Cramér-Rao Bounds for Channel Estimation in Amplify-and-Forward Cooperative Communication Systems

Wei-Lun Lin*, Chi-Hua Huang†, Po-Yu Li‡, and Char-Dir Chung§
* Graduate Institute of Communication Engineering, National Taiwan University, Taipei
E-mail: weilunlin@ntu.edu.tw Tel: +886-2-3366-3075 Fax: +886-2-2368-3824
† Graduate Institute of Communication Engineering, National Taiwan University, Taipei
E-mail: f95942046@ntu.edu.tw Tel: +886-2-3366-3075 Fax: +886-2-2368-3824
‡ Graduate Institute of Communication Engineering, National Taiwan University, Taipei
E-mail: r97942127@ntu.edu.tw Tel: +886-2-3366-3075 Fax: +886-2-2368-3824
§ Department of Electrical Engineering
and Graduate Institute of Communication Engineering, National Taiwan University, Taipei
E-mail: cdchung@cc.ee.ntu.edu.tw Tel: +886-2-3366-3075 Fax: +886-2-3366-3596

***Abstract*—The Cramér-Rao bounds of centralized and distributed channel estimations are studied for the cooperative communication systems with single amplify-and-forward relay and two transmission phases. Particularly, two cooperation protocols are considered with full cooperation in both transmission phases and partial cooperation in the listening phase, respectively. It is shown that centralized channel estimation is a better choice than distributed channel estimation for both full and partial cooperation protocols.**

I. Introduction

Cooperative diversity is an efficacious method to realize spatial diversity gain through the distributed relay terminals acting as a virtual antenna array in a communication network. The cooperation network has been shown to provide the same diversity order as an equivalent multiple-input-multiple-output system [1] and higher capacity than the conventional point-to-point communication network [2] when some specific cooperation protocols are adopted.

A typical class of protocols [3]-[4] supporting the cooperation network with three terminals consisting of a source, a relay, and a destination has been extensively studied based on a two-phase transmission, namely a listening phase followed by a cooperation phase, within each signaling frame as shown in Fig. 1. In these protocols, the source transmits the desired signal to the relay in the listening phase, and during the cooperation phase the relay processes the received signal in the listening phase by either amplify-and-forward (AF) or decode-and-forward (DF) operation and transmit the outcome to the destination. Depending on the protocol [3] implementing full or partial cooperation, the source transmits the desired signal to the destination in both listening and cooperation phases or in listening or cooperation phase, respectively, as illustrated by the dashed links in Fig. 1.

Coherent detection is considered in most cooperative communication systems employing the above-mentioned protocols

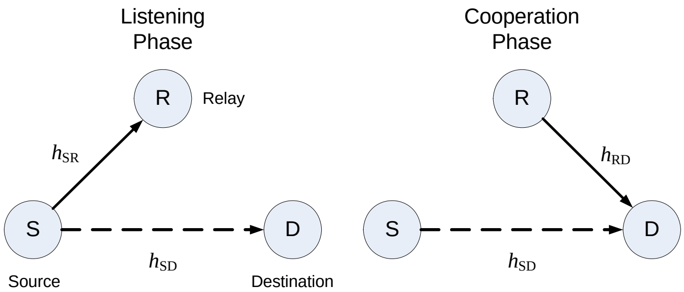

Fig. 1. Signal transmission model for a cooperative communication system with single relay.

[3]-[5] under the system setup that the complex channel gains h_{SD}, h_{SR}, and h_{RD} corresponding to source-to-destination ($S \longrightarrow D$), source-to-relay ($S \longrightarrow R$), and relay-to-destination ($R \longrightarrow D$) links are perfectly estimated. For the DF relaying systems, this setup can be best approached by employing the conventional direct-link channel estimation (DLCE) mechanism [5]-[7] to separately estimate h_{SD} and h_{RD} at destination and h_{SR} at relay in that coherent detection is carried out at both DF relay and destination. However, DLCE is not adequate for the AF relaying systems employing the cooperation protocols, since all the received signals at both AF relay and destination contain useful channel gain information and have to be taken into account during channel estimation. In the latter case, two alternative channel estimation mechanisms, namely centralized channel estimation (CCE) and distributed channel estimation (DCE) [7]-[11], are viable choices in order to achieve better estimation performance. Specifically, channel gains for the three transmission links are estimated all at destination for CCE and distributively at relay and destination for DCE, under an ideal circumstance that a reliable feedforward control channel [7]-[9] is required

to send full or partial $S \longrightarrow R$ channel state information from relay to destination for both CCE and DCE mechanisms.

