# Compressive Sensing Based Fingerprinter Positioning with MCA-based Pre-processing

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***Abstract*—The sparse nature of location finding in the spatial domain makes it possible to exploit the compressive sensing (CS) theory for wireless location. CS-based location algorithm can largely reduce the number of measurements while achieve a high level of localization accuracy, which makes the CS-based solution very attractive for indoor positioning. In this paper, a novel CS-based fingerprinting location algorithm with minor component analysis (MCA) is proposed by us. MCA theory is firstly introduced into CS to solve the coherence of Received Signal Strength (RSS) measurements in wireless location scenario. MCA-based pre-processing can better satisfy the restricted isometry property (RIP) condition by finding the minor components of RSS measurements. Analytical studies and simulations are provided to indicate that the proposed novel method using MAC significantly outperforms that with orthogonalization pre-processing, and also has lower complexity.**

## I. Introduction

Wireless location has drawn increasing attentions in the past few decades, mainly driven by the commercial and military potentials and government regulations[1][2][3]. Accurate and timely location information plays a prime role in personal and commercial applications including indoor positioning, equipment monitoring, and radio frequency identification (RFID)-based tracking. However, it is usually difficult to provide a satisfactory level of accuracy in most applications due to very complex indoor environment. Besides, the cost also always restricts the application of the positioning techniques. Compared with other measurement-based algorithms (e.g., time-of-arrival (TOA) or angle-of arrival (AOA) measurements), Received Signal Strength (RSS)-based localization algorithms have been extensively studied as an inexpensive solution for indoor positioning systems in recent years [4][5]. The key challenges for the RSS-based positioning systems comes from two aspects:1) the variations of RSS due to the radio channel impediments such as shadowing, multipath, and the orientation of the wireless device, etc. 2) only a small number of RSS measurements in real situations. These factors greatly increase the difficulty of realizing accurate indoor positioning objective.

In general, two main approaches are proposed to solve the mentioned two difficulties in existing indoor positioning techniques. One method uses a prior theoretical or empirical radio propagation model to formulate the RSS position dependency for location estimation, which are unreliable due to the dynamic and unpredictable nature of indoor radio propagation [6] [7] . Another method known as fingerprinter is proposed [8], which includes the k-nearest neighbor algorithm (kNN) [9] and fingerprinter algorithm using Bayesian theory or kernel function [5]. The position of the target to be located is estimated by comparing its online RSS readings with offline observations. However, an accurate location scheme requires a large grid size, while on each grid point an RSS measurement is needed. In fact, the number of the available RSS measurements is always limited in the realistic environment, which results in the bad positioning performance. Besides, this method is highly dependent on the environment, any significant change to the topology implies a costly new recalibration.

Compressive sensing (CS) theory can recover signals that are sparse or compressible under a certain basis with far fewer noisy measurements than the Nyquist sampling theorem [10][11], so it provides a very good approach to solve the above problem. Since the location of a target is unique in the discrete spatial domain at a certain time, the localization problem can be modeled as an ideal 1-sparse vector. Motivated by the idea, the researchers [Chen Feng and Shahrokh Valaee el ta.] propose a CS-based multiple target localization algorithm in [12][13], which exploits far fewer number of RSS measurements to realize the good location accuracy. Compared with traditional fingerprinting algorithm, it reduces the number of measurements in a logarithmic sense, while achieves a high localization accuracy.

Although the CS-based solution is very meaningful, two basic components need be held in CS: sparsity and incoherence. The RSS measurements from the different points are coherent in spatial domain under realistic envirnoment, so the CS-based solution has to solve one core problem that the sparsity basis and the measurement matrix must be spatially incoherent. The data of the offline stage and the online stage need be pre-processed to achieve the incoherent effects. The matrix orthogonalization transformation is adopted to reduce the correlation between the measurements matrix and the sparse matrix. Through the deeper analysis of simulation results, we discover that the orthogonalization –based pre-processing is not ideal so that the performance of CS-based location algorithm described in [12][13] is not as good as expected. The main reason is that the RSS fingerprints matrix isn't generally a square matrix so that its orthogonalization processing only can realize the normalization and correlation reducing of matrix row vectors, but the column vectors are not normalized and still strongly coherent. However, the incoherence of the column vectors is very important to recover the sparse signals, which greatly impact the positioning accuracy.

