New Targets Number Estimation Method Based on Spatial Smoothing with Auxiliary Vectors

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Abstract— **A Multiple Targets Detection method based on Spatial Smoothing (MTDSS) is proposed to solve the problem of source number estimation under color noise background. Forward-backward smoothing by using signal vectors on specific elements as auxiliary vectors is computed. By spatial smoothing with auxiliary vectors, the correlated signals are decorrelated, and the color noise is partially alleviated. The correlation matrix of forward-backward smoothed data is computed. Gerschgorin radii of the smoothed correlation matrix are computed by unitary transformation. By exploring the inherent property of the covariance matrix, a threshold based on the Gerschgorin radii is set to estimate the number of sources. Simulation results validate that MTDSS has effective performance under the condition of colored noise background and coherent sources.**

I. INTRODUCTION

Source number is a crucial factor in array signal processing. Multiple targets detection methods under white noise background have been widely explored, but this class of methods suffer from serious degradation when the noise environment becomes complex.

Previous source number estimation methods under complex noise background are to estimate the source number and the directions as a whole [1-3]. This class of methods is not convergent and computationally unattractive which is hard to apply in real time system. Some ameliorated methods are proposed to solve the problem of source number estimation under color noise background [4-8]. A method based on the ratio of minimum description length (MDL) criterion is proposed by smoothing the eigen values of the covariance matrix [4]. This method can deal with the problem of multiple targets detection in color noise environment, but does not perform well when the correlation of color noise is strong and signals are closed spaced. A method is proposed by revising the eigen values by filtering the array receiving data with the noise subspace projection matrix [5]. The influence of color noise is alleviated, so detection performance is improved. But this method has large computational burden. A method under color noise background is proposed by ubiety transformation of the Gerschgorin radii location [6], this method usually over estimates the number of signals. A multiple targets detection method is proposed based on K-means clustering algorithm by extracting signal eigenvalues from the all the eigenvalues [7]. K-means clustering algorithm is an iterative algorithm which performs efficiently under color noise background, but can not work effectively when the signals are closely spaced and the signals are correlated with each other. A nonparametric method for estimating the number of signals without eigendecomposition (MENSE) is proposed in [8]. The number of signals is revealed in the rank of the QR upper-trapezoidal factor of the autoproduct of a combined hankel matrix. The disadvantage of MENSE is that there exists large false probability, and is ineffective under complex noise background.

In this paper we proposed a multiple targets number detection method based on spatial smoothing (MTDSS). The array receiving data on the first and last elements are used as the auxiliary vectors, and the forward-backward covariance matrix is computed. A threshold criterion based on the Gerschgorin radii of the covariance matrix is set to estimate the target number. Simulation on 10 element array show that MTDSS can estimate the number of correlated signals and multiple targets under color noise background effectively. The detection performance is better than other methods existing.

II. SIGNAL MODEL

Assume there are q narrow band signals arrive at an M element array, the received data model can be written as follows

$$\mathbf{x}(t)=\sum_{i=1}^{q}\mathbf{a}(\theta_i)s_i(t)+\mathbf{w}(t)=\mathbf{A}(\Theta)\mathbf{s}+\mathbf{w},\ t=1,2,...,N \tag{1}$$

Where $\mathbf{x}(t)$ is the received data on the array. $\mathbf{a}(\theta_i)=[1,e^{j\omega_0\tau(\theta_i)},...,e^{j\omega_0(M-1)\tau(\theta_i)}]^{\mathrm{T}},i=1,2,...,q$ is the steering vector of the ith signal, and θ_i is its direction-of-arrival (DOA). $\mathbf{A}(\Theta)=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),...,\mathbf{a}(\theta_q)]$ is the steering matrix. $\mathbf{s}=[s_1(t),s_2(t),...,s_q(t)]^{\mathrm{T}}$ is the amplitude of the narrow band signal at time slot t. $\mathbf{w}=[w_1(t),w_2(t),...,w_M(t)]^{\mathrm{T}}$ is the additive noise, $t(t=1,2,...,N)$ and N denote the time slot and sample number respectively, $(\cdot)^{\mathrm{T}}$ denotes transposition.

