



PSD-based Doppler Estimation for TDD Systems

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Abstract—We present a new Doppler estimation for time division duplex (TDD) systems using power spectral density (PSD) based method. First, a detailed analysis of the effect of TDD frame structure is proposed. Based on the results of analysis, we proposed a method of getting the best detection of Doppler spread by adaptive thresholding and a method of eliminating the partial aliasing of PSD by linear interpolation. Then we compared with the existing methods, the new algorithm is accurate and robust to noise. It is also shown that the PSD-based algorithm has good performance for wide range of Doppler spread and suitable for next generation wireless communication systems, such as TD-LTE.

I. INTRODUCTION

In mobile communication systems, the Doppler spread, or equivalently, the mobile speed, is an important parameter of mobile fading channel. The faster the User Equipment (UE) moves, the larger Doppler spread is, and the quicker the channel changes. Based on the requirement of next-generation wireless systems, high mobile speed between 15 and 120 km/h should be supported with high performance. And mobility across the cellular network shall be maintained at the speed from 120 km/h to 350 km/h (or even up to 500 km/h depending on the frequency band). Hence, the knowledge of the Doppler effect at the Base Station (BS) can help to optimize the mobile transmission system and compensate the synchronization distortion. Moreover, the knowledge of Doppler effect at the BS is helpful for the channel scheduling, link adaptation and channel estimation.

There are six major classes of Doppler estimation techniques: Power spectral density (PSD)-based method, Level crossing rate (LCR)-based method, zero crossing rate (ZCR)based method, auto correlation function (ACF)-based method, covariance (COV)-based method and pattern recognition (PR)based method.

PSD-based method is robust to both Rice factor and angle of arrival of the line of sight (LOS) component, but this method is only suited for frequency division duplex (FDD) systems [1]. LCR-based and ZCR-based methods are simple and easy to be implemented. However, these estimators suffer from severe estimation error when the signal-to-noise ratio (SNR) is low [2] [3]. ACF-based method is efficient in classifying the speed of UE into slow, medium, or fast. However, better resolution of the speed is not achievable [4]. COV-based method is sensitive to noise, Rice factor and the angle of arrival of LOS component [5]. PR-based method is computationally intensive [6]. Furthermore, most algorithms mentioned above are adapted for FDD system, where frames (or slots) are continuous in time domain for a particular UE. This is very different from time division duplex (TDD) system, where frame allocations are dynamic rather than continuous in TDD system. The dynamic feature causes the decrease of sampling rate in time domain or the non-uniform distribution of sampling points which makes the performance of the existing algorithms, as shown later, poor in practice. To the best of our knowledge, there is no new Doppler estimation algorithm for uplink in TDD system.

In this paper, we propose a novel algorithm, which, as shown later, utilizes power spectrum density (PSD) of the received signal to estimate the Doppler spread by adaptive thresholding and linear interpolation. The proposed method could improve the accuracy of Doppler estimation for TDD system.

The rest of this paper is organized as follows. The system models are described in Section II. Our modified PSD-based algorithm is presented in Section III. Simulation results and discussion are given in Section IV and the paper is concluded in Section V.

II. SYSTEM MODEL

For a single user, the allocations of uplink frame are not continuous in the TDD system. For example, the TD-LTE system supports seven kinds of uplink-downlink configurations, the maximum ratio of uplink to downlink sub-frame is 3 to 1, the minimum ratio of uplink to downlink sub-frame is 1 to 8 [7]. The increase of the time interval between two adjacent uplink frames results in the decrease of the sampling rate in time domain and the non-uniform distribution of sampling points. If we reduce the sampling frequency, the estimation range of Doppler estimator will be decreased. On the other hand, if we maintain a constant sampling frequency, we cannot get any information about downlink channel impulse response (CIR) during uplink time frame.

The signal model at receiver side can be expressed as follows:

$$y(t) = \sum_{l=0}^{L-1} h_{TDD}(t, l) x(t - \tau_l) + n(t),$$
(1)

where $h_{TDD}(t, l)$ and τ_l are the complex random amplitude and tap delay of the *l*-th path of the multipath channel, and n(t) is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2 . Equation (1) also can be written as:

$$y(t) = \left[\sum_{l=0}^{L-1} h_{FDD}(t, l) x(t - \tau_l)\right] w(t) + n(t), \qquad (2)$$

where $h_{TDD}(t, l)$ is the CIR in FDD and w(t) is introduced to model the TDD frame. The amplitude of w(t) is one for uplink frame and zero for downlink frame in the uplink system. w(t)can be expressed as:

$$w(t) = \sum_{k=-\infty}^{\infty} rect\left(\frac{t}{N_{UL}T_s} - \frac{k}{(N_{UL} + N_{DL})T_s}\right)$$
$$= \sum_{k=-\infty}^{\infty} rect\left(\frac{t}{T_{UL}} - \frac{k}{T}\right).$$
(3)