Several channel estimation schemes based on CCE or DCE are proposed in [9]-[11], but the limiting performance characteristics of CCE and DCE in terms of Cramér-Rao bound (CRB) for the AF cooperative communication systems employing the above-mentioned cooperation protocols still remain unexplored. Although an attempt in [11] tries to obtain the CRB of CCE for the AF system employing the protocol with full cooperation under the assumption that the source-to-relay gain h_{SR} is complex Gaussian, the obtained CRB result is not accurate for deterministic channel estimation. To fill up the void of limiting performance characteristics, this letter studies the CRBs of both CCE and DCE mechanisms for the AF cooperative communication systems employing full cooperation protocol (FCP) and partial cooperation protocol (PCP).

Nomenclature: Superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote conjugate, transpose, and conjugate transpose, respectively. $E\{\cdot\}$ denotes the expectation. $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ are the real and imaginary parts, respectively, of x. The boldface lowercase letter denotes a column vector and the boldface uppercase letter a matrix. $\mathbf{I}_N$ and $\mathbf{O}_N$ are the $N \times N$ identity and all-zero matrices, respectively. $\mathbf{A}^{-1}$, $tr(\mathbf{A})$, and $[\mathbf{A}]_{ij}$ are the inverse, the trace, and the (i,j)-th entry, respectively, of $\mathbf{A}$.

II. Cooperation Protocols and Channel Estimation Mechanisms

Consider the cooperative communication system based on a two-phase transmission within each signaling frame and a store-and-forward relay, as depicted in Fig. 1. The desired signal is transmitted from a source (S) to a destination (D) through the cooperation of a relay (R). Assume that perfect signal synchronization is achieved in the cooperation network and channels are constant but unknown in all communication links and transmission phases. Under the setup that any terminal can not transmit and receive simultaneously, different cooperation protocols [3] and channel estimation mechanisms are described as follows.

A. Cooperation Protocols

FCP realizes full cooperation in both listening and cooperation phases. In the listening phase, the source broadcasts the desired signal to both relay and destination. In the cooperation phase, the source re-transmits the signal to the destination and the relay amplifies the received signal in the listening phase with a relay gain α and transmits the amplified signal to the destination, where α is a positive real number.

PCP realizes partial cooperation under the scenario that the source engages in signal reception from other terminal in the network in the cooperation phase and thus is unable to transmit. In the listening phase, the source broadcasts the desired signal to both relay and destination. In the cooperation phase, only the relay transmits the amplified signal with gain α to the destination.

By analogizing the distributed terminals to the spatially distributed antennas, FCP and PCP can be deemed as multiple-input-multiple-output and single-input-multiple-output systems, respectively, in a distributed fashion. When considering perfect channel knowledge in the coherent receiver at destination, comparisons have been made on coherent demodulation among both protocols in [3] where FCP is shown to perform better than PCP in terms of both achievable transmission rate and error performance, whereas PCP is more efficient in terms of energy consumption at source than FCP, since the source transmits twice for FCP and only once for PCP.

Apart from FCP and PCP, a different partial cooperation protocol is also provided in [3] where the destination engages in signal transmission to other terminal in the network during the listening phase and thus is unable to receive. Unlike FCP and PCP, such a protocol is devoid of any diversity gain resulting in poor performance in both achievable transmission rate and error performance, and is not considered herein.

