Design of Efficient Clustering Dictionary for Sparse Representation of General Image Super Resolution

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***Abstract*—Designing an efficient over-complete dictionary is an important issue for developing a learning based system of super-resolution. To obtain fast solution, the size of dictionary needs to be reduced. However it may lower the performance as dictionary maybe incomplete. To address this issue, in this paper, we propose an improvement of dictionary learning for image super-resolution based on sparse representation. The proposed training process method can generate an efficient clustering of an over-complete dictionary without reducing its size but with low computational cost when applied with basis pursuit denosing of sparse representation solution. The performance of the proposed dictionary is satisfactory in terms of computational time reduction with comparable RMSE (root mean square error) when compared with other known methods.**

I. Introduction

Super-resolution is a technique to reverse image capturing process. Super-resolution can be used to generate high-resolution image given a single or multiple low-resolution images. This process is needed in many digital image and video processing applications especially for real time video surveillance. Video surveillance is a system that requires fast processing time.

Super-resolution is becoming needed in video surveillance to generate high resolution image in order to improve accuracy. This is because, in general, most video camera used for surveillance system give low resolution image. The quality of low resolution image is still deteriorated in low lighting condition, environmental noises, and camera angle position. Thus, fast and accurate super-resolution method is urgently needed as a preprocessing step for video surveillance data to further analysis.

There are three categories of super-resolution method; single image interpolation, multiple images reconstruction, and learning based super-resolution. Learning based super-resolution method will generate high resolution image by learning fine details in both low and high resolution images. This method can generate better quality high resolution image than other methods. From this reason, this method becomes more attractive in present days.

There are two main steps in learning based method of super-resolution: a training step and a testing step. The training step is the time to design a dictionary matrix from a set of image data set. The resulted dictionary is then used in the testing step. A low resolution image is transformed to high resolution image with the dictionary as a reference.

Designing an efficient dictionary and arriving at the optimized solution are the key combination inefficient learning-based super-resolution. Flexibility, simplicity, efficiency, and well-defined objective are desirable properties for a successful dictionary [1], [2], [3]. As an example, Aharon, et. al. [1] proposed Generalized K-Means via Singular Value Decomposition or K-SVD as a basic of clustering dictionary. However, the computational cost to design the dictionary is very high and take long time to converge in testing step. These limitations make the dictionary inefficient. To improve the dictionary, more efficient over-complete dicionary is needed. The goal is to have small dimension dictionary with fast training process and with the optimized solution as much as possible.

Thus, in this paper, we focus on the design of an efficient dictionary for generating high-resolution image from a single low-resolution image using learning based approach, especially via sparse representation [4]. Then we used this proposed dictionary in this approach to enhance detail characteristic of the object's features effectively and efficiently.

The remainders of this paper are organized as follows. In section II, we explain briefly about sparse representation of image super-resolution with elastic-net as a solution. The review of dictionary learning algorithms for image super-resolution is explained in section III. Section IV shows the results and analysis. Conclusion of this paper is given in the last section.

II. Sparse Representation of Image Super-Resolution

When we take a picture by camera, the captured image $\boldsymbol{Y}$ is gotten from real view high-resolution image $\boldsymbol{X}$ which is decimated by factor $\boldsymbol{L}$ and is added blur and noise by $\boldsymbol{V}$ filter. By this statement, we can represent the reconstruction constraint, as in (1).

$$Y = LVX \tag{1}$$

Super-resolution is how to reverse the process. The main task is how to generate high resolution image Y given low resolution image X. super-resolution is aimed to bring back missing information on low resolution image. One great method in learning based super-resolution is using sparse representation to recover missing information in caputuring process.

Sparse representation will help to present most or all information from a pair of low and high resolution image using linear combination of a small number of elements, or called atoms [4]. These atoms are chosen from an over-complete dictionary matrix $\boldsymbol{D}$.