Based on the deeper investigations, we propose a novel CS-based fingerprinter positioning algorithm with minor component analysis (MCA) in this paper. MCA theory is firstly introduced into CS theory to solve the coherence between the measurements matrix and the sparse matrix in location scenario. MCA-based pre-processing can better satisfy the RIP condition by finding the minor components of RSS measurements from the grid points and the RPs [14]. Analytical studies and simulations are provided to indicate that the proposed novel method using MCA significantly outperforms that with orthogonalization pre-processing in [12][13].

The remainder of this paper is organized as follows: Section II briefly introduces the CS theory and the CS-based localization problem. In section III, CS-based wireless positioning algorithm is presented. Some corresponding theoretical analysis is also described in this section. Section VI gives the simulation results and analysis. Section V concludes the full paper.

## II. MODEL RESTATE AND ANALYSIS

### A. *Brief Review of Compressive Sensing*

Compressive sensing is put forward to represent and capture a signal at a rate significantly below the Nyquist rate [10]. The transformed sparse signal can be recovered by some dedicated algorithms, such as orthogonal matching pursuit (OMP) [15] or basis pursuit (BP) [16] algorithms. Then the original signal can be obtained by the inverse transformation processor of the recovered sparse signals. The common CS model in the discrete domain is as follows:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{s} \tag{1}$$

where $\mathbf{s}$ is the original discrete-time signal that can be seen as a $N\times 1$ vector in $\mathbb{R}^N$. Each column of sparsity matrix $\mathbf{\Psi}$ is an orthogonal basis, which is a $N$ by $N$ matrix. Then $\mathbf{x} = \mathbf{\Psi}\mathbf{s}$ is a $K$-sparse transformed signal. $\mathbf{\Phi}$ is the $M$ by $N$ measurements matrix, with $K < M << N$. $\mathbf{y}$ is the last measurements vector in $\mathbb{R}^M$ .The critical points in CS theory is the design of the sparsity matrix $\mathbf{\Psi}$ ,which is correlated to the character of the original signal [17]. A well-designed sparsity matrix can insure the higher sparsity of the transformed signal .Then the measurements matrix is chosen to be uncorrelated with the sparsity matrix.

### B. *CS-based Localization problem*

Considering a case, where $K$ targets with unknown locations are located in an isotropic area that is divided into a discrete grid with $N$ points. The localization area includes $N\times N$ grids. Wireless nodes take RSS measurements from $M$ arbitrary reference points referred as RPs over the grids. RPs mainly realize the measurement function, and may be the points with known positions or with unknown positions. The goal is to determine the locations of the $K$ targets simultaneously and accurately, using only a small of noisy RSS measurements and simple operation. It is noticed that $K \ll N$, $M \ll N$.

The localization problem can be well formulated as a sparse matrix recovery problem in the discrete spatial domain.

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{\Theta} + \boldsymbol{\varepsilon} \tag{2}$$

a) $\mathbf{\Phi}$ is a $M$ by $N$ measurement matrix. Only a small number of measurements are collected from $N$ grid points on arbitrary $M$ RPs. Each row of $\mathbf{\Phi}$ is a $1\times N$ vector containing only one nonzero value of 1.The location of 1 in the row vector represents the location of one RP in the localization area.

b) $\mathbf{\Psi}$ is a N by N sparsity basis matrix. The element $\Psi_{i,j}$ = $RSS(d_{i,j})$ ,records the RSS reading on grid point $i$ from the target located at grid point $j$, for all $1 \le i \le N$, $1 \le j \le N$. $RSS(d)$ denotes the radio propagation channel model including path loss, fast fading, and shadowing etc. $d_{ij}$ denotes the real transmitter-receiver distance from the grid point $i$ to the grid point $j$. The calculation of RSS measurements can utilize the given indoor radio propagation channel model in WLAN.

c) $\mathbf{\Theta}$ is a $N\times K$ matrix ,with one nonzero value of 1 in each of its column vector . $\mathbf{\Theta}$ denotes the location of the targets over the grid. So there are $K$ targets located in the area.

d) $\boldsymbol{\varepsilon}$ is the unknown measurements noise.

e) The $M \times K$ matrix $\mathbf{y}$ ,is the compressive noisy RSS measurements from $K$ targets on $M$ RPs, with each row vector indicating one measurement value. The number of measurements obeys $M = O(K \log(N/K))$.