This paper explores the multiple target detection method with coherent signals and color noise. Some basic assumptions are given as follows

1) The steering vector $\mathbf{a}(\theta_i),i=1,2,...,q$ is unknown;

2) The signals $s_1(t),s_2(t),...,s_q(t)$ are coherent narrow band signals, which satisfy

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$$s_k(t) = \beta_k s_1(t), k = 1,2,...,q \tag{2}$$

Where β_k is the complex attenuation coefficient with $\beta_k \neq 0$ and $\beta_1 = 1$;

3) $s_1(t), s_2(t),..., s_q(t)$ are signals with sufficient long time correlations. Assume there exists K, which satisfies $K \geq L \geq 1$, to insure $[\mathbf{Q}_L^s, \mathbf{Q}_{L+1}^s,..., \mathbf{Q}_K^s]$ is column full rank, where

$$\mathbf{Q}_l^s = E[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t-l)] \tag{3}$$

4) The correlated time of spatial color noise is shorter than the correlated signals, and $\forall l \geq L$ to insure the correctness of equation below

$$\mathbf{Q}_l^n = E[\mathbf{w}(t)\mathbf{w}^{\mathrm{H}}(t-l)] = 0 \tag{4}$$

5) Assume the number of subarray element is p, and the number of signals is q, they must satisfy the inequation $q < p$.

Where $(\cdot)^{\mathrm{H}}$ denotes conjugate transpose.

Assumption 1 indicates method in this paper needs not to know the prior directions of the signals. Assumption 2 describes the correlated relations between coherent signals. Assumption 3 and assumption 4 indicate that correlation time of noise is shorter than signal. Assumption 5 ensures the number of signals is smaller than the number of subarray element. In order to make the detectable condition agree with MENSE, the subarray size is chosen to be $q = M/2$. The subarray size can be adjusted according to real condition, and the maximum detectable number of signals is $p-1$.

III. SOURCE NUMBER ESTIMATION BASED ON SPATIAL SMOOTHING

A. Spatial smoothing with auxiliary vector

Assume the number of array element is M, and the size of overlapping subarray is $p(p<M)$. The number of overlapping subarray is $L = M - p + 1$, and the lth forward and backward subarray comprises $\{l, l+1,..., l+p-1\}$ and $\{M-l+1, M-l,..., L-l+1\}$ element for $l = 1,2,...,L$. The data vectors of the lth forward and backward subarrays are

$$\begin{aligned}\mathbf{x}_{fl}(t) &= [x_l(t), x_{l+1}(t),..., x_{l+p-1}(t)]^{\mathrm{T}} \\ &= \tilde{\mathbf{A}}\mathbf{D}^{l-1}\mathbf{s}(t) + \mathbf{w}_{fl}(t)\end{aligned} \tag{5}$$

$$\begin{aligned}\mathbf{x}_{bl}(t) &= [x_{M-l+1}(t), x_{M-l}(t),..., x_{L-l+1}(t)]^{\mathrm{T}} \\ &= \tilde{\mathbf{A}}\mathbf{D}^{-(M-l)}\mathbf{s}(t) + \mathbf{w}_{bl}(t)\end{aligned} \tag{6}$$

Where $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}(\theta_1), \tilde{\mathbf{a}}(\theta_2),..., \tilde{\mathbf{a}}(\theta_q)]$ is the submatrix of $\mathbf{A}(\Theta)$ consisting of the first p rows, and $\tilde{\mathbf{a}}(\theta_i) = [1, e^{j\omega_0\tau(\theta_i)},..., e^{j\omega_0(p-1)\tau(\theta_i)}]^{\mathrm{T}}$ for $i = 1,2,...,q$. $\mathbf{D} = diag[e^{j\omega_0 t(\theta_1)}, e^{j\omega_0 t(\theta_2)},..., e^{j\omega_0 t(\theta_q)}]$ is a diagonal matrix. The noise vectors of the lth forward and backward overlapping subarray are $\mathbf{w}_{fl}(t) = [w_l(t), w_{l+1}(t),..., w_{l+p-1}(t)]^{\mathrm{T}}$ and $\mathbf{w}_{bl}(t) = [w_{M-l+1}(t), w_{M-l}(t),..., w_{L-l+1}(t)]^{\mathrm{T}}$ respectively.