A. Sparse Representation of Image Super-Resolution

The problem of super-resolution is how to recover as much as possible information from degraded high resolution image. In learning based super-resolution, it stores information in one matrix, called dictionary. Suppose $\boldsymbol{D} \in \Re^{n \times k}$ is an over-complete dictionary that can be represented in sparse representation, as in (2).

$$y_i = D\beta_i \ \ , \ \ i = 1, 2, ..., N \tag{2}$$

Where vector $\beta \in \Re^k$ contains the representations coefficients of the signal y. The error of the sparest solution of the (2) represented by P_0, must be less than or equal to ϵ.

$$\min_{\beta} ||\beta||_0 \quad s.t. \quad ||y - D\beta||_2 \leq \epsilon \tag{3}$$

Two ways to solve the representation to find β are Matching Pursuit and Basis Pursuit. Matching Pursuit is a greedy algorithm that finds one atom at a time and select atom sequentially to get the sparest solution based on ℓ_0-norm. Basis Pursuit suggests a convexification of the problems mentioned in (2) and (3), by replacing the ℓ_0-norm with an ℓ_1-norm. Because of its advantages, stability and high estimation accuracy, we use Basis Pursuit to solve super-resolution problem.

Vector $\beta \in \Re^k$ as a representations coefficients of the low resolution signal can be found using least square method. Because of the complexity of the signal, constraints are needed when least square is applied. Ridge regression, Least Absolute Shrinkage Selection (Lasso - ℓ_1-norm) [4], or combination between tow constrains will make least square more powerful to find β.

B. Elastic Net for Sparse Representation Solution

Least squares is easy to understand for smaller models, but not for larger models. Also, it has a high-variance even though the bias is low. It has effect on difficulties to interprate the model of super-resolution since the numbuer of agressors is large. The challenge is how to minimize mean squared error by appropriate trade off between bias and variance.

The first idea is called ridge regression as in (4). Ridge regression shrinks the regression coefficients by imposing a penalty on their size. The limitation of this penalty is inputs must be standardized, because ridge solutions are not equivariant under scaling of the inputs.

$$\beta_{EN} = \underset{\beta}{\operatorname{argmin}} ||y - \beta D||^2 + \lambda_2 ||\beta||_2^2 \tag{4}$$

Another idea is called Lasso as in (5). Lasso does shrinkage and variable selection simultaneously for better prediction and model interpretation. Limitations of Lasso are it will selects at most n variables before it saturates and it can not do group selection [5].

$$\beta_{EN} = \underset{\beta}{\operatorname{argmin}} ||y - \beta D||^2 + \lambda_1 |\beta|_1 \tag{5}$$

The common problem in image processing is that there are very large number of pixels as a variables compared to number of blocks as an observation i.e. $p \gg n$. With this condition, if we only use one constraint i.e. Lasso, we will find some drawbacks. The drawbacks are the unfulfilled requirement to select at most n variables before it saturates and the inability to do group selection. If there is a group of variables among which the pairwise correlations are very high, then the Lasso tends to arbitrarily select only one variable from the group [6]. Elasticnet as a formula that combine Lasso and Ridge Regression as can be used to solve this problem.

$$\beta_{EN} = \underset{\beta}{\operatorname{argmin}} ||y - \beta D||^2 + (1 - \alpha)\lambda_2 ||\beta||_2^2 + \alpha\lambda_1 |\beta|_1 \tag{6}$$

As shown in (6), Elastic Net, as a new method, is powerful to solve this problem, especially to generate high resolution image [7], [8]. Elastic Net is combination of ℓ_1 and ℓ_2 penalty where λ_1 and λ_2 are positive weights. If $\lambda_1 = 0$, Elastic Net will be Ridge Regression and if $\lambda_2 = 0$, Elastic Net will be Lasso. Scale factor α is used to avoid double shrinkage to reduce bias, so better performance will be achieved. The elastic net produces a sparse model with good prediction accuracy, while encouraging a grouping effect.

III. Dictionary for Sparse Representation of Image Super-Resolution

Dictionary is a main part in learning based system as well as in sparse representation method. The way to design a dictionary can be classified into two basic approaches: analytic approach and learning-based approach. In the analytic approach, the dictionary model is represented by mathematical formulation with an efficient analytic construction. This approach such as Wavelets, Curvelets, and Edgelets, will conduct a fast numerical calculation in highly structured dictionary [2].

The dictionary with learning based approach is deduced from training process of a set of examples. The advantages of this approach are the more detailed information brought in that dictionary matrix, significantly better performance in applications, and promising better results. However, this approach always generate an unstructured dictionary, which is more costly to apply, and complexity limitation keep a tight rein on the size of the dictionary [2].

