

Preamble Based Joint Channel and CFO Estimation for MIMO-OFDM Systems with Null Subcarriers

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Abstract—In this paper, challenges regarding the provision of channel state information (CSI) and carrier frequency synchronization for multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) systems with null subcarriers are addressed. We propose a novel maximum likelihood (ML) based scheme that estimates the aggregate effects of the channel and carrier frequency offset (CFO) by using two successive OFDM preambles. In the presented scheme, CFO is estimated by considering the phase rotation between two consecutive received OFDM preambles. Both channel and CFO mean squared errors (MSE) are used to evaluate the performance of our proposed scheme. Simulation results indicate that the BER performance of the proposed estimator is comparable to that of the system with known channel state information and the CFO MSE performance achieves the Cramer-Rao bound (CRB) of the fully loaded OFDM system.

I. INTRODUCTION

The advantages offered by combining orthogonal frequency division multiplexing (OFDM) with multiple-input multiple-output (MIMO) techniques are manifold. The most remarkable of them are robustness of OFDM systems against frequency selective fading channels, obtained by converting the OFDM channel into flat fading subchannels [1] and the significant information capacity gain together with improved BER performance of the MIMO systems [2], [3].

In contrast to these appealing attributes, OFDM systems perform poorly under the influence of carrier frequency offset (CFO) [4]. CFO may damage the orthogonality of subcarriers and lead to inter-carrier interference (ICI) that results in severe degradation of the system's performance [5]–[8]. Likewise, a MIMO system with N_t transmit and N_r receive antennas necessitates $N_t \times N_r$ channels to be estimated [1], while for a single input single output (SISO) system only one channel is to be estimated.

To obtain better quality of the high rate communications systems, efficient channel estimation and frequency synchronization techniques are crucial. When the OFDM-based wireless systems operate in a slow fading multiple-access environment, the use of preambles to facilitate channel estimation have been well discussed in IEEE 802.11a/g standard [9]. The short preambles in a WLAN system can be used to estimate and correct the coarse CFO. However, residual CFO always remains, producing a non negligible phase shift between consecutive OFDM blocks which can significantly deteriorate the signal detection. The accuracy of CFO estimate can be

improved by succeeding residual CFO estimation based on the long preambles.

Several techniques for ICI suppression and channel estimation have been predominantly developed for single input single output (SISO)-OFDM systems [10]–[13], and the reference therein. In [10], [11], algorithms that employ a priori known training sequences are proposed, while in [12], the redundancy in cyclic prefix is utilized and in [13], scattered pilots and virtual carriers based CFO tracking algorithms are proposed.

If we extend the preamble based techniques to MIMO systems, the same short training symbols can be sent from only one antenna. At the receiver, the known SISO synchronization algorithm can accomplish the packet detection and frequency synchronization [14]. However, for the long preambles, in order to mitigate the effect of co-channel interference between the transmitting antennas, it is necessary for the training signals from each antenna to be orthogonal. The orthogonality of the training sequences for MIMO-OFDM preambles can be established by special codes, such as Phase-Shift (PS) proposed in [1], [15]. These techniques were adopted in [14] to design long preambles for a MIMO-OFDM system. Also in [4], [16], the orthogonality of the training symbols is achieved by ensuring that the training symbols of one antenna are disjoint from the training symbols of any other antenna.

Channel and CFO estimation methods for MIMO-OFDM systems have been studied as well, e.g. in [2], [6], [17], [18]. In [17], a technique for jointly estimating the channel and CFO in a MIMO system using block type pilots is proposed. To estimate CFO, the method utilizes both grid search and Newton method which increases the complexity of the algorithm. In [6], a method that utilizes pilot symbols to estimate the residual CFO is proposed, however the method assumes known channel state information which is not available in practical systems.

In this paper, we focus on channel estimation and CFO synchronization of the residual CFO in time domain for MIMO-OFDM systems. We derive the ML estimator that utilizes two successive preambles to estimate the channel as well as the CFO by considering the phase rotation between the successive OFDM blocks or frames. Similar ML techniques, that involve repetition of training symbols between two successive OFDM blocks for SISO systems are proposed in [10], [11]. In [11], a maximum-likelihood CFO estimator uses the phase difference between channel estimates of two successive

OFDM symbols. Here we provide a mathematical derivation of the ML estimator that captures the aggregate effects of the channel and the CFO. Unlike [11], the proposed CFO estimator uses the phase difference between two successive received OFDM symbols. Thus, CFO can be estimated without a priori knowledge of the channel.

