(j-2)} & \dots & P^{(j-2)(k-j-1)} \\ \underbrace{0 \quad 0 \quad \dots \quad 0 \quad I}_{j} & & & & & \underbrace{\dots \quad P^{(j-2)(k-j)}}_{k-j} & \end{bmatrix} \quad (3)$$

Code rate can be calculated by equation (4).

$$R = \frac{k-j}{k} = 1 - \frac{j}{k} \quad (4)$$

A quasi-cyclic property is processed by array code and has a linear encoding complexity.

B. LDPC Encoding

The LDPC encoding has a linear complexity. The encoding transforms a message into a codeword. For systematic code, this means adding parity bits. A simple scheme is to exploit the relationship between codeword and the parity-check matrix $\mathbf{H}$. We define a codeword with c dimension of $1 \times n$ and the corresponding matrix parity check $\mathbf{H}$ with dimension of $(n-k) \times n$ and $\mathbf{0}$ are a vector with dimension of $(n-k) \times n$. The encoding process uses modulo-2 addition or exclusive-or (XOR) operation. The relationship between the party matrix $\mathbf{H}$ and codeword c can be written as equation (5).

$$\mathbf{cH^T = 0} \quad (5)$$

Equation (6) is to transpose both sides of equation (5).

$$\mathbf{Hc^T = 0^T} \quad (6)$$

We define a systematic codeword c as the follows,

$$\mathbf{c} = \left[p_1 \;\; p_2 \; \dots . p_{n-k} \mid m_1 \; m_2 \dots . m_k \right] = [\mathbf{p \; m}] \quad (7)$$

Where the parity bits are positioned at the front part and the message bits are at the back part. In this scheme, the encoding can be done efficiently. The transpose of codeword c in equation (8) can be written as

$$\mathbf{c^T} = \begin{bmatrix} \mathbf{p} \\ \mathbf{m} \end{bmatrix} \quad (8)$$

Substituting equation (8) into equation (6), the parity-check equation can be derived, for example, given that a parity-check matrix $\mathbf{H}$ of a (3, 7) code is

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}. \quad (9)$$

Equation (10) is rewritten as

$$\mathbf{Hc^T = 0^T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \hline m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (10)$$

Then the parity-check equations are

$$\begin{aligned} p_1 + m_2 + m_3 + m_4 &= 0 \\ p_2 + m_1 + m_2 + m_4 &= 0, \\ p_3 + m_1 + m_3 + m_4 &= 0 \end{aligned} \quad (11)$$

where '+' is a modulo-2 addition or an XOR operation. The parity bits can be then found from

$$\begin{aligned} p_1 &= m_2 + m_3 + m_4 \\ p_2 &= m_1 + m_2 + m_4 . \\ p_3 &= m_1 + m_3 + m_4 \end{aligned} \quad (12)$$

C. Sub-block Encoder Designed

The encoder complexity can be reduced by encoding systematic codeword c sub-block. It was defined by using the parity bits are positioned at the front part, the back part and the middle part which is between the message bits as the follows,

$$\mathbf{c} = \left[p_1 \;\; m_1 \; m_2 \;\; p_2 \; m_3 \; m_4 \;\; p_3 \right] \quad (13)$$

In this scheme, the encoding can protect and keep quality of the received image.

III. METHODOLOGY

The configuration of image transmission system is shown in Fig. 1. We use three high-rate QC LDPC codes, $R = 0.7294$, $R = 0.8438$ and $R = 0.9000$, with iterative decoding. The high rates are suitable for wireless data communication under acceptable conditions. In order to evaluate the performance of these error correcting codes, first the source image is encoded by using JPEG compression. After JPEG encoder, image compressed is encoded by sub-block design of QC-LDPC codes. The data are then modulated using binary phase shift keying (BPSK) scheme before it is transmitted via additive white Gaussian noise (AWGN) channel. The reverse process is performed by the receiver.

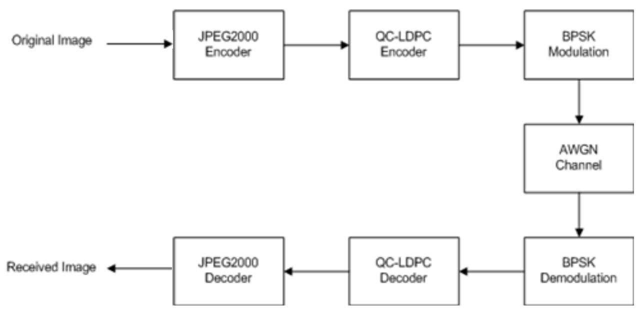

Fig. 1 Block diagram for image transmission system.

IV. SIMULATION RESULTS

In this section, the simulation results of the image transmission using the proposed QC-LDPC code as channel coding with BPSK modulation via AWGN channel at various noise levels are shown.

We examine the system performance with subjective received images visualization, bit-error-rate (BER) and peak-signal-noise-ratio (PSNR), respectively.

In the simulation, 512×512 pixel 8-bit grayscale Lena image and Barbara image are tested with various levels of noises in order to evaluate the performance of the proposed channel coding. Fig. 2 shows the bit error rate curves of the QC-LDPC code using 20 iterations. Fig. 3 shows the bit error rate curves of sub-block encoder design of the QC-LDPC code using 20 iterations. As the result of BER in fig2 and fig 3, we can see that performance of sub-block design in fig. 3 better than fig. 2. The peak signal-to-noise ratios of the received images are show in fig.4. Fig. 5 shows the comparison of the received images.