A. Dictionary Learning Review

In order to have more flexibility through sparse representation, Donoho [9] proposed combination of Wavelets and Edgelets to generate an over-complete dictionary.

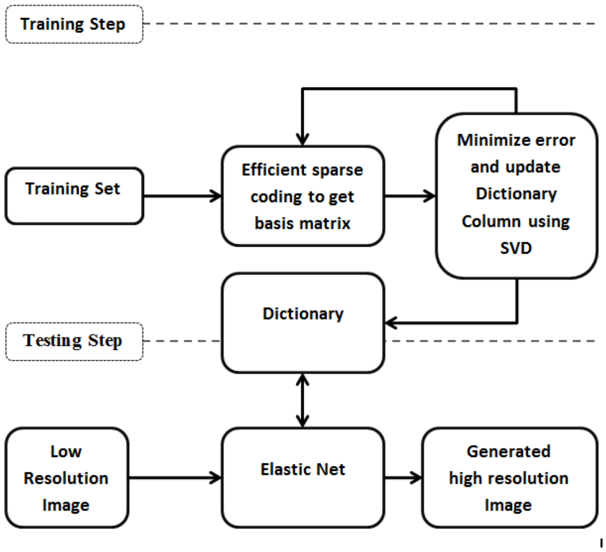


Fig. 1. Block diagram of the training step and testing step super-resolution using proposed method and elastic net.

Another approach is based on clustering for adapting dictionaries to achieve sparse representation of signals, proposed by Liao et. al. [10]. The method, called K-LMS algorithm, is aimed to design an overcomplete dictionary. It generalize the K-Means clustering process, for adapting dictionaries to achieve sparse representation of signals.

Aharon et. al. [1] proposed dictionary for sparse representation based on K-means clustering process. They introduced new method in sparse representation called K-SVD. K is come from K-Means and SVD is Singular Value Decomposition. This dictionary was purposed to work on both orthogonal matching pursuit and basis pursuit.

B. *Efficient Clustering Dictionary*

An efficient dictionary learning and the efficient solution step are an ideal combination to achieve best performance. In this paper, we propose an efficient clustering dictionary, a new combination and optimization of a dictionary training learned from the drawbacks of K-SVD. The main idea of building an efficient super-resolution based on its sparse representation is shown in Fig. 1. Based on the figure, we devided super-resolution process into two steps, training step and testing step.

The proposed dictionary learning method is matched to basis pursuit and then synchronized to elastic-net as an efficient solution of sparse representation as shown in Algorithm 1. We used efficient sparse coding based on basis pursuit denoising which is equipped with feature exctraction and combined with decomposition process to reduce redundant data.

Algorithm 1
Learning Based Image Super-Resolution Algorithm

A. Training Step

1. Initialize Dictionary $D_{i \times j}$
2. Applying efficient sparse coding
   Using basis pursuit denoising and feature extraction
3. Dictionary Column Update
   Reducing estimated error using Singular Value Decomposition
4. Repeat 2 and 3 until convergence (stopping rule) or reaching maximum iteration
5. Save dictionary D

B. Solution (Testing) Step

1. Testing input: a low resolution image Y
2. Upsize Y using Bicubic interpolation
3. For each block pixel y taken starting from the upper left corner with 1 pixel overlap in each direction,
   a. Solve the optimization problem with D and y in (4): $\beta_{EN} = \underset{\beta}{\text{argmin}} ||y - \beta D||^2 + (1-\alpha)\lambda_2 ||\beta||_2^2 + \alpha\lambda_1 |\beta|_1$
   b. $\beta*$ = Normalized Coefficients β
4. Output: super-resolution $X*$

In the training step, the process was started from pairs of low and high resolution image patches denoted as Dl and Dh. Matrix of Dh is set as an initial dictionary. Since Dh is the initial dictionary as problem stated in (2), the sparsest basis matrix as a correlation between Dl and Dh can be solved using efficient sparse coding of basis pursuit denoising. Using basis pursuit denoising in (7), basis matrix can be generated.

$$\min_{\beta_i} \|y - D\beta_i\|_2^2 \quad s.t. \quad \|\beta_i\|_0 \leq \epsilon \tag{7}$$

