Cramér-Rao Bounds for Channel Estimation in Amplify-and-Forward Cooperative Communication Systems

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Abstract—The Cramér-Rao bounds of centralized and distributed channel estimations are studied for the cooperative communication systems with single amplify-and-forward relay and two transmission phases. Particularly, two cooperation protocols are considered with full cooperation in both transmission phases and partial cooperation in the listening phase, respectively. It is shown that centralized channel estimation is a better choice than distributed channel estimation for both full and partial cooperation protocols.

I. INTRODUCTION

Cooperative diversity is an efficacious method to realize spatial diversity gain through the distributed relay terminals acting as a virtual antenna array in a communication network. The cooperation network has been shown to provide the same diversity order as an equivalent multiple-input-multiple-output system [1] and higher capacity than the conventional point-to-point communication network [2] when some specific cooperation protocols are adopted.

A typical class of protocols [3]-[4] supporting the cooperation network with three terminals consisting of a source, a relay, and a destination has been extensively studied based on a two-phase transmission, namely a listening phase followed by a cooperation phase, within each signaling frame as shown in Fig. 1. In these protocols, the source transmits the desired signal to the relay in the listening phase, and during the cooperation phase the relay processes the received signal in the listening phase by either amplify-and-forward (AF) or decode-and-forward (DF) operation and transmit the outcome to the destination. Depending on the protocol [3] implementing full or partial cooperation, the source transmits the desired signal to the destination in both listening and cooperation phases or in listening or cooperation phase, respectively, as illustrated by the dashed links in Fig. 1.

Coherent detection is considered in most cooperative communication systems employing the above-mentioned protocols [3]-[5] under the system setup that the complex channel gains $h_{SD}$, $h_{SR}$, and $h_{RD}$ corresponding to source-to-destination ($S \rightarrow D$), source-to-relay ($S \rightarrow R$), and relay-to-destination ($R \rightarrow D$) links are perfectly estimated. For the DF relaying systems, this setup can be best approached by employing the conventional direct-link channel estimation (DLCE) mechanism [5]-[7] to separately estimate $h_{SD}$ and $h_{RD}$ at destination and $h_{SR}$ at relay in that coherent detection is carried out at both DF relay and destination. However, DLCE is not adequate for the AF relaying systems employing the cooperation protocols, since all the received signals at both AF relay and destination contain useful channel gain information and have to be taken into account during channel estimation. In the latter case, two alternative channel estimation mechanisms, namely centralized channel estimation (CCE) and distributed channel estimation (DCE) [7]-[11], are viable choices in order to achieve better estimation performance. Specifically, channel gains for the three transmission links are estimated all at destination for CCE and distributively at relay and destination for DCE, under an ideal circumstance that a reliable feedforward control channel [7]-[9] is required...
to send full or partial $S \rightarrow R$ channel state information from relay to destination for both CCE and DCE mechanisms.

Several channel estimation schemes based on CCE or DCE are proposed in [9]-[11], but the limiting performance characteristics of CCE and DCE in terms of Cramér-Rao bound (CRB) for the AF cooperative communication systems employing the above-mentioned cooperation protocols still remain unexplored. Although an attempt in [11] tries to obtain bound (CRB) for the AF cooperative communication systems, DCE are proposed in [9]-[11], but the limiting performance for relay to destination for both CCE and DCE mechanisms.

By analogizing the distributed terminals to the spatially distributed antennas, FCP and PCP can be deemed as multiple-input-multiple-output and single-input-multiple-output systems, respectively, in a distributed fashion. When considering perfect channel knowledge in the coherent receiver at destination, comparisons have been made on coherent demodulation among both protocols in [3] where FCP is shown to perform better than PCP in terms of both achievable transmission rate and error performance, whereas PCP is more efficient in terms of energy consumption at source than FCP, since the source transmits twice for FCP and only once for PCP.