Define data sequences on the first and the Mth elements as auxiliary vectors. The correlation results between the lth forward and backward overlapping subarrays and the auxiliary vectors are written as $\phi_{fl} = E[\mathbf{x}_{fl}(t)\mathbf{x}_M^{\mathrm{H}}(t)]$, $\bar{\phi}_{fl} = E[\mathbf{x}_{fl}(t)\mathbf{x}_1^{\mathrm{H}}(t)]$, $\phi_{bl} = E[\mathbf{x}_{bl}(t)\mathbf{x}_1^{\mathrm{H}}(t)]$ and $\bar{\phi}_{bl} = E[\mathbf{x}_{bl}(t)\mathbf{x}_M^{\mathrm{H}}(t)]$ respectively. By combining the correlation results, we get the combined correlation matrix as

$$\Phi \overset{\Delta}{=} [\Phi_f, \bar{\Phi}_f, \Phi_b, \bar{\Phi}_b] \tag{7}$$

Where $\Phi_f, \bar{\Phi}_f, \Phi_b, \bar{\Phi}_b$ are defined as $\Phi_f = [\phi_{f1}, \phi_{f2},..., \phi_{f(L-1)}]^{\mathrm{TT}}$, $\bar{\Phi}_f = [\bar{\phi}_{f2}, \bar{\phi}_{f3},..., \bar{\phi}_{fL}]$, $\Phi_b = [\phi_{b1}, \phi_{b2},..., \phi_{b(L-1)}]^{\mathrm{T}}$ and $\bar{\Phi}_b = [\bar{\phi}_{b2}, \bar{\phi}_{b3},..., \bar{\phi}_{bL}]^{\mathrm{T}}$ respectively. And the dimension of the combined matrix Φ is $(M-p) \times 4p$.

The final covariance matrix is computed by the combined matrix, and it is written as follows

$$\varphi = \Phi\Phi^{\mathrm{H}} \tag{8}$$

The covariance matrix φ is a central symmetry matrix, and $\varphi = \varphi^{\mathrm{H}}$, is a Hermitian matrix.

Colored noise is decorrelated by spatial smoothing with auxiliary vectors, and coherent signals are decorrelated by averaging the results of each overlapping subarray.

B. Multiple targets detection based on spatial smoothing

As the correlation time of signal is longer than noise, this information can be used to detect the targets number. As in real applications, both the correlation of noise and signals sequences are strong, MENSE can not work effectively under these bad conditions. A new method is proposed by further processing the spatial smoothing data, and a multiple targets detection method based on spatial smoothing is proposed.

The covariance matrix φ in (8) is a square matrix with dimension of $(M-p) \times (M-p)$. Divide the matrix into small blocks, and $\varphi = \begin{bmatrix} \varphi_0 & \varphi_1^{\mathrm{H}} \\ \varphi_1 & \varphi_{(M-p)\times(M-p)} \end{bmatrix}$, compute the eigen decomposition of φ_0

$$\varphi_0 = \mathbf{U}_1 \Sigma \mathbf{U}_1^{\mathrm{H}} \tag{9}$$

$\mathbf{U}_1 = [\mathbf{u}_1, \mathbf{u}_2,..., \mathbf{u}_{M-p-1}]$ is the eigenvector of φ_0. The transform matrix is defined as

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \tag{10}$$

Multiply the spatial smoothing matrix φ with transform matrix, the result can be written as

$$\begin{aligned}\mathbf{U}^{\mathrm{H}}\varphi\mathbf{U} &= \begin{bmatrix} \mathbf{U}_1^{\mathrm{H}}\varphi_0\mathbf{U}_1 & \mathbf{U}_1^{\mathrm{H}}\varphi_1 \\ \varphi_1^{\mathrm{H}}\mathbf{U}_1 & \varphi_{(M-p)\times(M-p)} \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1 & \cdots & 0 & \rho_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \lambda_{M-p-1} & \rho_{M-p-1} \\ \rho_1^* & \cdots & \rho_{M-p-1}^* & \varphi_{(M-p)\times(M-p)} \end{bmatrix}\end{aligned} \tag{11}$$

Where $\lambda_1, \lambda_2, ..., \lambda_{M-p-1}$ are Gerschgorin centers, and $\rho_1, \rho_2, ..., \rho_{M-p-1}$ are the Gerschgorin radius corresponding to the Gerschgorin center.