To improve regular basis pursuit denoising, in this paper, we add ℓ_1 penalty, as in (8). This constrain is aimed to do feature extraction from the training set.

$$\min_{\beta_i} \|y - D\beta_i\|_2^2 + \lambda|\beta|_1 \quad s.t. \quad \|\beta_i\|_0 \leq \epsilon \tag{8}$$

Like a clustering step of the generalized K-means, dictionary column update is performed. Firstly, in dictionary column update, we tried to colculate residual error of the dictionary. The error value could be found by calculating the difference between Y and multiplication of basis matrix and dictionary element. This calculation was done column by column. Error in Dictionary D at column j is expressed in (9). And then, minimum error at column j could be achieved by finding minimum value of each element in Dictionary D at the column.

$$E_j = Y \quad - \quad \sum_{i \neq j} d_i \beta_i^T \tag{9}$$

$$\min_{D_j} \|D_j \beta_j^T - E_j\|_F^2 \quad \forall i, \|\beta_i\|_0 \leq \epsilon \tag{10}$$

The next step to update the dictionary column by column is performing singular value decomposition (SVD). As suggested in [1], residual error E_j is decomposed to find alternative value of D_j and β_j. SVD will effectively minimize the error by finding the closest rank-1 matrix (in Forbenius norm) that aproximate E_j.

We performed both processes, sparse coding process and decomposition process, iteratively. We also tried to keep the result didn't diverge. We did this by holding elements on initial dictionary while updating basis matrix, and keep value of β while updating dictionary column. The detail algorithm of this proposed dictionary training is shown in Algorithm 2.

Algorithm 2

Detail of Dictionary Learning Algorithm in Training Step

1. Initialize Dictionary $D_{i\times j}$
   a. Prepare pairs of high and low resolution image patches as training tet Dh and Dl
   b. Set Dh as an initial Dictionary
   c. Dl as a signal is prepared to generate basis matrix (β)
2. Applying efficient sparse coding
   a. Using Basis Pursuit Denoising to approximate minmum value of basis matrix: $||\beta_i||_0 \leq \epsilon \quad s.t. \quad \min_{\beta_i} ||y - D\beta_i||_2^2$
   b. Do the same on all rows of Dh
3. Dictionary Column Update
   a. Compute error $E_j = Y \quad - \quad \sum_{i \neq j} d_i \beta_i^T$
   b. Apply SVD to get minimum error value E_j=$\boldsymbol{U}\Delta V^T$
   c. Do the same on all columns of Dh
4. Repeat 2 and 3 until convergence or reaching maximum iteration.
5. Save Dh as the Dictionary D

IV. Experimental Results

In this section, we discuss about our experiment on designing an over-complete dictionary for super-resolution using efficient clustering algorithm as an improvement of optimized K-SVD. We also show the performance measurement of the proposed dictionary algorithm in term of speed and root mean square error measurement, compare with original K-SVD [1].

A. Preparing Data Set

The training set for generating dictionary was taken from Caltech-256 image data set [11]. We choose 25 categories (bear, dolphin, duck, fern, grapes, hibiscus, hummingbird, iris, killer-whale, owl, palm tree, penguin, people, racoon, rainbow, sheet music, skycraper, snake, stained glass, sunflower, swan,