The w(t) is sum of rect(t) with downlink-to-uplink switchperiod $T=(N_{UL} + N_{DL})T_s$ and maximum frame duration for uplink $T_{UL}=N_{UL}T_s$, where N_{UL} is the number of uplink frame and N_{DL} is the number of downlink frame. T_s denotes the frame duration and rect(t) denotes rectangular function, i.e. rect(t)=0, if $|t|>\frac{1}{2}$, otherwise, rect(t)=1.

As it is shown in Fig. 1, the impact of wireless channel response in TDD frame structure equals to the channel response of FDD structure pass through a transmission system, where $h_{FDD}(t)$ is CIR in FDD system and $h_{TDD}(t)$ is the actual observed CIR at receiver of communication system in TDD. According to (1) and (2), the results of CIR passing through TDD system can be expressed as:

$$h_{TDD}(t) = h_{FDD}(t)w(t)$$
$$= h_{FDD}(t)\sum_{k=-\infty}^{\infty} rect(\frac{t}{T_{UL}} - \frac{k}{T}).$$
(4)

The autocorrelation function of system output is

$$R_{H_{TDD}}(\tau) = R_{H_{FDD}}(\tau)R_W(\tau), \tag{5}$$



Fig. 1. The impact of CIR in the TDD systems

and its power spectral density is

$$S_{H_{TDD}}(f) = \frac{1}{2\pi} S_{H_{FDD}}(f) * S_W(f)$$

$$= \frac{1}{2\pi} S_{H_{FDD}}(f) * \left(\frac{T_{UL}}{T}\right)^2 \times$$

$$\sum_{k=-\infty}^{\infty} sinc^2 \left(\frac{\pi n T_{UL}}{T} \delta(f - \frac{n}{T})\right)$$

$$= \frac{T_{UL}^2}{2\pi T^2} \sum_{k=-\infty}^{\infty} sinc^2 \left(\frac{\pi n T_{UL}}{T} S_{H_{FDD}}(f - \frac{n}{T})\right)$$

$$= \sum_{k=-\infty}^{\infty} a_n S_{H_{FDD}}(f - \frac{n}{T})), \qquad (6)$$

where

$$a_n = \frac{T_{UL}}{2\pi T^2} sinc^2 \left(\frac{\pi n T_{UL}}{T}\right)$$

= $\frac{1}{2\pi (1 + T_{DL}/T_{UL})^2} sinc^2 \left(\frac{\pi n}{1 + T_{DL}/T_{UL}}\right).$ (7)

Equation (6) shows the relationship between FDD and TDD system. The PSD of channel for TDD system may be aliased based on the downlink-to-uplink switch-period T, and the gain of aliased a_n is depended on the ratio of uplink to downlink frame T_{DL}/T_{DL} . If $T_{DL}/T_{DL}=0$, (5) and (6) express the PSD of channel for FDD system.

As shown in Fig. 2, a_n in (7) rapidly decreased when n is increased. The difference of $a_n, n = 1, 2, ...$, depends on the T_{DL}/T_{DL} and it will be more remarkable when the numbers of uplink frames are more than the numbers of downlink frames.

III. PSD-BASED ALGORITHM

The most commonly used Doppler power spectrum model for the mobile radio channel is Jakes' Model, where the autocorrelation function for the time variant transfer function can be expressed as:

$$R_X(\tau) = J_0(2\pi f_d \tau),\tag{8}$$



Fig. 2. Normalized a_n in different T_{DL}/T_{DL} ratio

where $J_0(x)$ is the zero-order Bessel function of the first kind and

$$f_d = v f_c / c, \tag{9}$$

where f_d is the maximum Doppler frequency, v is the vehicle speed, f_c fc is the carrier frequency, and c is the speed of light. The PSD of the mobile radio channel is

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-jw\tau} d\tau = \begin{cases} \frac{1}{\pi f_d \sqrt{1 - (\frac{f - f_c}{f_d})^2}}, & \text{if } |f| \le f_d, \\ 0, & \text{if } |f| > f_d. \end{cases}$$
(10)

Therefore, for FDD system, the PSD of channel has maximum values at frequencies of $f_c \pm f_d$. The frequency component, $f=f_c+f_d$, is always greater than $f=f_c-f_d$. We use the maximum value of (10) at the highest frequency component $f=f_c+f_d$ to estimate f_d .