The proposed scheme utilizes a grid search within the acquisition range to obtain the suboptimal value of the CFO estimate. Simulation results are provided to demonstrate the MSE as well as the BER performance of our proposed estimator for different quadrature amplitude modulation (QAM) schemes.

Notations: The following notations will be used throughout this article, the frequency domain and time domain vectors will be represented by the upper and lower case letters respectively, while the superscript $(\cdot)^T$ and $(\cdot)^H$ will denote transpose and Hermitian transpose respectively.

II. MIMO-OFDM SYSTEM MODEL

Let us consider a MIMO-OFDM wireless system with N_t transmit and N_r receive antennas over frequency selective channels. The frequency domain representation of the k th transmitted OFDM symbol with N number of subcarriers at the p th transmit antenna can be written as the vector $\mathbf{X}_k^p = [X_{k,0}^p, X_{k,1}^p, \dots, X_{k,N-1}^p]^T$, and the corresponding time domain signal is given by

$$\mathbf{x}_k^p = \mathbf{F} \mathbf{X}_k^p \quad (1)$$

where \mathbf{F} is an $N \times N$ DFT matrix with $(m+1, n+1)$ th entry $[\mathbf{F}]_{m,n} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi mn}{N}}$ and $\mathbf{x}_k^p = [x_{k,0}^p, x_{k,1}^p, \dots, x_{k,N-1}^p]^T$.

We assume that the discrete-time baseband equivalent channel between each transmit-receive antenna has FIR of maximum length L , and remains constant in at least one OFDM block, i.e., is quasi-static. Let us denote the channel from the p th transmit antenna to the q th receive antenna as $\mathbf{h}^{(q,p)} = [h_0^{(q,p)}, \dots, h_{L-1}^{(q,p)}]^T$ and its frequency response as

$$\mathbf{H}^{(q,p)} = \mathbf{F}_L \mathbf{h}^{(q,p)} \quad (2)$$

with $\mathbf{F}_L = [\mathbf{f}_0, \dots, \mathbf{f}_{L-1}]$ representing the N rows and the first L columns of the DFT matrix \mathbf{F} .

The CFO between the transmitter and the receiver antenna q is normalized by the subcarrier spacing (also referred to as inter-carrier spacing), and is denoted by ϵ_q where ϵ_q is assumed to be in $(-0.5, 0.5]$. Practically, the instability of the transmit/receiver oscillators influence the maximum frequency offset. This implies that the CFO can be different at each receive antenna due to, for example having different local oscillator at each RF chain, however the model can also be used for the case of having common CFO for all receive antennas.

Assume that the insertion of a long enough cyclic prefix (CP) at the transmitter maintains the orthogonality of the subcarriers after transmission. Then, at the receiver, after discarding the cyclic prefix, the complex envelope of the baseband received signal in an OFDM block including CFO

can be described as [17], [19]

$$\mathbf{y}_k^q = \sum_{p=1}^{N_t} \mathbf{D}(\epsilon_q) \mathbf{H}_c^{(q,p)} \mathbf{x}_k^p + \mathbf{v}_k^q \quad (3)$$

where \mathbf{y}_k^q is the k th received OFDM symbol and $\mathbf{H}_c^{(q,p)}$ is the circulant channel matrix whose first column is $[\mathbf{h}^{(q,p)T}, 0, \dots, 0]^T$, and $\mathbf{D}(\epsilon_q) = \text{diag}(1, e^{j2\pi\epsilon_q \frac{1}{N}}, \dots, e^{j2\pi\epsilon_q \frac{N-1}{N}})$, is an $N \times N$ diagonal matrix that describes the phase rotating effect by the frequency offset on each time domain OFDM symbol.

We can also represent (3) as

$$\mathbf{y}_k^q = \sum_{p=1}^{N_t} \mathbf{D}(\epsilon_q) \mathbf{F}^H \mathbf{D}(\mathbf{X}_k^p) \mathbf{H}^{(q,p)} + \mathbf{v}_k^q \quad (4)$$

where $\mathbf{D}(\mathbf{X}_k^p)$ is a diagonal matrix with vector \mathbf{X}_k^p as its diagonal elements.