Apart from FCP and PCP, a different partial cooperation protocol is also provided in [3] where the destination engages in signal transmission to other terminal in the network during the listening phase and thus is unable to receive. Unlike FCP and PCP, such a protocol is devoid of any diversity gain resulting in poor performance in both achievable transmission rate and error performance, and is not considered herein.

B. Channel Estimation Mechanisms

For the purpose of CCE and DCE, the source transmits a block of $N$ training signals denoted by $s = [s_0, s_1, \ldots, s_{N-1}]^T$ in each designated transmission phase. In the listening phase, the received signals at relay and destination are respectively given by

$$r_i = h_{SR}s + n_{ri},$$  \hspace{1cm} (1)

and

$$d_c = h_{SD}s + n_{dc},$$  \hspace{1cm} (2)

for both FCP and PCP, while in the cooperation phase the received signal at destination is given by

$$d_c = ah_{RD}r_i + h_{SR}s + n_{dc} = (h_{SD} + ah_{RD}h_{SR})s + ah_{RD}n_{ri} + n_{dc}.$$  \hspace{1cm} (3)

for FCP and

$$d_c = ah_{RD}r_i + n_{dc} = ah_{RD}h_{SR} + ah_{RD}n_{ri} + n_{dc}.$$  \hspace{1cm} (4)

For PCP. Here, for convenience, $n_{dc}$ denotes the identically-independent-distributed complex additive white Gaussian noise vector with zero mean and covariance $N_0I_N$ at relay ($x = r$) or destination ($x = d$) in the listening ($z = l$) or cooperation ($z = c$) phase. Based on these signal models, the complex channel gains $h_{SR}, h_{SD},$ and $h_{RD}$ can be estimated by CCE and DCE mechanisms as follows.

For CCE, the destination universally estimates $h_{SR}, h_{SD},$ and $h_{RD}$ based on the received vectors $r_i, d_l,$ and $d_c$ under the assumption that the received vector $r_i$ at relay is perfectly fedforward to destination.

For DCE, the relay estimates $h_{SR}$ based on $r_i,$ and the destination estimates $h_{SD}$ and $h_{RD}$ based on $d_l$ and $d_c,$ respectively, under the assumption that the estimation result of $h_{SR}$ at relay is perfectly fedforward to destination.

Note that CCE requires the received vectors at both relay and destination, which would incur larger feed-forward communication overhead than DCE [8].
III. CRBs for AF Cooperative Communication Systems

In this section, the CRB expressions for CCE and DCE mechanisms are analyzed. To facilitate the analysis, a general CRB formula is first given in the following for estimating a complex M-dimensional parameter vector \( \theta = [\theta_0, \theta_1, \ldots, \theta_{M-1}]^T \) based on a complex K-dimensional observation vector \( y \). The CRB expressions for both specific mechanisms are then derived accordingly.

A. General Formula for CRB

The CRB of \( \theta \) is denoted by CRB(\( \theta \)) and defined by the diagonal entries of the inverse of the Fisher information matrix (FIM) \( F(\theta) \) as [6]

\[
\text{CRB}(\theta) = \left[ F^{-1}(\theta) \right]_{\theta_0, \theta_1, \ldots, \theta_{M-1}}.
\]

(5)

Here, \( F(\theta) \) is defined by

\[
F(\theta) = E \left\{ \frac{\partial}{\partial \theta^T} \ln f(y|\theta) \frac{\partial}{\partial \theta} \ln f(y|\theta)^T \right\}
\]

when the condition

\[
E \left\{ \frac{\partial}{\partial \theta^T} \ln f(y|\theta) \frac{\partial}{\partial \theta} \ln f(y|\theta)^T \right\} = 0
\]

is satisfied [6], where \( f(y|\theta) \) represents the conditional likelihood density of \( y \) given \( \theta \). If the condition in (7) is not satisfied, a real vector \( \theta' = [\theta_0', \theta_1', \ldots, \theta_{2M-1}'] \) is considered and composed by stacking the real and imaginary parts of the complex parameters \( \theta_m \)’s with \( \theta_{2m} = \text{Re}\{\theta_m\} \) and \( \theta_{2m+1} = \text{Im}\{\theta_m\} \) for \( m \in \{0, 1, \ldots, M-1\} \). Based on \( \theta' \), \( F(\theta') \) is given by [6]