However, for TDD system, from (6) we know that the PSD of channel may be aliased. The maximum value of (6) is not corresponding to the frequency component $f=f_c+f_d$. Two methods are proposed to estimate the Doppler spread in this scenario. One is to obtain the best detection of f_d by adaptive thresholding according to aliased PSD. The other is to partly eliminate spectrum aliased by linear interpolation.

A. Adaptive Thresholding Based on ML Criterion

Taking the derivative of $S_{H_{FDD}}(f)$ with respect to f, we get

$$\frac{dS_{H_{FDD}}(f)}{df} = \begin{cases} \frac{f}{\pi f_d^3 [1 - (f/f_d)^2]^{\frac{3}{2}}}, & \text{if } |f| \le f_d, \\ 0, & \text{if } |f| > f_d. \end{cases}$$
(11)

The maximum value of $\frac{dS_{H_{FDD}}(f)}{df}$ is at the point $f = \pm f_d$, and then $\frac{dS_{H_{FDD}}(f)}{df}|_{f=\pm f_d} = S_{H_{FDD}}(f_d)\delta(f\pm f_d)$. The derivative of $S_{H_{TDD}}(f)$ can be expressed as:

$$\frac{dS_{H_{TDD}}(f)}{df} = \sum_{n=-\infty}^{\infty} a_n S_{H_{FDD}}(f_d) \delta(f \pm f_d - \frac{n}{T}).$$
(12)

When n=0, the $\left|\frac{dS_{H_{TDD}}(f)}{dt}\right|$ has the maximum, let

$$s_{0} = \max\{|\frac{dS_{H_{TDD}}(f)}{df}|\} = a_{0}|S_{H_{FDD}}(f_{d})|\delta(f \pm f_{d}),(13)|$$

$$f_0 = \arg\max_n \{ |\frac{dS_{H_{TDD}}(f)}{df}|, n = 0, 1, 2, ... \},$$
(14)

where f_0 denotes the Doppler spread. When $n \ge 1$, let

$$s_{1} = \max\{|\frac{dS_{H_{TDD}}(f)}{df}|\}$$

= max{a_{n}}|S_{H_{FDD}}(f_{d})|\delta(f \pm f_{d} - \frac{k}{T}), (15)

$$f_1 = \arg\max_n \{ |\frac{dS_{H_{TDD}}(f)}{df}|, n = 1, 2, ...\},$$
(16)

where f_1 denotes the value of aliased PSD.

From (13) and (15), we know that s_i depend on a_n . Hence, the difference between s_1 and s_2 will be remarkable, when the numbers of uplink frames are increased.

We consider the following two hypotheses for making decisions based on maximum likelihood (ML):

$$H_0: x(f) = s_0 + S'_N(f), H_1: x(f) = s_1 + S'_N(f),$$
(17)

where $S'_N(f)$ is the derivative of PSD of n(t).

If hypothesis H_0 is accepted, the corresponded frequency is the value of Doppler spread, otherwise, it is the value of aliased PSD.

For practical implementation, we use discrete derivative calculation, i.e.

$$\frac{df(x)}{dx} = \frac{f(x + \Delta x) - f(x)}{\delta x}.$$
(18)

In the ideal condition, where $S'_N(f)$ is zero, it is easy to detect s_0 since s_0 is always greater than s_1 . However, the square of spectrum is usually used to estimate the PSD, i.e.

$$S'_N(f) \approx n^2(f_1) - n^2(f_2),$$
 (19)

where $n(f_i)$, i = 1, 2, denotes the spectrum of n(t). The Fourier transform is a linear transform, which does not change the distribution, i.e., $n(f_i) \sim N(0, \sigma^2)$. Moreover, we assume f_i are independent random variables. Hence, the probability density function (pdf) of $S'_N(f)$ is [8]:

$$p(z) = \frac{1}{4\pi\sigma^4} K_0 \left(\frac{z}{4\sigma^4}\right),\tag{20}$$

where K_0 is the modified Bessel function of the second kind. The conditional probability density function of x(f) is:

$$p(x|s_i) = \frac{p(x|H_1)}{p(x|H_0)} = K_0 \left(\frac{x-s_1}{4\sigma^4}\right) / K_0 \left(\frac{x-s_0}{4\sigma^4}\right).$$
 (21)