B. Channel Estimation Mechanisms

For the purpose of CCE and DCE, the source transmits a block of N training signals denoted by $\mathbf{s} = [s_0, s_1, \ldots, s_{N-1}]^T$ in each designated transmission phase. In the listening phase, the received signals at relay and destination are respectively given by

$$\mathbf{r}_l = h_{SR}\mathbf{s} + \mathbf{n}_{\mathbf{r}_l} \tag{1}$$

and

$$\mathbf{d}_l = h_{SD}\mathbf{s} + \mathbf{n}_{\mathbf{d}_l} \tag{2}$$

for both FCP and PCP, while in the cooperation phase the received signal at destination is given by

$$\begin{aligned}\mathbf{d}_c &= \alpha h_{RD}\mathbf{r}_l + h_{SD}\mathbf{s} + \mathbf{n}_{\mathbf{d}_c} \\ &= (h_{SD} + \alpha h_{RD} h_{SR})\,\mathbf{s} + \alpha h_{RD}\mathbf{n}_{\mathbf{r}_l} + \mathbf{n}_{\mathbf{d}_c}\end{aligned} \tag{3}$$

for FCP and

$$\begin{aligned}\mathbf{d}_c &= \alpha h_{RD}\mathbf{r}_l + \mathbf{n}_{\mathbf{d}_c} \\ &= \alpha h_{RD} h_{SR}\mathbf{s} + \alpha h_{RD}\mathbf{n}_{\mathbf{r}_l} + \mathbf{n}_{\mathbf{d}_c}\end{aligned} \tag{4}$$

for PCP. Here, for convenience, $\mathbf{n}_{\mathbf{x}_z}$ denotes the identically-independent-distributed complex additive white Gaussian noise vector with zero mean and covariance $N_0\mathbf{I}_N$ at relay ($\mathbf{x} = \mathbf{r}$) or destination ($\mathbf{x} = \mathbf{d}$) in the listening ($z = l$) or cooperation ($z = c$) phase. Based on these signal models, the complex channel gains h_{SR}, h_{SD}, and h_{RD} can be estimated by CCE and DCE mechanisms as follows.

For CCE, the destination universally estimates h_{SD}, h_{SR}, and h_{RD} based on the received vectors $\mathbf{r}_l$, $\mathbf{d}_l$, and $\mathbf{d}_c$ under the assumption that the received vector $\mathbf{r}_l$ at relay is perfectly fedforward to destination.

For DCE, the relay estimates h_{SR} based on $\mathbf{r}_l$, and the destination estimates h_{SD} and h_{RD} based on $\mathbf{d}_l$ and $\mathbf{d}_c$, respectively, under the assumption that the estimation result of h_{SR} at relay is perfectly fedforward to destination.

Note that CCE requires the received vectors at both relay and destination, which would incur larger feed-forward communication overhead than DCE [8].

III. CRBs for AF Cooperative Communication Systems

In this section, the CRB expressions for CCE and DCE mechanisms are analyzed. To facilitate the analysis, a general CRB formula is first given in the following for estimating a complex M-dimensional parameter vector $\boldsymbol{\theta} = [\theta_0, \theta_1, ..., \theta_{M-1}]^T$ based on a complex K-dimensional observation vector $\mathbf{y}$. The CRB expressions for both specific mechanisms are then derived accordingly.

A. General Formula for CRB

The CRB of $\boldsymbol{\theta}$ is denoted by $CRB(\boldsymbol{\theta})$ and defined by the diagonal entries of the inverse of the Fisher information matrix (FIM) $\mathbf{F}(\boldsymbol{\theta})$ as [6]

$$\begin{aligned} CRB(\boldsymbol{\theta}) \;\; = \;\; & \big[[\mathbf{F}^{-1}(\boldsymbol{\theta})]_{00}, [\mathbf{F}^{-1}(\boldsymbol{\theta})]_{11}, \ldots \\ & , [\mathbf{F}^{-1}(\boldsymbol{\theta})]_{(M-1)(M-1)}\big]^T . \end{aligned} \tag{5}$$