Suppose that there are some null subcarriers and let N_a be the number of active subcarriers. Then, given $\mathbf{X}_{k,a}^p$ as a transmitted OFDM symbol in frequency domain at the active subcarriers, the received signal can be expressed as

$$\mathbf{y}_k^q = \sum_{p=1}^{N_t} \mathbf{D}(\epsilon_q) \mathbf{F}_a^H \mathbf{D}(\mathbf{X}_{k,a}^p) \mathbf{H}_a^{(q,p)} + \mathbf{v}_k^q \quad (5)$$

where $\mathbf{H}_a^{(q,p)}$ is a channel coefficient vector at the active subcarriers and \mathbf{F}_a is an $N_a \times N$ DFT sub-matrix corresponding to N_a number of active subcarriers. We can rewrite (5) as

$$\mathbf{y}_k^q = \sum_{p=1}^{N_t} \mathbf{D}(\epsilon_q) \mathbf{F}_a^H \mathbf{D}(\mathbf{X}_{k,a}^p) \mathbf{F}_{L,a} \mathbf{h}^{(q,p)} + \mathbf{v}_k^q \quad (6)$$

where $\mathbf{F}_{L,a}$ is an $N_a \times L$ submatrix of \mathbf{F}_L corresponding to the active subcarriers. Likewise, the next received OFDM symbol for the transmitted active subcarriers $\mathbf{X}_{k+1,a}^p$ can be expressed as

$$\mathbf{y}_{k+1}^q = \sum_{p=1}^{N_t} e^{j\alpha\epsilon_q} \mathbf{D}(\epsilon_q) \mathbf{F}_a^H \mathbf{D}(\mathbf{X}_{k+1,a}^p) \mathbf{F}_{L,a} \mathbf{h}^{(q,p)} + \mathbf{v}_{k+1}^q \quad (7)$$

where $\alpha = \frac{2\pi(N+N_{cp})}{N}$, is one OFDM symbol duration including cyclic prefix of length N_{cp} .

In the following section, we will derive the maximum likelihood (ML) estimator capable of decoupling the CFO and channel estimation from the two received successive long training symbols.

III. JOINT ML CHANNEL AND CFO ESTIMATION

Since the same channel and CFO estimation process is performed at each receive antenna, we only need to consider N_t transmit antennas and one receive antenna in deriving our ML estimator, that is, the system is modeled as a superposition of multiple-input single-output (MISO) systems [4], [20]. Thus, we can describe the first receive antenna and omit the receive antenna index.

From the expressions derived in Section II, stacking the

consecutive received signals leads to

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_{k+1} \end{bmatrix} = \sum_{p=1}^{N_t} \mathbf{A}_p \mathbf{h}^{(p)} + \tilde{\mathbf{v}}_k \quad (8)$$

where $\tilde{\mathbf{v}}_k = [\mathbf{v}_k^T, \mathbf{v}_{k+1}^T]^T$ and

$$\mathbf{A}_p = \begin{bmatrix} \mathbf{D}(\epsilon) \mathbf{F}^H \mathbf{D}(\mathbf{X}_k^p) \\ e^{j\alpha\epsilon} \mathbf{D}(\epsilon) \mathbf{F}^H \mathbf{D}(\mathbf{X}_{k+1}^p) \end{bmatrix} \mathbf{F}_{L,a}.$$

Eq. (8) can also be represented as

$$\tilde{\mathbf{y}}_k = \mathbf{A}(\epsilon) \mathbf{h} + \tilde{\mathbf{v}}_k \quad (9)$$

where $\mathbf{A}(\epsilon) = [\mathbf{A}_1, \dots, \mathbf{A}_{N_t}]$ and $\mathbf{h}^T = [\mathbf{h}^{(1)T}, \dots, \mathbf{h}^{(N_t)T}]$ is channel vector of length $N_t L \times 1$.