The objective measured quality of the received images can be evaluated by PSNR calculated from the following equation (13).

$$PSNR = 10\log\frac{K^2}{MSE} \qquad (13)$$

where K is maximum grayscale value of the image.
The mean squared error (MSE) can be determined by the following equation (14).

$$PSNR = \frac{1}{L \times P}\sum_y\sum_x\left[g(x,y) - \hat{g}(x,y)\right]^2 \qquad (14)$$

where $g(x,y), \hat{g}(x,y)$ represent the grayscale values of any pixels in an original image and a recovered image, respectively. L, P represent the width and height of the image, respectively.

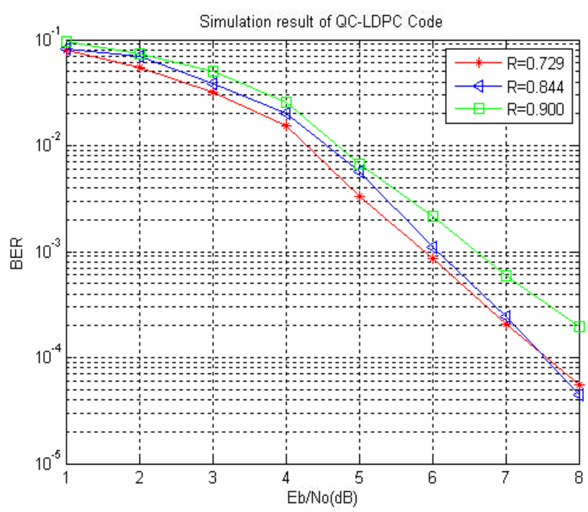

Fig. 2 Image transmission BER of QC-LDPC Code using 20 iterations.

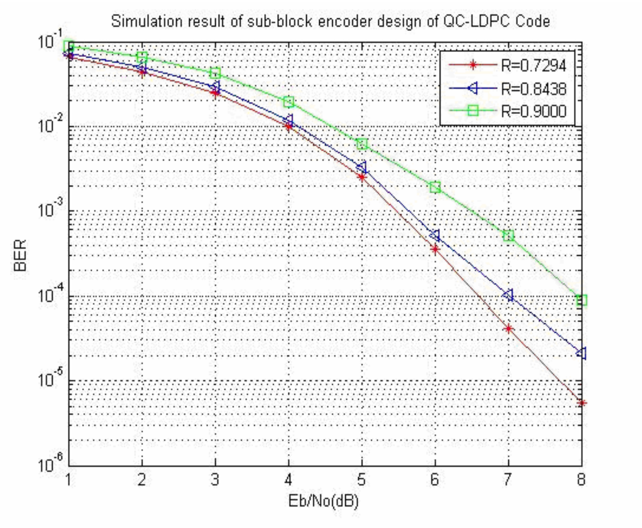

Fig. 3 Image transmission BER of sub-block encoder design of QC-LDPC Code using 20 iterations.

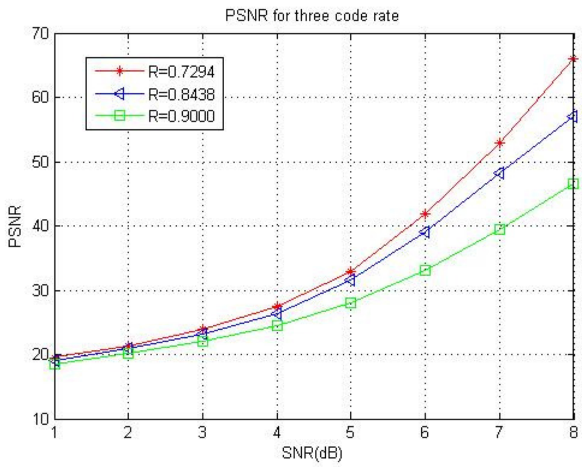

Fig. 4 PSNR of three code rates of sub-block encoder design of QC-LDPC Code.

(a) R = 0.7294 at 5 dB (b) R = 0.7294 at 5 dB

(c) R = 0.8348 at 5 dB (d) R = 0.8348 at 5 dB

(e) R = 0.9000 at 5 dB (f) R = 0.9000 at 5 dB

Fig. 5 Comparison of the transmitted images for three rates at the same SNR.

From the simulation results, it shows that the QC-LDPC codes perform well with iteratively decoded and achieves a good performance in image transmission over AWGN channel. In the condition with the sub-block codeword design, the useful information speed of Code 1 (R=0.7294) is 13.5% lower than that of Code 2 (R= 0.8438) and 6% lower than that of Code 3 (R=0.900). However, the transmission performance of Code 1 is better than Code 2 and that of Code 2 is better than Code 3. The BER curves are shown in Fig. 3. Moreover, we can see that when BER is 10^{-4}, Code 1 has higher coding gain of about 0.5 dB than Code 2, and a higher coding gain of about 1 dB than Code 3. The PSNR curve of the recovered images at three rates can be shown in Fig. 4. In comparison of the transmitted images at $E_b / N_0 = 5\ dB$, the PSNR of Code 1, Code 2 and Code 3 is about 33.7, 32.5 and 28.7, respectively. It is shown in Fig 5. The proposed sub-block encoder designs of QC-LDPC codes give a good performance to keep a good quality of the transmitted image.

V. CONCLUSIONS

The sub-block encoder design of QC-LDPC code, a channel coding applied to image transmission via AWGN channel has been presented in this paper. The simulation results show a good performance for correcting data error at a low SNR. The proposed sub-block encoder design of QC-LDPC code reduces bit error at various levels of noise and keeps sufficiently quality of the received image. The objective performance is evaluated by BER and PSNR.

ACKNOWLEDGMENT

This paper is finished finely. Thanks to friends and seniors who help and give me all supports. Finally, thanks to everyone who took part in this manuscript.

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