Here, $\mathbf{F}(\boldsymbol{\theta})$ is defined by

$$\mathbf{F}(\boldsymbol{\theta}) = E\left\{ \frac{\partial}{\partial \boldsymbol{\theta}^*} \ln f(\mathbf{y}|\boldsymbol{\theta}) \frac{\partial}{\partial \boldsymbol{\theta}^*} \ln f(\mathbf{y}|\boldsymbol{\theta})^H \right\} \tag{6}$$

when the condition

$$E\left\{ \frac{\partial}{\partial \boldsymbol{\theta}^*} \ln f(\mathbf{y}|\boldsymbol{\theta}) \frac{\partial}{\partial \boldsymbol{\theta}^*} \ln f(\mathbf{y}|\boldsymbol{\theta})^T \right\} = 0 \tag{7}$$

is satisfied [6], where $f(\mathbf{y}|\boldsymbol{\theta})$ represents the conditional likelihood density of $\mathbf{y}$ given $\boldsymbol{\theta}$. If the condition in (7) is not satisfied, a real vector $\boldsymbol{\theta}^r = [\theta_0^r, \theta_1^r, ..., \theta_{2M-1}^r]$ is considered and composed by stacking the real and imaginary parts of the complex parameters θ_m's with $\theta_{2m}^r = \mathrm{Re}\{\theta_m\}$ and $\theta_{2m+1}^r = \mathrm{Im}\{\theta_m\}$ for $m \in \{0, 1, ..., M-1\}$. Based on $\boldsymbol{\theta}^r$, $\mathbf{F}(\boldsymbol{\theta}^r)$ is given by [6]

$$[\mathbf{F}(\boldsymbol{\theta}^r)]_{ij} = tr\left\{ \mathbf{C}_{\mathbf{y}}^{-1} \frac{\partial \mathbf{C}_{\mathbf{y}}}{\partial \theta_i^r} \mathbf{C}_{\mathbf{y}}^{-1} \frac{\partial \mathbf{C}_{\mathbf{y}}}{\partial \theta_j^r} \right\} + 2\mathrm{Re}\left\{ \frac{\partial \mathbf{u}^H}{\partial \theta_j^r} \mathbf{C}_{\mathbf{y}}^{-1} \frac{\partial \mathbf{u}}{\partial \theta_j^r} \right\} \tag{8}$$

when $f(\mathbf{y}|\boldsymbol{\theta}^r)$ is a complex Gaussian likelihood density with mean $\mathbf{u}$ and covariance $\mathbf{C}_{\mathbf{y}} \triangleq E\{(\mathbf{y}-\mathbf{u})(\mathbf{y}-\mathbf{u})^H\}$. Following (5), $CRB(\boldsymbol{\theta}^r)$ is obtained as the diagonal entries of $\mathbf{F}^{-1}(\boldsymbol{\theta}^r)$. Then, $CRB(\boldsymbol{\theta})$ is given by $CRB(\theta_m) = CRB(\theta_{2m}^r) + CRB(\theta_{2m+1}^r)$ for $m \in \{0, 1, ..., M-1\}$.

B. CRBs for CCE

The channels are estimated only at destination for CCE. Now, $\mathbf{y}$ is composed by stacking the received vectors with dimension $K = 3N$ as $\mathbf{y} = [\mathbf{r}_l^T, \mathbf{d}_l^T, \mathbf{d}_c^T]^T = \mathbf{u} + \mathbf{n}$, where the vectors $\mathbf{u}$ and $\mathbf{n}$ represent the signal and noise parts, respectively, of $\mathbf{y}$. For FCP and PCP, $\mathbf{u}$ is given by $[h_{SR}\mathbf{s}^T, h_{SD}\mathbf{s}^T, (h_{SD} + \alpha h_{RD} h_{SR})\mathbf{s}^T]^T$ and $[h_{SR}\mathbf{s}^T, h_{SD}\mathbf{s}^T, \alpha h_{RD} h_{SR}\mathbf{s}^T]^T$, respectively. $\mathbf{n}$ is given by $[\mathbf{n}_{\mathbf{r}_l}^T, \mathbf{n}_{\mathbf{d}_l}^T, \alpha h_{RD}\mathbf{n}_{\mathbf{r}_l}^T + \mathbf{n}_{\mathbf{d}_c}^T]^T$ for both FCP and PCP. When $\boldsymbol{\theta} = [h_{SD}, h_{SR}, h_{RD}]^T$, $f(\mathbf{y}|\boldsymbol{\theta})$ is complex Gaussian distributed with mean $\mathbf{u}$ and covariance $\mathbf{C}_{\mathbf{y}}$ given by

$$\mathbf{C}_{\mathbf{y}} = N_0 \begin{bmatrix} \mathbf{I}_N & \mathbf{O}_N & \alpha h_{RD}^* \mathbf{I}_N \\ \mathbf{O}_N & \mathbf{I}_N & \mathbf{O}_N \\ \alpha h_{RD} \mathbf{I}_N & \mathbf{O}_N & (1 + \alpha^2 |h_{RD}|^2) \mathbf{I}_N \end{bmatrix}. \tag{9}$$