If the noise is i.i.d. white Gaussian, then the ML estimate of the channel \mathbf{h} and CFO ϵ are obtained by minimizing

$$\|\tilde{\mathbf{y}}_k - \mathbf{A}(\epsilon) \mathbf{h}\|^2. \quad (10)$$

If $\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon) > 0$, then, for a given ϵ , the ML estimate of \mathbf{h} is given by

$$\hat{\mathbf{h}} = [\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon)]^{-1} \mathbf{A}(\epsilon)^H \tilde{\mathbf{y}}_k. \quad (11)$$

Next, we minimize $\|\tilde{\mathbf{y}}_k - \mathbf{A}(\epsilon) \hat{\mathbf{h}}\|^2$ to obtain the ML estimate of $\hat{\epsilon}$. Substituting (11) to (10), results into

$$\begin{aligned} & \left\| \left(\mathbf{I} - \mathbf{A}(\epsilon) (\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon))^{-1} \mathbf{A}(\epsilon)^H \right) \tilde{\mathbf{y}}_k \right\|^2 = \\ & \|\tilde{\mathbf{y}}_k\|^2 - \|\mathbf{A}(\epsilon) (\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon))^{-1} \mathbf{A}(\epsilon)^H \tilde{\mathbf{y}}_k\|^2 \end{aligned} \quad (12)$$

Since the first term of the R.H.S. of the equation above is constant, for the CFO estimation, we need to maximize

$$\begin{aligned} & \|\mathbf{A}(\epsilon) (\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon))^{-1} \mathbf{A}(\epsilon)^H \tilde{\mathbf{y}}_k\|^2 \\ & = \tilde{\mathbf{y}}_k^H \mathbf{A}(\epsilon) (\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon))^{-1} \mathbf{A}(\epsilon)^H \tilde{\mathbf{y}}_k. \end{aligned} \quad (13)$$

In [10], Moose et al. proposed a CFO estimator that uses two consecutive preambles where the estimated CFO is given by

$$\hat{\epsilon} = \frac{1}{\alpha} \angle (\mathbf{y}_k^H \mathbf{y}_{k+1}). \quad (14)$$

which is the maximization of [10]

$$\text{Re}\{e^{-j\alpha\epsilon} \mathbf{y}_k^H \mathbf{y}_{k+1}\}. \quad (15)$$

The complexity of the estimator in [10] is lower than the proposed estimator. However, the accuracy of the estimator is not guaranteed for OFDM systems with null subcarriers. In practical systems, several subcarriers of in the OFDM symbol are null to avoid interference between adjacent OFDM symbols. This may restrict the use of the estimator in [10] for practical OFDM systems .

Since the proposed and the estimator in [10] are not linear in ϵ , we resort to numerical optimization. More specifically, we utilize a grid search within the acquisition range to obtain the (sub-)optimal value of the CFO estimate. We only consider the residual CFO estimation, which means the CFO is small enough for the grid search to work.

Note that, to avoid interference in MIMO-OFDM systems,

the training signals between the transmit antennas need to be orthogonal. Several techniques have been developed to ensure the orthogonality of the training sequences for MIMO-OFDM. In [14], the orthogonality of the MIMO long training preambles is established by the Phase-Shift (PS) codes proposed in [1], [15] where all the symbols have the same power, while in [16], the orthogonality of the training symbols is achieved by ensuring that, the training symbols of one antenna are disjoint from the training symbols of any other antenna in the frequency domain and power of each training symbols is obtained by minimizing the channel MSE with respect to the total power of symbols in a preamble. The channel MSE is given as

$$E\{\|\hat{\mathbf{h}} - \mathbf{h}\|^2\} = \frac{\sigma_v^2}{N} \text{trace} \left[(\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon))^{-1} \right]. \quad (16)$$

Design of disjoint training set is also present in [20] under the assumption that all subcarriers in an OFDM block are used as pilot tones. However, in most realistic cases some of the subcarriers at the edge of the spectrum are nulled to avoid interference between adjacent OFDM blocks. This limits the adoption of the design technique in [20] in practical systems.

In the following Section we will demonstrate the performance of the proposed CFO estimator by numerical simulations for a normalized CFO between $(-0.5, 0.5]$.