\[
[F(\theta')]_{ij} = \text{tr} \left\{ C_y \frac{\partial C_y}{\partial \theta_i} C_y^{-1} \frac{\partial C_y}{\partial \theta_j} + 2\text{Re} \left\{ \frac{\partial u_i}{\partial \theta_j} C_y^{-1} \frac{\partial u_i}{\partial \theta_j} \right\} \right\}
\]

(8)

when \( f(y|\theta') \) is a complex Gaussian likelihood density with mean \( u \) and covariance \( C_y = E[(y-u)(y-u)^H] \). Following (5), CRB(\( \theta' \)) is the diagonal entries of \( F^{-1}(\theta') \). Then, CRB(\( \theta \)) is given by CRB(\( \theta_m \)) = CRB(\( \theta'_{2m} \)) + CRB(\( \theta'_{2m+1} \)) for \( m \in \{0, 1, \ldots, M-1\} \).

B. CRBs for CCE

The channels are estimated only at destination for CCE. Now, \( y \) is composed by stacking the received vectors with dimension \( K = 3N \) as \( y = [r_1^T, d_1^T, d_2^T]^T = u + n \), where the vectors \( u \) and \( n \) represent the signal and noise parts, respectively, of \( y \). For FCP and PCP, \( u \) is given by \([h_{SR} s^T, h_{SD} s^T, (h_{RD} h_{SR}) s^T]\) and \([h_{SR} s^T, h_{SD} s^T, ah_{RD} h_{SR} s^T]\), respectively. \( n \) is given by \([n_1^T, n_2^T, ah_{RD} n_1^T + n_2^T]^T \) for both FCP and PCP. When \( \theta = [h_{SD}, h_{SR}, h_{RD}]^T \), \( f(y|\theta) \) is complex Gaussian distributed with mean \( u \) and covariance \( C_y \) given by

\[
C_y = N_0 \begin{bmatrix}
I_N & O_N & ah_{RD}^* I_N \\
O_N & I_N & O_N \\
ah_{RD} I_N & O_N & (1 + a^2|h_{RD}|^2) I_N
\end{bmatrix}
\]

(9)

\( C_y^{-1} \) can be obtained according to the formula provided in [12, Sec. 0.7.3] and given by

\[
C_y^{-1} = N_0^{-1} \begin{bmatrix}
(1 + 2|a|^2|h_{RD}|^2) I_N & O_N & -ah_{RD}^* I_N \\
O_N & I_N & O_N \\
-ah_{RD} I_N & O_N & I_N
\end{bmatrix}
\]

(10)

For both FCP and PCP, \( f(y|\theta) \) satisfies the condition in (7) and \( F(\theta) \) in (6) is thus derived as

\[
F(\theta) = \begin{bmatrix}
2N_\theta & 0 & ah_{SR} N_\theta \\
0 & N_\theta & 0 \\
ah_{SR}^* N_\theta & 0 & 2N_\theta (1 + |h_{SR}|^2)
\end{bmatrix}
\]

(11)

for FCP, and

\[
F(\theta) = \begin{bmatrix}
N_\theta & 0 & 0 \\
0 & N_\theta & 0 \\
0 & 0 & 2N_\theta (1 + |h_{SR}|^2)
\end{bmatrix}
\]

(12)

for PCP, where \( c = s^H s/(N \cdot N_0) \) denotes the transmitted symbol-to-noise power ratio (SNR). As a result, CRB(\( \theta \))’s are easily obtained by inverting \( F(\theta) \)’s according to [12, Sec. 0.7.3] and provided in Table I.

\[
\gamma = \frac{s^H s}{(N \cdot N_0)}
\]

C. CRBs for DCE

The channels are estimated distributively at relay and at destination for DCE. Based on the received vectors at relay and at destination, the CRBs of the channel gains for FCP and PCP are derived as follows.