TABLE I

CRBs for CCE and DCE. For convenience, we let $CRB(x) = \frac{1}{N\gamma} \cdot \frac{1}{\Gamma_x}$, $\Delta = 1 + \alpha^2 |h_{RD}|^2$, $G_0 = 2$ and $G_1 = \alpha^2 |h_{RD}|^2 (2 + \alpha^2 |h_{RD}|^2)$ for FCP, and $G_0 = 1$ and $G_1 = \alpha^2 |h_{RD}|^2$ for PCP.

Γ_x	$Mecha-nism$	FCP	PCP
$\Gamma_{h_{SD}}$	CCE	$1 + \frac{1}{1+\gamma \lvert h_{SR} \rvert^2}$	1
	DCE	$1 + \frac{1}{1+\left(\gamma \lvert h_{SR} \rvert^2 \left(\frac{\Delta}{\alpha^2 \lvert h_{RD} \rvert^2}\right)+2\right)\Delta}$	1
$\Gamma_{h_{SR}}$	CCE & DCE	1	
$\Gamma_{h_{RD}}$	CCE	$\alpha^2 \left(\frac{\lvert h_{SR} \rvert^2}{G_0} + \frac{1}{\gamma} \right)$	
	DCE	$\left(\frac{\lvert h_{SR} \rvert^2}{\lvert h_{RD} \rvert^2 + \frac{G_0}{\alpha^2}} \right) \left(1 + \frac{1}{1+\gamma \lvert h_{SR} \rvert^2 \left(1+\frac{1}{G_1}\right)} \right)$	

$\mathbf{C}_{\mathbf{y}}^{-1}$ can be obtained according to the formula provided in [12, Sec. 0.7.3] and given by

$$\mathbf{C}_{\mathbf{y}}^{-1} = N_0^{-1} \begin{bmatrix} \left(1 + \alpha^2 |h_{RD}|^2\right) \mathbf{I}_N & \mathbf{O}_N & -\alpha h_{RD}^* \mathbf{I}_N \\ \mathbf{O}_N & \mathbf{I}_N & \mathbf{O}_N \\ -\alpha h_{RD} \mathbf{I}_N & \mathbf{O}_N & \mathbf{I}_N \end{bmatrix}. \tag{10}$$

For both FCP and PCP, $f(\mathbf{y}|\boldsymbol{\theta})$ satisfies the condition in (7) and $\mathbf{F}(\boldsymbol{\theta})$ in (6) is thus derived as

$$\mathbf{F}(\boldsymbol{\theta}) = \begin{bmatrix} 2N\gamma & 0 & \alpha h_{SR} N\gamma \\ 0 & N\gamma & 0 \\ \alpha h_{SR}^* N\gamma & 0 & \alpha^2 N \left(1 + \gamma |h_{SR}|^2\right) \end{bmatrix} \tag{11}$$

for FCP, and

$$\mathbf{F}(\boldsymbol{\theta}) = \begin{bmatrix} N\gamma & 0 & 0 \\ 0 & N\gamma & 0 \\ 0 & 0 & \alpha^2 N \left(1 + \gamma |h_{SR}|^2\right) \end{bmatrix} \tag{12}$$

for PCP, where $\gamma = \mathbf{s}^H\mathbf{s}/(N \cdot N_0)$ denotes the transmitted symbol signal-to-noise power ratio (SNR). As a result, $CRB(\boldsymbol{\theta})$'s are easily obtained by inverting $\mathbf{F}(\boldsymbol{\theta})$'s according to [12, Sec. 0.7.3] and provided in Table I.

C. CRBs for DCE

The channels are estimated distributively at relay and at destination for DCE. Based on the received vectors at relay and at destination, the CRBs of the channel gains for FCP and PCP are derived as follows.