Because of the space correlation between the RPs and the grid points, solving equation (2) to reconstruct $\mathbf{\Theta}$ doesn't satisfy the incoherence condition of CS theory. Reference [12] also presents a data pre-processing method as follows:

$$\mathbf{y}' = T\mathbf{y} = T(\mathbf{\Phi}\mathbf{\Psi}\mathbf{\Theta} + \boldsymbol{\varepsilon}) \tag{3}$$

where $T$ denotes a linear transformation operation. Let $\mathbf{R} = \mathbf{\Phi}\mathbf{\Psi}$, $\mathbf{Q} = orth(\mathbf{R}^{\mathrm{T}})^{\mathrm{T}}$, $T = \mathbf{Q}\mathbf{R}^{\dagger}$, where $orth$(A) is an orthogonal basis for the range of A, $\{\bullet\}^{\mathrm{T}}$ denotes the transpose operation, and $\{\bullet\}^{\dagger}$ returns the pseudo-inverse operation. Thus

$$\mathbf{y}' = T\mathbf{y} = \mathbf{Q}\mathbf{\Theta} + T\boldsymbol{\varepsilon} = \mathbf{Q}\mathbf{\Theta} + \boldsymbol{\varepsilon}' \tag{4}$$

The simulation results have shown that the algorithm can obtain better performance than the other fingerprinting algorithms, such as *kNN* algorithm. Moreover, it greatly reduces the number of measurements in a logarithmic sense.

### C. Restricted Isometry Property in CS Theory

However, a major problem still remains through such data pre-processing. As we know, another very important restrict obeyed by CS theory is called the restricted isometry property (RIP) [18][19] .Given any set $F$ of column indices, where $F \subset \{1,2,3,...N\}$ . The formulation to express RIP in the CS localization model is as follows:

$$(1-\delta_k) \le \frac{\|\mathbf{Q}_F \mathbf{\Theta}_J\|_2}{\|\mathbf{\Theta}_J\|_2} \le (1+\delta_k) \tag{5}$$

where $\|\cdot\|_q$ denotes the $l_q$ -norm. $\mathbf{\Theta}_J$ denotes the vector corresponding to the column indices $J$ in $\mathbf{\Theta}$ ,where J $\in$ $F$. $\delta_k$ is the smallest quantity satisfying the above formulation with the value between 0 and 1, for all the subsets $F \subset \{1,2,3,...N\}$ with the cardinality $|F| \le K$ and all coefficient sequences $(\Theta_j)_{J \in F}$ . $\delta_k$ is also correlated with the value of $K$ , namely the sparsity of $\mathbf{\Theta}$ . $\delta_k$ determines the orthogonality of the column vectors of the measurements matrix with the column suffix in the subset $F$ .The smaller $\delta_k$ is, the higher accuracy $\mathbf{\Theta}$ can be reconstructed. However, it is easy to prove that the matrix $\mathbf{Q}$ doesn't satisfy this property. Because the signal to be recovered are 1-sparse column vectors of $\mathbf{\Theta}$ , the $l_2$ -norm of each column of $\mathbf{Q}$ should be nearly close to 1 but the fact isn't the same as it. Under this condition, the normalization of the columns of $\mathbf{Q}$ and $\mathbf{y}'$ is an indispensible procedure. Then the general structure of the positioning model can be shown in Fig.1. Since the signal to be recovered is a 1-sparse signal, the normalization of the columns of measurements matrix $\mathbf{R}$ and the online readings $\mathbf{y}$ is essential to satisfy the constraints in (5).

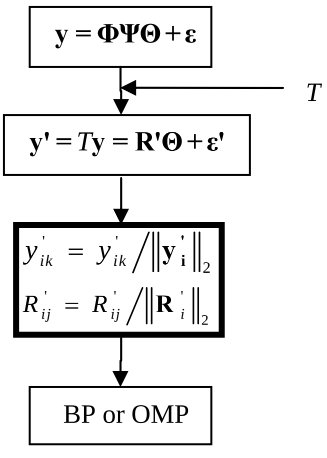


Fig.1 The general structure of data processing

Since $\mathbf{R} = \mathbf{\Phi\Psi}$ , with the size $M \times N$ , isn't a square matrix, orthogonalization can't absolutely ensure the orthogonality of its column vectors. $M$ is chosen to be far smaller than $N$ in the real localization situation. Moreover, if the data processing is based on the principal of linear transformation, the space correlation can't be lowered to a satisfactory degree.

## III. CS-BASED LOCATION ALGORITHM WITH MCA

Before we introduce the proposed novel approach, some analysis about the factors influencing the accuracy of fingerprint localization is described to solve the existing problem in the CS model.