Consider the combined correlation matrix in (7), and the covariance matrix of combined matrix in (8) can be expanded as

$$\varphi = \Phi_f \Phi_f^{\mathrm{H}} + \overline{\Phi}_f \overline{\Phi}_f^{\mathrm{H}} + \Phi_b \Phi_b^{\mathrm{H}} + \overline{\Phi}_b \overline{\Phi}_b^{\mathrm{H}} \quad (12)$$

Define a new matrix Δ which is one section of polynomial in (12)

$$\Delta = \Phi_f \Phi_f^{\mathrm{H}} \quad (13)$$

Now we consider the property of noise subspace in [9], and gain the facts as follows

$$\delta\Delta = \Delta - \Delta = 0\left[\sqrt{\frac{\log\log(p(M-p))}{p(M-p)}}\right] a.s. \quad (14)$$

Where $a.s.$ denotes almost sure convergent with probability 1. $0(\cdot)$ denotes infinitesimal of same order. As other sections of polynomial in (12) $\overline{\Phi}_f \overline{\Phi}_f^{\mathrm{H}}$, $\Phi_b \Phi_b^{\mathrm{H}}$, $\overline{\Phi}_b \overline{\Phi}_b^{\mathrm{H}}$ all have the similar propriety as matrix Δ, we introduce the multiple targets detection criterion

$$H(k) = |\rho_k| - coe\sqrt{\frac{\log\log(p(M-p))}{p(M-p)}} \sum_{i=1}^{M-p-1} |\rho_i|, \quad (15)$$
$$k = 1, 2, ..., M-p-1$$

Where coe is the coefficient that can be adjusted. N is the snapshots that is used. Sort H in descent order, and compute the first k that satisfy $H(k) < 0$, the estimated signal number is $\hat{q} = k - 1$.

The multiple targets detection criterion in (15) is the criterion of MTDSS method. Spatial smoothing can de-correlate the coherent signals, and the power of color noise is partly restrained. After spatial smoothing, the deference between signal and noise power becomes larger, and the corresponding Gerschgorin radii becomes larger. So it is easier for MENSE to estimate the target number.

IV. SIMULATION RESULTS

We devise simulation experiments based on correlated signals and color noise background, and the model of correlated signals is the same as in [8]. The covariance matrix of spatial color noise is written as

$$(\mathbf{Q})_{mn} = \sigma_n^2 \beta^{|m-n|} \exp(j2\pi(m-n)) \quad (16)$$

Where $(\cdot)_{mn}$ is the element of the mth row and nth column of noise covariance matrix, σ_n^2 is the noise power, and β is the correlation factor of color noise.

A. Multiple targets detection with correlated signals

This section explores the detection performance of MTDSS with multiple correlated signals. A 10 element uniform linear array (ULA) with half wavelength inter-element spacing is employed in this section. 3000 snapshots are used in each simulation. x coordinate denotes signal-to-noise power ratio in dB.

Fig. 1 shows the detection performance of MTDSS, MENSE and MDL on two correlated signals. The correlated factor of correlated narrow band signal is 0.4. The DOAs of incident signals are 0° and 5°. Simulation results of fig. 1 show that MTDSS performs better than MENSE and MDL method. When the probability of detection reaches 90%, MTDSS works better than MENSE and MDL of 2dB and 8dB respectively.

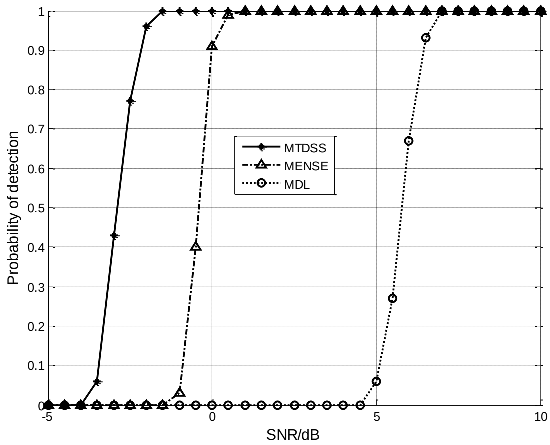

Fig. 1 Comparison of the detection performance on two correlated signals

Fig. 2 shows the detection performance of MTDSS, MENSE and MDL on three correlated signals. The correlated factor of correlated narrow band signal is 0.4. The DOAs of incident signals are 0°, 5° and 10°. Simulation results of fig. 2 show that MTDSS performs better than MENSE and MDL method under the condition of three targets. When the probability of detection reaches 90%, MTDSS works better than MENSE of 4dB. MDL can not estimate number of signals correctly when signal power is smaller than 5dB.