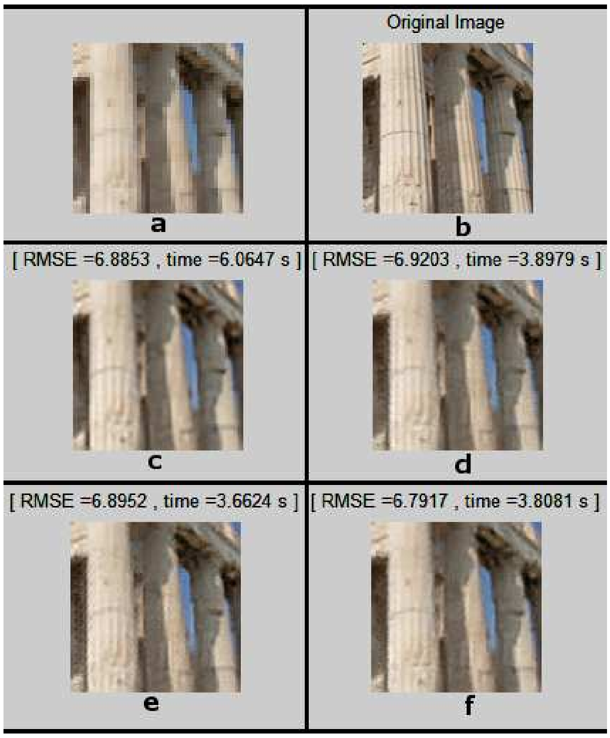


Fig. 2. Sample images: (a) low resolution image (40 × 40 pixels), (b) original high resolution image (120 × 120 pixels) , and generated high resolution image (120 × 120 pixels) using Elastic Net and various dictionary (c) Original-KSVD, (d) proposed 10 iterations, (e) proposed 20 iterations, (f) proposed 30 iterations.

teapot, tennis court, tomato, pisa tower) with 10 images per category. This data set was prepared to be trained in joint learning process. The resolution of images from the data set were down graded to obtain the low resolution version of the images. Then, pairs of patches from low and high resolution images were randomly taken to obtain two correlated training set matrix Dl and Dh. Subsequently, both Dh and Dl were trained in joint learning dictionary process.

B. Training Dictionary

At first dictionary was trained using original version of KSVD. In this process, the optimization loop was conducted over fifty times. The dictionaries were trained in various size, e.g. 256, 512, 768, 1024, 1280, 1536, 1792, and 2048. The proposed dictionary, efficient clustering algorithm, was applied to generate the dictionary using three different iterations. These efficient clustering dictionary training was performed in 10 loops, 20 loops, and 30 loops.

C. Elastic Net as a Suitable Solution for Proposed Dictionary

Elastic Net is an improvement of ordinary least square method by adding combination of ℓ_1 and ℓ_2 penalty. It is a fast solution to solve sparse representation on finding β in (3). Since the dictionary is trained under basis pursuit denoising, Elastic Net will be the most suitable solution.

From fig. 2 we can see that Elastic Net can deal with original-KSVD and our proposed dictionary. High resolution images generated using Elastic Net with these dictionaries have competitive RMSE. Time consumed by Elastic Net to generate high resolution image using our proposed dictionary is faster than using original-KSVD.

D. Dictionary Performance Against Various Pictures

All four dictionaries resulted from the algorithms explained in subsection IV.B were tested using Elastic Net to generate high resolution image from single low resolution image. The testing images were taken from other random images which were not included in the training process. Some of the results of this testing process using 1024-sized dictionary is shown in Table 1. The test was performed by 1.37 GHz of Intel Core i7 processor and 4 GB of memory.

TABLE I
TESTING TIME AND RMSE OF THE GENERATED IMAGE USING VARIOUS DICTIONARY AND TRAINING ITERATION

Image	Dictionary (# of training iteration)	Test Time (s)	RMSE
Map	KSVD (50 iters)	23.25	6.3489
	Proposed (10 iters)	21.46	6.2911
	Proposed (20 iters)	21.30	**6.2897**
	Proposed (30 iters)	**19.48**	6.3041
Lena	KSVD (50 iters)	7.20	5.6480
	Proposed (10 iters)	7.48	5.5995
	Proposed (20 iters)	6.32	**5.5525**
	Proposed (30 iters)	**6.03**	5.6974
Books	KSVD (50 iters)	17.08	7.9396
	Proposed (10 iters)	7.08	7.8346
	Proposed (20 iters)	8.10	**7.7943**
	Proposed (30 iters)	**6.95**	7.8617
Parthenon	KSVD (50 iters)	4.24	7.0464
	Proposed (10 iters)	3.54	6.8952
	Proposed (20 iters)	**3.29**	6.9203
	Proposed (30 iters)	3.42	**6.8788**
Leaf	KSVD (50 iters)	21.77	**2.8770**
	Proposed (10 iters)	15.75	2.8794
	Proposed (20 iters)	**15.07**	2.8898
	Proposed (30 iters)	18.09	2.8884

From Table 1, it can be seen that searching time of the Elastic Net solution in proposed dictionary algorithm is faster than in original K-SVD. The dictionary itself was trained in less number of iterations (only 10, 20, and 30 times). However, the RMSE of the high resolution image generated by this dictionary was comparable and, in many cases, was lower than KSVD.