Obviously, h_{SR} for both FCP and PCP is estimated at relay through the direct $S \longrightarrow R$ transmission link, which corresponds to the conventional point-to-point channel estimation mechanism, i.e., DLCE with training vector $\mathbf{s}$. Here, the channel parameter vector $\boldsymbol{\theta}$ is set to $[h_{SR}]$ and estimated through observation vector $\mathbf{y} = \mathbf{r}_l$. Specifically, $f(\mathbf{y}|\boldsymbol{\theta})$ is complex Gaussian distributed with mean $\mathbf{u} = h_{SR}\mathbf{s}$ and covariance $\mathbf{C}_\mathbf{y} = N_0\mathbf{I}_N$, which satisfies the condition in (7). Thus, $\mathbf{F}(\boldsymbol{\theta})$ can be immediately obtained by substituting $f(\mathbf{y}|\boldsymbol{\theta})$ into (6) as $N\gamma$ and it results in $CRB(\boldsymbol{\theta}) = (N\gamma)^{-1}$, which is also the CRB for DLCE.

Next, we consider $CRB(\boldsymbol{\theta})$ with $\boldsymbol{\theta} = [h_{SD}, h_{RD}]^T$ and $\mathbf{y} = [\mathbf{d}_l^T, \mathbf{d}_c^T]^T$. In the case, $f(\mathbf{y}|\boldsymbol{\theta})$ is complex Gaussian distributed with covariance

$$\mathbf{C}_\mathbf{y} = N_0 \begin{bmatrix} \mathbf{I}_N & \mathbf{O}_N \\ \mathbf{O}_N & (1+\alpha^2 |h_{RD}|^2)\mathbf{I}_N \end{bmatrix} \tag{13}$$

for both FCP and PCP, and with mean $\mathbf{u} = \left[h_{SD}\mathbf{s}^T, (h_{SD} + \alpha h_{RD} h_{SR})\,\mathbf{s}^T\right]^T$ for FCP and $\mathbf{u} = \left[h_{SD}\mathbf{s}^T, \alpha h_{RD} h_{SR}\mathbf{s}^T\right]^T$ for PCP. However, this likelihood density does not satisfy the condition in (7). Instead, $\boldsymbol{\theta}^r = [\operatorname{Re}\{h_{SD}\}, \operatorname{Im}\{h_{SD}\}, \operatorname{Re}\{h_{RD}\}, \operatorname{Im}\{h_{RD}\}]^T$ is considered. Note that $f(\mathbf{y}|\boldsymbol{\theta}^r)$ is also Gaussian distributed with $\mathbf{u}$ and $\mathbf{C}_\mathbf{y}$ identical to those of $f(\mathbf{y}|\boldsymbol{\theta})$. By putting $\mathbf{u}$ and $\mathbf{C}_\mathbf{y}$ into (8), $\mathbf{F}(\boldsymbol{\theta}^r)$ is derived for both FCP and PCP as

$$\mathbf{F}(\boldsymbol{\theta}^r) = 2N\gamma \begin{bmatrix} \mathbf{F}_0 & \mathbf{F}_1 \\ \mathbf{F}_2 & \mathbf{F}_3 \end{bmatrix} \tag{14}$$

where $\mathbf{F}_0 = (1 + (1+\alpha^2 |h_{RD}|^2)^{-1})\mathbf{I}_2$ and

$$\mathbf{F}_1 = \mathbf{F}_2^T = (L_0/\alpha) \begin{bmatrix} \operatorname{Re}\{h_{SD}\} & -\operatorname{Im}\{h_{SD}\} \\ \operatorname{Im}\{h_{SD}\} & \operatorname{Re}\{h_{SD}\} \end{bmatrix} \tag{15}$$

for FCP, $\mathbf{F}_0 = \mathbf{I}_2$ and $\mathbf{F}_1 = \mathbf{F}_2 = \mathbf{O}_2$ for PCP, and