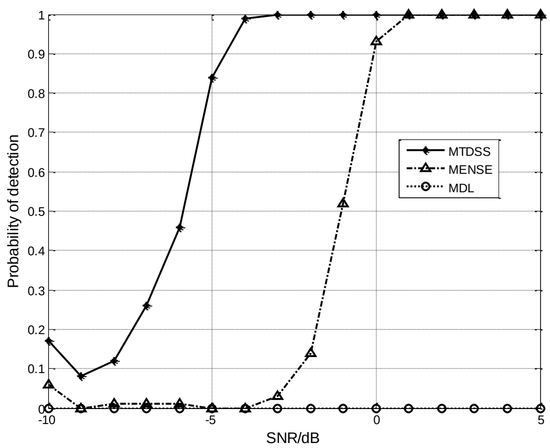

Fig. 2 Comparison of the detection performance on three correlated signals

B. Multiple targets detection under color noise background

This section explores the detection performance of MTDSS under color noise background. A 10 element ULA with half wavelength inter-element spacing is employed in this section. 3000 snapshots are used in each simulation.

Fig. 3 compares the detection performance of MTDSS, MENSE and MDL on two signals under color noise background. Signals are independent narrow band signals. The DOAs of signal are -1° and 4°. The correlated factor of color noise is 0.01. Simulation results of fig. 3 show that MTDSS performs better than MENSE method of 3dB under color noise background when the probability of detection reaches 90%. MDL method can not estimate number of signals correctly when the signal power is smaller than 10dB under color noise background.

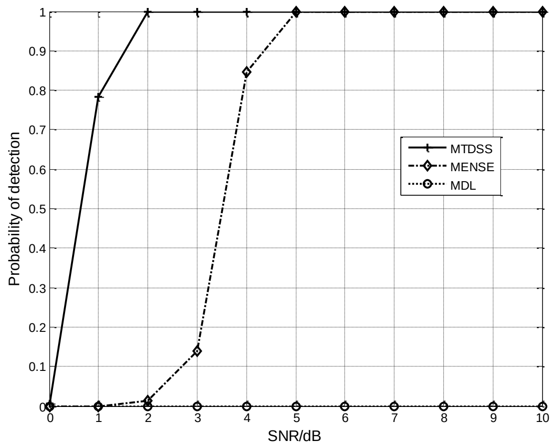

Fig.3 Comparison of the detection performance on two signals under color noise background

Fig. 4 compares the detection performance of MTDSS, MENSE and MDL on three signals under color noise background. Signals are independent narrow band signals. The DOAs of signal are -1°, 4° and 10°. The correlated factor of color noise is 0.01. Simulation results of fig. 4 show that MTDSS performs better than MENSE method of 2dB under color noise background when the probability of detection reaches 90%. MDL method can not estimate number of signals correctly when the signal power is smaller than 5dB under color noise background.

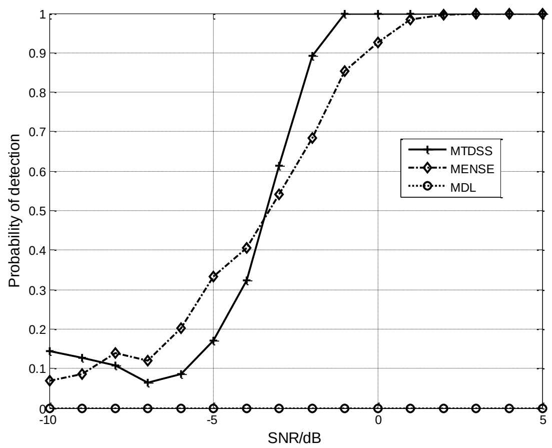

Fig. 4 Comparison of the detection performance on three signals under color noise background

V. CONCLUSIONS

A multiple targets detection method based on spatial smoothing (MTDSS) is proposed in this paper. The received data is spatial smoothed by received data vectors on some specific elements as auxiliary vectors. Gerschgorin radii of the spatial smoothed data covariance matrix are computed. The inherent property of the covariance matrix is explored. A threshold based on Gerschgorin radii is proposed to estimate the number of signals. The influence of correlated signals and color noise eliminated by spatial smoothing, and MTDSS performs robust with correlated signals and color noise background. Simulation results validate the efficiency of MTDSS on correlated signals and color noise background. MTDSS can detect multiple targets which are strongly correlated with each other, and multiple targets under color noise background. The detection performance of MTDSS is better than other methods existing.

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