Theoritically, if the training steps (efficient sparse coding and dictionary column update) are performed iteratively, residual error value will become smaller. Smaller residual error value means the dictionary become more effective to be used. The performance of dictionary is better as the number of iterations are increased. In summary, the proposed method has advantages in term of fast processing time and comparable quality of the generated high resolution image given the input of a single low resolution one.

E. Dictionary Performance Against Codebook Size

In order to analyze the performance of the proposed dictionary, various size of the dictionaries had been comparized to generate high resolution image. Graphics shown in Fig. 4 illustrate the results of the RMS error and the time to generated high resolution images generated by 10, 20, and 30 interations of the proposed dictionary compared with 50 times iteration of KSVD dictionary.

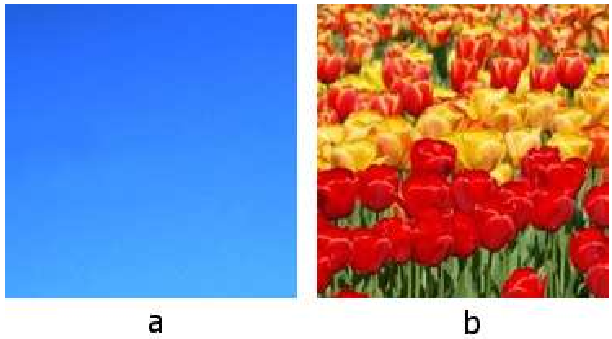


Fig. 3. Sample of simple image (a) and complex image (b) to show correlation between image complexity and dictionary performance.

In terms of processing time, proposed dictionary was seem to give faster result. From 256 to 2038-sized dictionary with 10, 20, and 30 iterations could generate high resolution images faster than 50 times iterations of KSVD dictionary. The slower time only occur on small size proposed dictionaries with 10 iterations applied on several images. If the dictionary was trained in 20 or 30 interations, the time to generate high resolution image is usuallyfaster than KSVD trained in 50 interations. The lower time cost to generate high resolution image, makes the proposed dictionary algorithm more preferable to be used in image super-resolution application. This is because people want to get high resolution image faster.

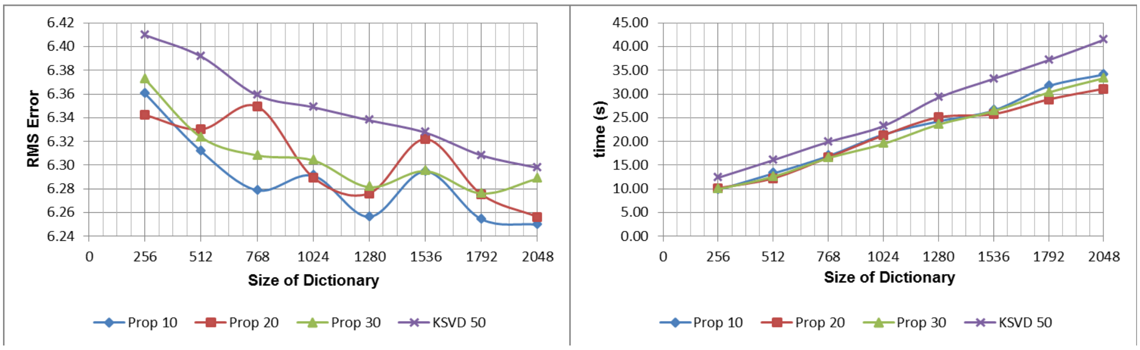
Generated high resolution images form proposed dictionary also have comparable RMSE as KSVD. Moreover, for most of the images, proposed dictionary give result with lower error than KSVD. This is because proposed algorithm is conducted to store adequate information in better representation.

Size of the dictionary will determine the performance of the system. Larger dictionaries always use more time to generate high resolution image than smaller dictionary. Larger dictionaries almost always give better result with lower error.