Comparing with the range-based localization algorithm using path-loss model of RSS values, the range-free methods using fingerprint of RSS in the CS model is more vulnerable to the variation of time and space. Generally speaking, $N$ is chosen to be as large as possible for obtaining high accuracy of localization. But the RSS values collected at two adjacent grid points is more likely to be confused especially when the power of noise gets close to the difference of the average RSS values of the two grid points. So some previous algorithm, such as *kNN*, which uses the Euclidean distance to calculate the weight of each RP, may not obtain high accuracy of positioning if the number of RPs is not enough under a low signal-noise-ratio (SNR).

As a result, the data pre-processing must be combined with the recovering algorithm in CS model. A widely applied approach to estimate the entries of nonzero value in the sparse signal is the computation of the projection, which is as follows:

$$S \leftarrow \left\{ i \in \{1,2,3,...,N\} : \frac{|<\mathbf{y}', \mathbf{R}_i>|}{|<\mathbf{y}', \mathbf{R}_i>|_{\max}} \ge \gamma \right\} \tag{6}$$

where $\gamma$ is the threshold correlated to the sparsity of the signal, index *i* represents the $i^{th}$ RP. $\mathbf{R}_i$ is the $i^{th}$ column vector of the original measurement matrix. The indices in the sets $S$ are the probable entries of nonzero values in the signal to be recovered. So the data pre-processing should aims at maximizing the following:

$$\arg\max \frac{\sum_{i \ne j, i, j \in S} |<\mathbf{y}', \mathbf{R}'_i> - <\mathbf{y}', \mathbf{R}'_j>|}{\|S\|_{\ell_0}} \tag{7}$$

Based on the above analysis, the MCA-based data pre-processing method is proposed to lower the space correlation between the grid points and RPs by discarding part of the RSS variables with high variation received at each RP. MCA theory is widely utilized to find the minor components of interrelated variables in signal processing. It is contrary to the theory of principal components analysis (PCA) [20], which retains the principal components with the largest variance. The minor components are uncorrelated and of smaller variance in decreased dimension. MCA can reduce the dimension of a set of variables which are interrelated, by discarding as much as possible of the components with large variation present in the data set. Those components are more susceptible to the fluctuation of the environment. Besides, the minor components of a set of given variables can be achieved by transforming it into the direction with small variation in the low dimension space.

Given a vector $\mathbf{x} = \{x_1, x_2, ..., x_p\}$ with $p$ variables, we are inclined to study the covariance matrix which contains both the variance and correlation of these variables. By subtracting the mean value of the samples from each variable in $\mathbf{x}$, the covariance matrix can be calculated by

$$\mathbf{C} = E\{\mathbf{x}\mathbf{x}^\mathbf{T}\} \tag{8}$$

where $\mathbf{C}$ is a $p \times p$ matrix, with its diagonal elements being th e variance and the other elements being the covariance betwee n the $p$ variables. $E\{\bullet\}$ denotes the expectation operator. We define the minor components of $\mathbf{x}$ as $\boldsymbol{\eta} = \{\eta_i\}_{i=1,2,...,q}$ .The transformation coefficients are denoted as $\mathbf{U}$ , and $\boldsymbol{\mu}_\mathbf{i}$ = $\{u_{i1}, u_{i2}, \ldots, u_{ip}\}$ is the $i^{\text{th}}$ column vector of $\mathbf{U}$ .

$$\eta_i = \boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{x} = \mu_{i1}x_1 + \mu_{i2}x_2 + \mu_{i3}x_3 + ... + \mu_{ip}x_p, 1 \le i \le q \tag{9}$$

The problem is to search the transformation coefficients $\mathbf{U}$ that can make the minor components to have the minimum variance :

$$\arg\min \mathrm{var}[\eta_i] = E\{\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{x}\mathbf{x}^\mathbf{T}\boldsymbol{\mu}_\mathbf{i}\} = \boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_\mathbf{i} \tag{10}$$

However, with the restrict of RIP the transformation coefficients $\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}$ should satisfy:

$$\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\boldsymbol{\mu}_\mathbf{i} = 1 \tag{11}$$

Then the optimization problem to find $\mathbf{U}$ can be solved by

$$\underset{i=1\sim q}{\arg\min}\left\{\Gamma_i = \boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_i + \lambda(\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\boldsymbol{\mu}_\mathbf{i} - 1)\right\} \tag{12}$$