Then, the likelihood ratio is given by:

$$\Lambda(x) = \frac{p(H_0)}{p(H_0)} = K_0 \left(\frac{|x - s_1|}{4\sigma^4}\right) / K_0 \left(\frac{|x - s_1|}{4\sigma^4}\right).$$
(22)

According to the principal of Bayes [9], i.e.

x

$$\Lambda(x_{th}) = \frac{p(H_0)}{1 - p(H_1)} \left(\frac{c_{10} - c_{00}}{c_{01} - c_{11}}\right),$$
(23)

we have

$$\frac{p(H_0)}{1-p(H_1)} \left(\frac{c_{10}-c_{00}}{c_{01}-c_{11}}\right) = K_0 \left(\frac{|x-s_1|}{4\sigma^4}\right) / K_0 \left(\frac{|x-s_1|}{4\sigma^4}\right).$$
(24)

We assume factor $c_{11}=c_{00}=0$ and $c_{01}=c_{10}=1$, prior probability $p(H_0)=p(H_1)=0.5$. Solving (24), we get the detection threshold: . . .

$$a_{th} = \frac{s_0 + s_1}{2}.$$
 (25)

If the derivative of PSD is greater than x_{th} , we declare the frequency component f_d as Doppler spread, otherwise as interference of aliased:

$$x(\hat{f}_d) \ge x_{th}.\tag{26}$$

$U_{\rm previous}$	D_0	D_1	 D_{i}	 $D_{\rm k}$	$U_{\rm current}$

uplink frame downlink frame

Fig. 3. TDD frame structure.

As was mentioned previously, s_i depend on T_{UL}/T_{DL} , the difference between s_1 and s_2 will be great when the uplink frames are increased. Hence, T_{UL}/T_{DL} determines the accuracy of detection, i.e., the accuracy of Doppler estimation.

B. Linear Interpolation Method

The Doppler spread results in the time-selective of the channel, the channel is said to be slow fading if the coherence time of the channel is much greater than the used signaling interval. When the Doppler spread is small, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of one or few symbols. We reconstruct the CIR during downlink frame by linearly interpolating the CIR during adjacent uplink frame. The reason why linear interpolation is chosen is that it does not require any statistic information.

As it is shown in Fig. 3, we combine the current uplink time channel response $U_{current}$ with previous uplink time channel response $U_{previous}$ to interpolate the uplink channel response during adjacent downlink frames $D_0, D_1, ..., D_k$. The *i*-th interpolated downlink time channel response D_i can be expressed as:

$$D_i = \frac{k-i}{k+i}U_{current} + \frac{i+1}{k+1}U_{previous}.$$
 (27)

However, when the Doppler spread is great than $f_{d,C}$, its corresponding coherence time Δt will be less than slot duration and the channel is fast fading. If we are still using interpolation by the channel data of uplink transmission time, it will not interpolate any approximate value for fading channel; on the contrary, it will bring new interference and worsen the accuracy of estimation. Consequently, linear interpolation method is useful for Doppler estimation under slow speed condition.

C. PSD-based Estimator with Adaptive Thresholding and Linear Interpolation

As mentioned above, the linear interpolation is only suitable for slow fading channel. The Doppler spread estimation is accomplished by the following steps:

1) Step1: We carry out the estimation of $\hat{f}_d = f_d^{(1)}$ by using adaptive thresholding method in (26), i.e.

$$x(f_d^{(1)}) \ge x_{th}^{(1)}.$$
(28)

2) Step2: We use linear interpolation when $f_d^{(1)}$ is less than $f_{d,C}$, and carry out a fine estimation of $\hat{f}_d = f_d^{(2)}$ by using adaptive thresholding in (26) again by the interpolated data,

$$x(f_d^{(2)}) \ge x_{th}^{(2)}.$$
(29)

3) Step3: The Doppler spread can be estimated as follows:

$$\hat{f}_d = \begin{cases} f_d^{(2)}, & \text{if } \hat{f}_d \le f_{d,C}, \\ f_d^{(1)}, & \text{if } \hat{f}_d > f_{d,C}. \end{cases}$$
(30)

D. Computation Complexity Analysis

The PSD-based algorithm includes three tasks: power spectral density calculation, difference calculation, linear interpolation. The calculation of power spectral density needs $C_M(N) = \frac{NlbN}{2} + N$ complex multiplications and $C_A(N) = NlbN$ complex additions as assumed N points FFT. The computation of difference needs N-1 complex additions. For the *k* downlink frames, the computation complexities of linear interpolation are $C_M(N) = \lceil 2k(N-1)/(k+1) \rceil$ complex multiplications and $C_A(N) = \lceil k(N-1)/(k+1) \rceil$ complex additions.