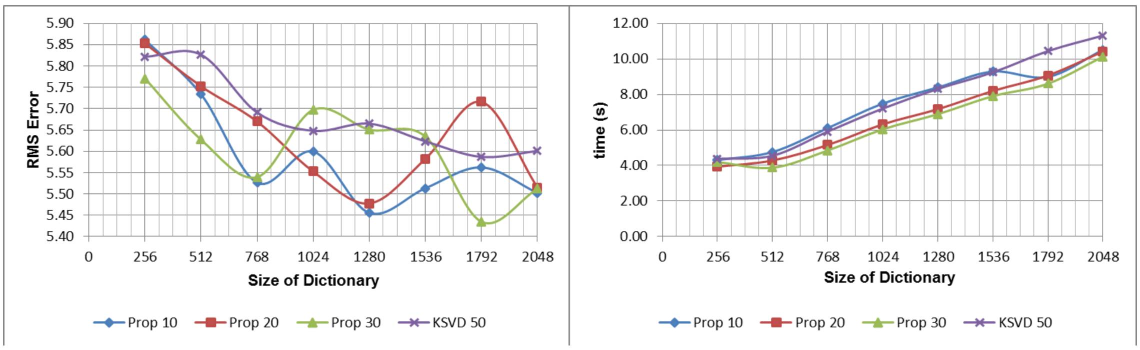
F. Analysis of Dictionary Testing Time and Measured Error

We define time used to test dictionary to generate high resolution image as testing time. Because of the dictionary matrix is fixed, the solvable formula is also fixed (e.g. Elastic Net), thus the error of the generated high resolution image from one same low resolution image is always fixed. Even though the image is tested over and over again. But different things happen in a matter of testing time.

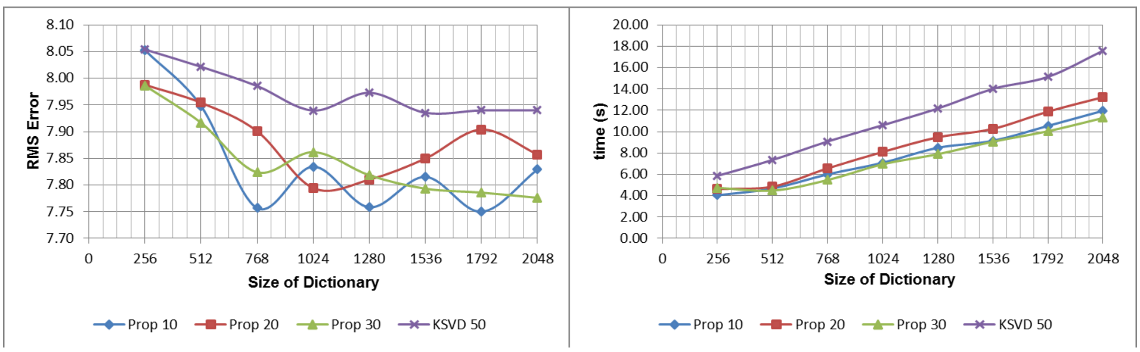
On the testing process, testing time is not fixed for same image. testing time depends on the speed of the computer and how many applications run at the same time. The testing time also depends on the software used to generate the high resolution image. Nevertheless, comparison of the time consumption among all of the dictionaries will always same.