Let's find $\boldsymbol{\mu}_\mathbf{1}$ at first by the partial derivatives, which is presented by

$$\frac{\partial\Gamma_1}{\partial\boldsymbol{\mu}_\mathbf{1}^\mathbf{T}} = \mathbf{C}\boldsymbol{\mu}_\mathbf{1} + \lambda_1\boldsymbol{\mu}_\mathbf{1} = 0 \Rightarrow \mathbf{C}\boldsymbol{\mu}_\mathbf{1} = -\lambda_1\boldsymbol{\mu}_\mathbf{1} = \lambda_1^{'}\boldsymbol{\mu}_\mathbf{1} \tag{13}$$

Then

$$\mathrm{var}[\eta_1] = \boldsymbol{\mu}_1^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_1 = \lambda_1^{'}\boldsymbol{\mu}_\mathbf{1}\boldsymbol{\mu}_1^\mathbf{T} = \lambda_1^{'} \tag{14}$$

According to equation (10), (13) and (14), it is known that when $\mathrm{var}[\eta_i]$ is minimal $\lambda_1^{'}$ 's value should be smallest . Thus, $\lambda_1^{'}$ should be selected as the smallest eigenvalue of $\mathbf{C}$ when $\mathbf{C}$ is not time-varying, and $\boldsymbol{\mu}_\mathbf{1}$ is the eigenvector corresponding to $\lambda_1^{'}$.

When $\boldsymbol{\mu}_2$ is solved， we have to consider the uncorrelation between $\boldsymbol{\mu}_\mathbf{1}$ and $\{\boldsymbol{\mu}_i\}_{2\le i\le q}$ .

$$\mathrm{cov}[\eta_1, \eta_2] = \mathrm{cov}[\boldsymbol{\mu}_\mathbf{1}^\mathbf{T}\mathbf{x}, \boldsymbol{\mu}_\mathbf{2}^\mathbf{T}\mathbf{x}] = \boldsymbol{\mu}_\mathbf{1}^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_2^\mathbf{T} = \lambda_1^{'}\boldsymbol{\mu}_\mathbf{1}^\mathbf{T}\boldsymbol{\mu}_2 \tag{15}$$

where $\lambda_1^{'}$ reflect the correlation of $\boldsymbol{\mu}_\mathbf{1}$ and $\boldsymbol{\mu}_2$ .In general, $\lambda_1^{'}$ is quite small, so the following constraints can be given as

$$\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_\mathbf{j} = 0 \qquad \boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\boldsymbol{\mu}_\mathbf{j} = 0 \ \ ,2 \le i \le j \le q \tag{16}$$

Based on the above analysis, the optimization problem of solving $\{\boldsymbol{\mu}_i\}_{2\le i\le q}$ can be strengthened as:

$$\min_{i=2,...,q}\{\Gamma_i = \boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_i + \lambda_i(\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\boldsymbol{\mu}_\mathbf{i} - 1) + \sum_{i>j\ge1} a_{ij}\boldsymbol{\mu}_\mathbf{i}^\mathbf{T}\mathbf{C}\boldsymbol{\mu}_\mathbf{j}\} \tag{17}$$

When $i$=2, $\boldsymbol{\mu}_2$ can be solved by the following

$$\frac{\partial\Gamma_2}{\partial\boldsymbol{\mu}_2^\mathbf{T}} = \mathbf{C}\boldsymbol{\mu}_\mathbf{2} + \lambda_2\boldsymbol{\mu}_\mathbf{2} + a_{21}\boldsymbol{\mu}_1 = 0 \tag{18}$$

Equation (18) is multiplying by $\boldsymbol{\mu}_\mathbf{1}^\mathbf{T}$ , we get that $a_{21} = 0$ , so

$$\mathbf{C}\boldsymbol{\mu}_2 = -\lambda_2\boldsymbol{\mu}_2 = \lambda_2^{'}\boldsymbol{\mu}_2 \tag{19}$$

$\lambda_2^{'}$ should be the second smallest eigenvalue of $\mathbf{C}$ and $\boldsymbol{\mu}_2$ is the eigenvector corresponding to $\lambda_2^{'}$ . Now the algorithm similar to recursion can find each $\boldsymbol{\mu}_\mathbf{i}$ .For, $i = 2 \sim q$ it is not difficult to demonstrate $\boldsymbol{\mu}_\mathbf{i}$ is the eigenvector with respect to the $i^{\text{th}}$ smallest eigenvalue $\lambda_i^{'}$ of $\mathbf{C}$ .