IV. SIMULATION RESULTS

We demonstrate the effectiveness of our proposed ML estimator through computer simulations. The parameters of the transmitted OFDM signal studied in our design examples are as in the IEEE 802.11a standard in [9, p.600], where an OFDM transmission frame with $N = 64$ is considered. Out of 64 subcarriers, 52 subcarriers are used as data subcarriers. Of the remaining 12 subcarriers, 6 are null in the lower frequency guard band while 5 are nulled in the upper frequency guard band and one is the central DC null subcarrier. Of the 52 used subcarriers, 4 are allocated as pilot subcarriers, while the remaining 48 are used for data transmission.

For the long preamble, all 52 active subcarriers are used as training symbols. To ensure orthogonality to multiple transmit antennas, we resort to the disjoint pilot sequence in [16] as well as training sequences presented in [14] where equal power is allocated to all active subcarriers and the orthogonality of the designed preamble is obtained by adopting the special phase shift codes proposed in [1], [15].

Fig. 1 shows the mean squared error (MSE) of the CFO estimator $\boldsymbol{\eta} = E\{\|\hat{\epsilon} - \epsilon\|^2\}$ against signal to noise ratio (SNR) for the proposed design (13), Moose estimator (15) and the CRB for the full loaded OFDM system OFDM system derived in [11]. The CRB allows to get an insight into the theoretical performance limit of the estimators. The performance of the proposed estimator are comparable to the CRB, suggesting that our proposed estimator performs well, even for low values of the SNR. At high SNR's Moose estimator performs equally well as the proposed estimator. However, Moose estimator does a poor job in estimation CFO at low signal to noise ratio