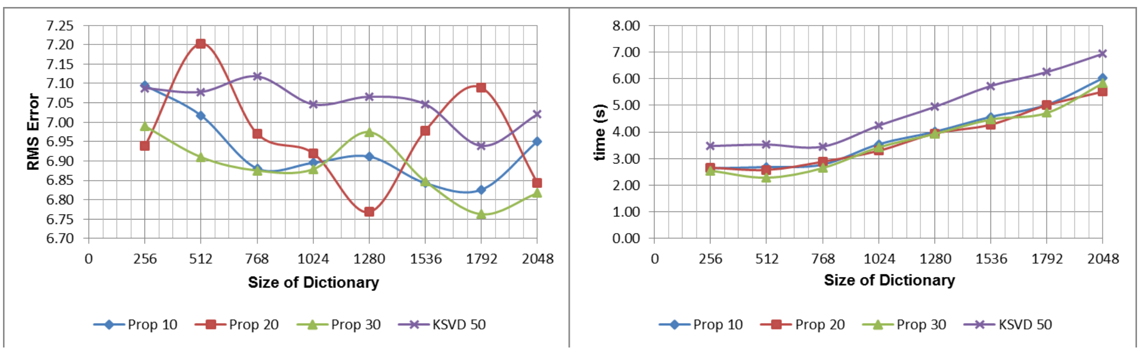
(a) Map



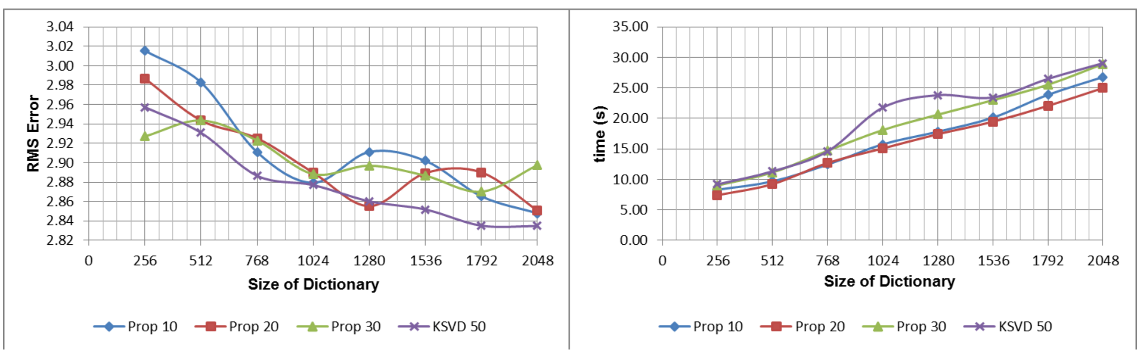
(b) Lena



(c) Books



(d) Parthenon



(e) Leaf

Fig. 4. Charts of the testing time and RMSE of the generated image using various dictionary and training iteration

Different size of the image will also give different testing time. But the complexity of the color and texture of the low resolution image will effect on RMS error much bigger than time consumption. This was evident when the dictionary is used to test the very simple and complex (colored and textured) image as shown in Fig. 3. The process to generate the simple image took 10.37 seconds and got RMS error 0.8084, while for the complex one took 10.71 seconds and got RMS error 7.4225.

From the calculated testing time of the process and the measured error generate high resolution image, the proposed dictionary gives better performance than KSVD. The proposed dictionary can be performed faster and give lower error of the generated high resolution images.

G. Analysis of Anomaly Across Dictionary Performances

As mentioned in Section IV, the larger dictionaries almost always give better result with lower error. This is theoritically correct, because the larger size of the dictionary means more adequate information are stored. However in some cases, the larger dictionaries can have more error than the smaller one. This incident is called anomaly. The anomaly can be seen from the graph in Fig. 4.b (Lena image) on the proposed dictionary with 20 iterations. When the size of dictionary is 1280, the RMS error is 5.4778, but when the size is increased to be 1792, the RMS error also increases to be 5.7159. In the testing time process, the uncertainty of the result sometimes happen in the unsupervised learning process. Other uncertainty results also can be seen in Table 1. Eventhough our proposed method can give faster results, occasionally higher training iteration cannot give faster testing time.

This anomaly indicates that improper size of the dictionary will give incorrect information. This error could be caused by some ambiguity among two or more observations that are stored in the codebooks. Since the observation data are randomly taken from block pixel in image data set, the degree of difference among all block in the training set cannot be controlled.

In order to achieve optimum we have to consider about the richness of the training set, patch sampling process, and how to prune the proposed training set. the other way to reduce the dictionary anomaly, clustering process in the training step should be optimized. The clustering process is performed to group common observation into the same cluster. In generalized clustering via SVD, each point will belong to a cluster with a certain intensity and clusters are not necessarily disjoint. An improvement of SVD applied in very large dimension of matrix is still needed to make the procedure feasible for the very large matrices.

V. Conclusions

An efficient dictionary is very important on super-resolution process. The way to design the dictionary will affect the solution step of the system. The proposed method, efficient clustering dictionary, gives better performance in term of faster solution speed and lower RMS error. By efficient clustering, the solution can find the best match between tested image and the dictionary codebook faster. The lower time cost makes the proposed dictionary algorithm more preferable to be used in image super-resolution application. This proposed algorithm is very promising to reduce anomaly performance of the dictionary size. Eventually, this dictionary is well suited with Elastic Net solution of sparse representation algorithm.

VI. Acknowledgment

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