$$\mathbf{F}_3 = \begin{bmatrix} \frac{2}{\gamma}L_0^2 L_1^2 + L\,|h_{SR}|^2 & \frac{2}{\gamma}L_0^2 L_1 L_2 \\ \frac{2}{\gamma}L_0^2 L_1 & \frac{2}{\gamma}L_0^2 L_2^2 + L_0\,|h_{SR}|^2 \end{bmatrix} \tag{16}$$

for both FCP and PCP, with $L_0 = (\alpha^{-2} + |h_{RD}|^2)^{-1}$, $L_1 = \operatorname{Re}\{h_{RD}\}$, and $L_2 = \operatorname{Im}\{h_{RD}\}$. Inverting $\mathbf{F}(\boldsymbol{\theta}^r)$ according to [12, Sec. 0.7.3], $CRB(h_{SD})$'s and $CRB(h_{RD})$'s for FCP and PCP are readily obtained and shown in Table I.

IV. Performance Comparison

In this section, the CRBs of CCE and DCE are compared for the AF cooperative communication systems employing FCP and PCP. As mentioned previously, the direct estimation over a single transmission link is identical to DLCE. For convenience, DLCE is used as a benchmark and the CRBs in Table I are expressed in the form of $CRB(x) = \frac{1}{N\gamma} \cdot \frac{1}{\Gamma_x}$, where $\frac{1}{N\gamma}$ is the CRB for DLCE with transmitted symbol SNR γ and training vector $\mathbf{s}$. Here, Γ_x denotes the CRB improvement factor in the sense that the mechanism with $\Gamma_x > 1$ outperforms the DLCE mechanism in the achievable CRB.

Observing Table I, we have several comments on the achievable CRBs for estimating the channel gains on three transmission links. First, for $S \longrightarrow R$ link, both CCE and DCE yield the same CRB as DLCE, regardless of the cooperation protocol. This indicates that $\mathbf{d}_l$ on $R \longrightarrow D$ link does not provide additional information on h_{SR} when $\mathbf{r}_l$ is available.

Second, for $S \longrightarrow D$ link, CCE with PCP and DCE with PCP are mechanisms with direct estimation on h_{SD} and thus also yield the same CRB as DLCE. Besides, CCE with FCP performs slightly better than DCE with FCP in the achievable CRB as shown in Table I, and the improvement factors for both estimation mechanisms are within $2 \geq \Gamma_{h_{SD}} > 1$ that outperforms DLCE. Particularly, this improvement comes from the fact that information on h_{SD} is observed twice through the two transmission phases and thus results in the best improvement in CRB ($\Gamma_{h_{SD}} \cong 2$) when $|h_{SR}|$ is virtually small and improvement decreases as $|h_{SR}|$ increases. This implies that the relayed signal from $S \longrightarrow R \longrightarrow D$ link can be deemed as interference for the estimation of h_{SD}. Thus, the weaker the interference, the lower the achievable CRB.

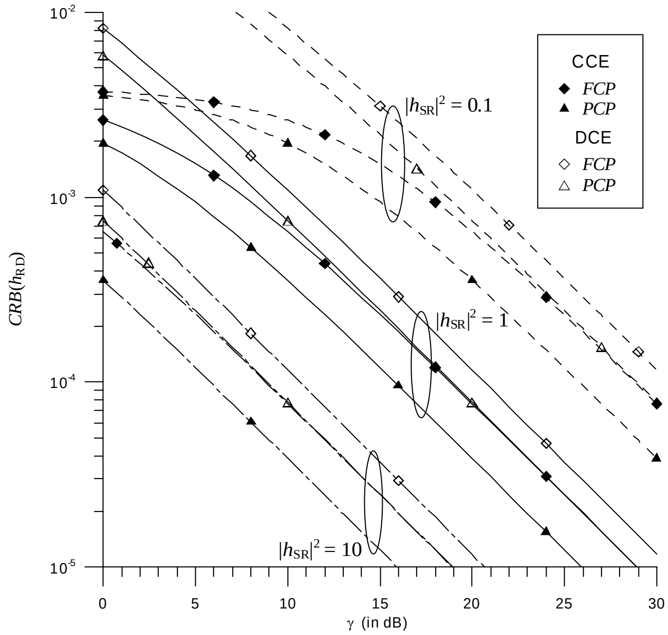

Fig. 2. CRBs for estimation on h_{RD} with $\alpha = 1$.