IV. SIMULATION RESULTS

In this section, we first evaluate the performance comparison of conventional and the proposed algorithm under the effect of noise and speed. Then we present the performance of our algorithm in different TDD configuration, i.e., uplink and downlink frame configuration. We consider the physical uplink shared channel (PUSCH) of TD-LTE uplink as simulation system, with subcarrier mapping for one user. The performance of the fd estimator is measured by the normalized mean square error (NMSE)

$$\varepsilon = \frac{1}{M} \sum_{i=0}^{M-1} \left(\hat{f}_d(i) / f_d - 1 \right),$$
 (31)

where M is the number of investigated \hat{f}_d samples for the evaluation, and in our simulation M=1000. Extended vehicular A (EVA) model is used as multi-path channel model [10]. The more detailed information is listed in Table I.

A. Performance Comparison of Conventional and the Proposed Algorithm Versus Speed

Fig. 4 shows the performance of proposed and three conventional Doppler estimation algorithms when the SNR is

Simulation Parameter Value Bandwidth for the uplink 5MHz Carrier frequency 2.6GHz FFT size 2048 Bandwidth for each subcarrier 15kHz Maximum vehicle speed 360km/h Maximum Doppler spread 866.7Hz Frame duration 10ms Sub-frame duration 0.5ms Number of symbols per slot 7 System sampling rate 30.72MHz PSD Sampling rate 2kHz 2048 PSD Sampling size

TABLE I SIMULATION PARAMETERS



Fig. 4. Performance comparison of conventional and the proposed algorithm versus speed.

20 dB and Uplink-downlink configuration is 0 as shown in Table II. The ACF, LCR and ZCR algorithms give the satisfying performance when the Doppler spread is below 150 Hz. When the Doppler spread becomes larger than 150 Hz, these algorithms are severely distorted, whereas the proposed algorithm still gives satisfying performance for any value of Doppler spread. The NMSE of the proposed algorithm is within 7.4721e-4.

B. Performance Comparison of Conventional and the Proposed Algorithm Versus SNR

Fig. 5 shows that the effect of SNR for estimation performance for Doppler spread $f_d = 100$ Hz. The LCR and ZCR algorithms are very sensitive to SNR and suffered from severe estimation error when the SNR is below 20 dB. On the contrary, both the proposed algorithm and ACF algorithm are robust to noise as long as SNR is greater than -4 dB.



Fig. 5. Performance comparison of conventional and the proposed algorithm versus SNR.



Fig. 6. Performance of the proposed algorithm versus TDD configuration.

C. Performance of the Proposed Algorithm Versus TDD Configuration

We carry out the performance for difference ratio of uplink to downlink in the TD-LTE system when the SNR is 20 dB. The proposed algorithm can estimate the Doppler frequency accurately over the range of 0-866 Hz in configuration 0, 1, 3, 4 and 6 as shown in Fig. 6. The estimator can also estimate accurately below 500 Hz and 100 Hz in configuration 2 and 6, respectively. The detailed uplink-downlink configurations and their corresponding simulation result are listed in Table II. The result shows that the accuracy of estimator depends on the ratio of uplink to downlink frame. The more we obtain information of channel impulse response in uplink time, the more accuracy of estimation is.

V. CONCLUSION

In TDD systems, the frame structure makes it difficult for long time sampling and leads to the alias of power spectral density. In this paper, we proposed a novel PSD-based algorithm of Doppler estimation by using the power spectral density with adaptive thresholding and linear interpolation. We carried out a detailed performance analysis of the proposed algorithm and compare it with the conventional algorithms. The simulation results show that PSD-based algorithm is better

TABLE II The Average of NMSE in Difference Ratio of Uplink to Downlink

Uplink-downlink configuration	Ratio of uplink to downlink	The average of NMSE
0	3:1	1.862×10^{-4}
1	1:1	9.438×10^{-4}
2	1:3	5.598×10^{-2}
3	1:2	3.825×10^{-2}
4	2:7	1.291×10^{-1}
5	1:8	4.031×10^{-1}
6	5:3	2.706×10^{-4}

than the three other exiting algorithms in terms of accuracy and robustness to noise. We investigated the effect introduced by TDD configuration, i.e., ratio of uplink to downlink frame. The results show that the PSD-based algorithm can be applied in different configuration ratios with satisfying performance. In addition, the algorithm has good performance for wide range of Doppler spread and suitable for next generation wireless communication systems, such as TD-LTE.

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