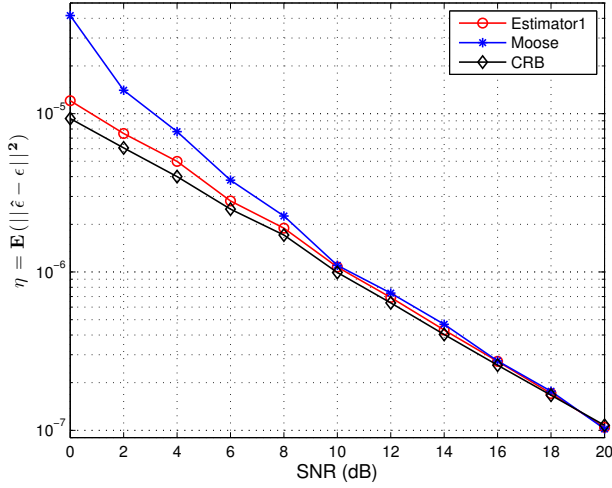


Fig. 1. Performance of the MSE of the CFO estimator for different SNRs

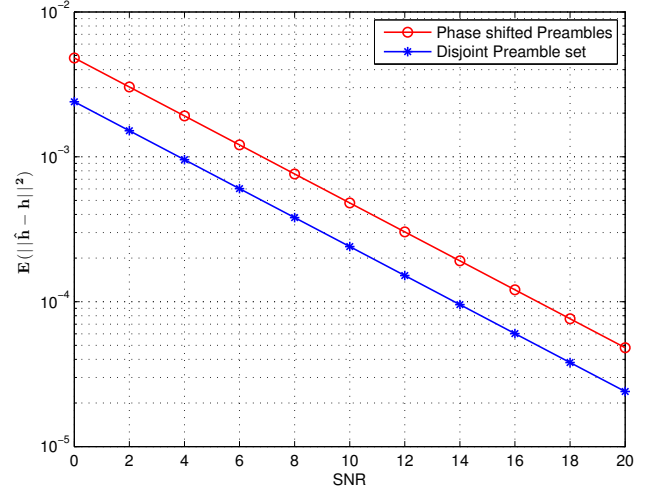


Fig. 3. Performance of the channel estimate MSE for different SNRs

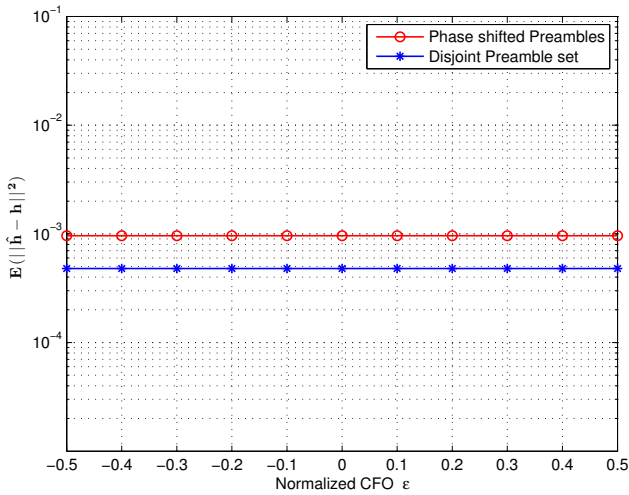


Fig. 2. Effects of different CFO's on channel estimate MSE

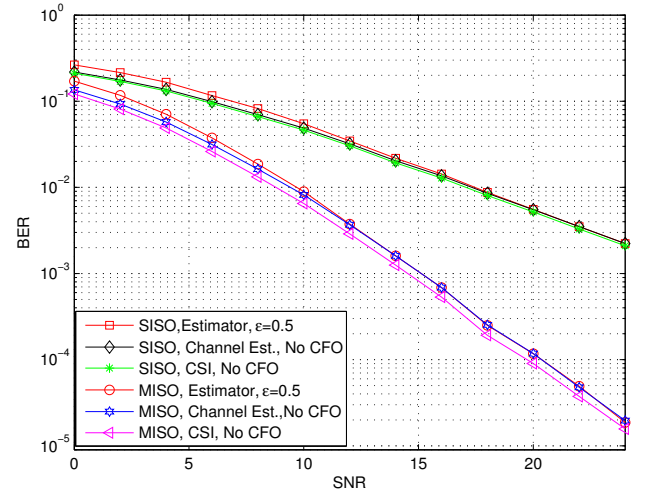


Fig. 4. Comparison of the BER performances for 16-QAM

(SNR). This corroborates the effectiveness of our proposed estimator over the Moose estimator.

To demonstrate the performance of our channel estimator we use both phase shifted preambles in [14] and the disjoint pilot set obtained by using the techniques in [16]. Fig. 2 depicts the effect of different CFO's on the channel estimation for $N_t = 2$. From the plots it is clear that both phase shifted preambles and the disjoint preamble set perform equally well for the whole range of normalized range of CFO $(-0.5, 0.5]$. The result verifies the robustness of the proposed channel estimator as its accuracy does not depend on the estimation and compensation of the residual CFO. The channel estimate remains the same regardless of the correctness of the CFO estimator.

For a given power per OFDM block, the disjoint set outperforms the phase shifted preambles by a small margin. This may be due to the fact that the minimization of the channel MSE

improves the condition number of the matrix $\mathbf{A}(\epsilon)^H \mathbf{A}(\epsilon)$. However, simulation results (which are not shown here) verified that there is no significant difference in bit error rate (BER) performance between the disjoint pilot set and the phase shifted preambles.

Fig. 3 shows the MSE of the channel estimator against SNR for the phase shifted preambles and the disjoint pilot symbols for $N_t = 2$. Likewise the disjoint preamble set outperforms the phase shifted preambles by a constant gap for different SNRs. This further demonstrates the effectiveness of our channel estimator.

Next, we demonstrate the performance of our joint channel and CFO estimator by considering the BER performance of both SISO and Alamouti STBC with two transmit antennas and one receive antenna (MISO). To obtain better BER performance, proper compensation of carrier frequency and accurate channel estimates are of primary importance.

The results in Fig. 4 show BER performance of our joint channel and CFO estimator together with the results of the known channel state information and the estimated channel without any residual CFO component. From the results it is clear that the BER performance of the estimated channel is comparable to that of the known channel state information for CFO free case. This demonstrates the effectiveness of our channel estimator. Also the performance of our joint channel and CFO estimator gives nearly the same BER performance as the estimated channel without any residual CFO for 16-QAM. The proposed approach maintains the bit error rate (BER) within 1dB of the value obtained from the CFO-free system for both SISO and MISO case. This further corroborates the significant performance of our estimator.

V. CONCLUSION

In this paper we addressed the problem of channel and CFO estimation for MIMO-OFDM systems with null subcarriers. Through numerical simulations, we have verified that the proposed scheme can be used to efficiently estimate the channel as well as carrier frequency offset in OFDM systems. Simulation results show that, the MSE of the proposed ML estimator achieves the derived CRB for the full loaded OFDM and the BER performance of the joint channel and CFO estimator are comparable to that of the known channel state information.

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