Third, for $R \longrightarrow D$ link, Fig. 2 is provided for making comparison among the CRBs of h_{RD} when $N = 256$ and $|h_{RD}|^2 = 1$ for DCE. Here, a unity relay gain ($\alpha = 1$) is considered, though higher α does help decreasing the achievable CRB of h_{RD} as observed in Table I. As shown in Fig. 2 CCE outperforms DCE for both FCP and PCP, since CCE gathers all received signals at both relay and destination to make estimation whereas DCE is merely based on the received signal at destination. Moreover, for both CCE and DCE, FCP is inferior to PCP in estimating h_{RD} as shown in Fig. 2 because for FCP the received signal at destination comes not only from $R \longrightarrow D$ link but also from $S \longrightarrow D$ link and the signal from $S \longrightarrow D$ link interferes virtually the estimation of h_{RD}. On the contrary to the estimation on h_{SD}, higher $|h_{SR}|$ obviously helps the estimation of h_{RD} in the cascaded $S \longrightarrow R \longrightarrow D$ link.

In summary, CCE is a better choice than DCE on the estimation of both h_{SD} and h_{RD} for the AF cooperative communication systems, though at a higher communication overhead.

V. Conclusions

The limiting performance characteristics of centralized and distributed channel estimations are studied in terms of CRB for the cooperative communication systems with single AF relay and two transmission phases. The CRB expressions corresponding to source-to-destination, source-to-relay, and relay-to-destination transmission links are analytically derived for full and partial cooperation protocols. It is shown that centralized channel estimation is a better mechanism than distributed channel estimation, especially in estimating the channel gains in source-to-destination and relay-to-destination links. Moreover, full cooperation protocol performs better than partial cooperation protocol in estimating the channel gain in source-to-destination link, while partial cooperation protocol performs better in estimating the channel gain in relay-to-destination link.

Acknowledgment

This work was supported by the NTU Excellent Research Project Contract 99R80305 and the Taiwan NSC Contracts 98-2218-E-002-020 and 100-2221-E-002-125-MY2.

References

[1] J. N. Laneman and G. W. Wornell, "Distributed space–time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[2] M. Gastpar and M. Vetterli, "On the capacity of wireless networks: The relay case," in *Proc. IEEE Int. Conf. on Computer Commun.*, New York, NY, Jun. 2002, vol. 3, pp. 1577–1586.

[3] R. U. Nabar, H. Bölcskei, and F. W. Kneubühler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099-1109, Aug. 2004.

[4] P. A. Parker, P. Mitran, D. W. Bliss, and V. Tarokh, "On bounds and algorithms for frequency synchronization for collaborative communication systems," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3742-3752, Aug. 2008.

[5] S. S. Ikki and M. H. Ahmed, "Exact error probability and channel capacity of the best-relay cooperative-diversity networks," *IEEE Signal Process. Lett.*, vol. 16, no. 12, pp. 1051–1054, Dec. 2009.

[6] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice-Hall: Englewood Cliffs, NJ, 1993.

[7] T. C.-Y. Ng and W. Yu, "Joint optimization of relay strategies and resource allocations in cooperative cellular networks," *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 328–339, Feb. 2007.

[8] K. T. Phan, L. B. Le, S. A. Vorobyov, and T. Le-Ngoc, "Power allocation and admission control in multiuser relay networks via convex programming: centralized and distributed schemes," *EURASIP J. Wireless Commun. and Networking*, vol. 2009, Article ID 901965.

[9] O. Amin, B. Gedik, and M. Uysal, "Channel estimation for amplify-and-forward relaying: Cascaded against disintegrated estimators," *IET Commun.*, vol. 4, no. 10, pp. 1207–1216, Jul. 2010.

[10] J.-J. Xiao, A. Ribeiro, Z.-Q. Luo, and G. B. Giannakis, "Distributed compression-estimation using wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 27–41, Jul. 2006.

[11] K. Kim and H. Park, "OFDM channel estimation for the amplify-and-forward cooperative channel," in *Proc. IEEE 65th Veh. Technol. Conf.*, Dublin, Apr. 2007, pp. 1642-1646.

[12] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.