Obviously, \( h_{SR} \) for both FCP and PCP is estimated at relay through the direct \( S \rightarrow R \) transmission link, which corresponds to the conventional point-to-point channel estimation mechanism, i.e., DLEC with training vector \( s \). Here, the channel parameter vector \( \theta \) is set to \([h_{SR}] \) and estimated through observation vector \( y = r_i \). Specifically, \( f(y|\theta) \) is
complex Gaussian distributed with mean $\mathbf{u} = h_{SR}\mathbf{s}$ and covariance $\mathbf{C}_Y = N_0\mathbf{I}_N$, which satisfies the condition in (7). Thus, $\mathbf{F}(\theta)$ can be immediately obtained by substituting $f(y(\theta))$ into (6) as $N\gamma$ and it results in $CRB(\theta) = (N\gamma)^{-1}$, which is also the CRB for DLCE.

Next, we consider $CRB(\theta)$ with $\theta = [h_{SD}, h_{RD}]^T$ and $y = [d_1^T, d_2^T]^T$. In the case, $f(y(\theta))$ is complex Gaussian distributed with covariance

$$\mathbf{C}_Y = N_0 \begin{bmatrix} \mathbf{I}_N & \mathbf{O}_N \\ \mathbf{O}_N & (1 + \alpha^2|h_{RD}|^2)\mathbf{I}_N \end{bmatrix}$$

for both FCP and PCP, and with mean $\mathbf{u} = [h_{SD}s^T, (h_{SD} + ah_{RD}h_{SR})s^T]^T$ for FCP and $\mathbf{u} = [h_{SD}s^T, ah_{RD}h_{SR}s^T]^T$ for PCP. However, this likelihood density does not satisfy the condition in (7). Instead, $\theta' = [\text{Re}\{h_{SD}\}, \text{Im}\{h_{SD}\}, \text{Re}\{h_{RD}\}, \text{Im}\{h_{RD}\}]^T$ is considered. Note that $f(y(\theta'))$ is also Gaussian distributed with $\mathbf{u}$ and $\mathbf{C}_Y$, identical to those of $f(y(\theta))$. By putting $\mathbf{u}$ and $\mathbf{C}_Y$ into (8), $\mathbf{F}(\theta')$ is derived for both FCP and PCP as

$$\mathbf{F}(\theta') = 2N\gamma \begin{bmatrix} \mathbf{F}_0 & \mathbf{F}_1 \\ \mathbf{F}_2 & \mathbf{F}_3 \end{bmatrix}$$

where $\mathbf{F}_0 = (1 + (1 + \alpha^2|h_{RD}|^2)^{-1})\mathbf{I}_2$ and

$$\mathbf{F}_1 = \frac{(L_0/\alpha)}{2} \begin{bmatrix} \text{Re}\{h_{SD}\} & -\text{Im}\{h_{SD}\} \\ \text{Im}\{h_{SD}\} & \text{Re}\{h_{SD}\} \end{bmatrix}$$

for FCP, $\mathbf{F}_0 = \mathbf{I}_2$ and $\mathbf{F}_1 = \mathbf{F}_2 = \mathbf{O}_2$ for PCP, and

$$\mathbf{F}_3 = \begin{bmatrix} \frac{2}{\gamma}L_2^2 + L_1 \|h_{SR}\|^2 & \frac{2L_2}{\gamma}L_1L_2 \\ \frac{2L_2}{\gamma}L_1 & \frac{2L_2^2}{\gamma}L_2 + L_0 \|h_{SR}\|^2 \end{bmatrix}$$

for both FCP and PCP, with $L_0 = (\alpha^{-2} + |h_{RD}|^2)^{-1}$, $L_1 = \text{Re}\{h_{RD}\}$, and $L_2 = \text{Im}\{h_{RD}\}$. Inverting $\mathbf{F}(\theta')$ according to [12, Sec. 0.7.3], $CRB(h_{SD})$'s and $CRB(h_{RD})$'s for FCP and PCP are readily obtained and shown in Table I.

IV. PERFORMANCE COMPARISON

In this section, the CRBs of CCE and DCE are compared for the AF cooperative communication systems employing FCP and PCP. As mentioned previously, the direct estimation over a single transmission link is identical to DLCE. For convenience, DLCE is used as a benchmark and the CRBs in Table I are expressed in the form of $CRB(\theta) = \frac{\Gamma_{RD}}{\gamma}$, where $\frac{\Gamma_{RD}}{\gamma}$ is the CRB for DLCE with transmitted symbol SNR $\gamma$ and training vector $\mathbf{s}$. Here, $\Gamma_{RD}$ denotes the CRB improvement factor in the sense that the mechanism with $\Gamma_{RD} > 1$ outperforms the DLCE mechanism in the achievable CRB.

Observing Table I, we have several comments on the achievable CRBs for estimating the channel gains on three transmission links. First, for $S \rightarrow R$ link, both CCE and DCE yield the same CRB as DLCE, regardless of the cooperation protocol. This indicates that $d_1$ on $R \rightarrow D$ link does not provide additional information on $h_{SR}$ when $R_1$ is available.

Second, for $S \rightarrow D$ link, CCE with PCP and DCE with PCP are mechanisms with direct estimation on $h_{SD}$ and thus also yield the same CRB as DLCE. Besides, CCE with FCP performs slightly better than DCE with FCP in the achievable CRB as shown in Table I, and the improvement factors for both estimation mechanisms are within $2 \geq \Gamma_{RD} > 1$ that outperforms DLCE. Particularly, this improvement comes from the fact that information on $h_{SD}$ is observed twice through the two transmission phases and thus results in the best improvement in CRB ($\Gamma_{RD} \equiv 2$) when $|h_{SR}|$ is virtually small and improvement decreases as $|h_{SR}|$ increases. This implies that the relayed signal from $S \rightarrow R \rightarrow D$ link can be deemed as interference for the estimation of $h_{SD}$. Thus, the weaker the interference, the lower the achievable CRB.

Third, for $R \rightarrow D$ link, Fig. 2 is provided for making comparison among the CRBs of $h_{RD}$ when $N = 256$ and $|h_{RD}|^2 = 1$ for DCE. Here, a unity relay gain ($\alpha = 1$) is considered, though higher $\alpha$ does help decreasing the achievable CRB of $h_{RD}$ as observed in Table I. As shown in Fig. 2 CCE outperforms DCE for both FCP and PCP, since CCE gathers all received signals at both relay and destination to make estimation whereas DCE is merely based on the received signal at destination. Moreover, for both CCE and DCE, PCP is inferior to PCP in estimating $h_{RD}$ as shown in Fig. 2 because for FCP the received signal at destination comes not only from $R \rightarrow D$ link but also from $S \rightarrow D$ link and the signal from $S \rightarrow D$ link interferes virtually the estimation of $h_{RD}$. On the contrary to the estimation on $h_{SD}$, higher $|h_{SR}|$ obviously helps the estimation of $h_{RD}$ in the cascaded $S \rightarrow R \rightarrow D$ link.

In summary, CCE is a better choice than DCE on the estimation of both $h_{SD}$ and $h_{RD}$ for the AF cooperative communication systems, though at a higher communication overhead.
V. CONCLUSIONS

The limiting performance characteristics of centralized and distributed channel estimations are studied in terms of CRB for the cooperative communication systems with single AF relay and two transmission phases. The CRB expressions corresponding to source-to-destination, source-to-relay, and relay-to-destination transmission links are analytically derived for full and partial cooperation protocols. It is shown that centralized channel estimation is a better mechanism than distributed channel estimation, especially in estimating the channel gains in source-to-destination and relay-to-destination links. Moreover, full cooperation protocol performs better than partial cooperation protocol in estimating the channel gain in source-to-destination link, while partial cooperation protocol performs better in estimating the channel gain in relay-to-destination link.

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