Now the main procedure of MCA will be combined with the data pre-processing model. $\mathbf{R} = \boldsymbol{\Phi}\boldsymbol{\Psi}$ is the $M \times N$ measurements matrix in the original CS model, where the $n^{\text{th}}$ column vector denotes the RSS measurements from $M$ RPs at the $n^{\text{th}}$ grid point. We need multiple RSS samples in time domain, so the column vector is extended into a sample matrix. For example, the $n^{\text{th}}$ $M \times 1$ column vector is changed as one $M \times L$ sample matrix $\mathbf{A}_\mathbf{n}$ , $1 \le n \le N$ . Each row of $\mathbf{A}_\mathbf{n}$ presents a sequence including $L$ RSS samples from a certain RPs at the $n^{\text{th}}$ grid point, where $L$ denotes the total sample times. The element $A_{ml}^n$ of $\mathbf{A}_\mathbf{n}$ denotes the RSS sample from the $m^{\text{th}}$ RP at the $n^{\text{th}}$ grid point on the $l^{\text{th}}$ sample time, where $1 \le m \le M$ , $1 \le l \le L$ .

For $1 \le n \le N$ , the calculation of the minor components of $\mathbf{R}$ will be depicted as follows:

1. Firstly, we calculate the mean value of the $m^{\text{th}}$ row vector of $\mathbf{A}_\mathbf{n}$ for $m = 1, 2, ..., M$ .

$$\overline{A}_m^n = \frac{1}{L}\sum_{l=1}^{L} A_{ml}^n \tag{20}$$

2. Every element of $\mathbf{A}_\mathbf{n}$ is subtracted by $\overline{A}_m^n$, then a new matrix $\hat{\mathbf{A}}_\mathbf{n}$ is obtained:

$$A_{ml}^n = A_{ml}^n - \overline{A}_m^n \tag{21}$$

3. The $M \times M$ covariance matrix of $\hat{\mathbf{A}}_{\mathbf{n}}$ is calculated in this step.

4. Let us calculate the eigenvalues $\lambda_m^{'}$ and the eigenvectors $\mathbf{e}_m$ of the covariance matrix of $\hat{\mathbf{A}}_{\mathbf{n}}$ .

5. We further rank the eigenvalues in the ascending order.

$$\lambda_1^{'} \le \lambda_2^{'} \le ... \le \lambda_M^{'} \qquad (22)$$

6. The accumulated contribution rate $\rho_W$ of the eigenvalues need to be computed here. $\rho_W$ presents the number of the minor components saved by us.

$$\rho_W = \sum_{w=1}^{W} \lambda_w \bigg/ \sum_{m=1}^{M} \lambda_M \le \alpha \qquad (23)$$

The threshold $\alpha$ of $\rho_W$ should be adjusted according to the accuracy requirement. When $\alpha$ is larger, the reserved eigenvalues are more.

7. It is assumed that $W$ eigenvalues are preserved for the selected value $\alpha$ .So we select the retained eigenvalues $\lambda_1^{'}, \lambda_2^{'}, ..., \lambda_W^{'}$ and the corresponding eigenvectors $\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_W$ .

8. According to $\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_W$ , we construct a $W \times M$ whitening matrix $\mathbf{E}^{\mathbf{w}}$. The $w^{\text{th}}$ row of $\mathbf{E}^{\mathbf{w}}$ is the normalized eigenvector $\mathbf{e}_w, 1 \le w \le W$ , an orthogonal basis uncorrelated with the other rows. Thus the minor components are

$$\mathbf{R}^n = \mathbf{E}^{\mathbf{w}} \mathbf{R}_n \qquad (24)$$

where $\mathbf{R}_n$ is the $n^{\text{th}}$ column vector of the original measurement matrix. $\mathbf{R}^n$ is the $W \times 1$ vector.

Finally, a $W \times N$ matrix $\hat{\mathbf{R}}$ consisting of $\mathbf{R}^n$ is acquired by the above processing. Here, $\hat{\mathbf{R}}$ very well satisfies the RIP condition, so $\mathbf{\Theta}$ can be effectively recovered given the compressive noisy measurements only via an $l_1$-minimization program. In this paper, BP algorithm is employed for the recovery problem from compressive noisy measurements.

Simultaneously, the online RSS values received at the targets to be located must be processed correspondingly in the proposed reconstruction algorithm. A major procedure in the proposed algorithm is to calculate the projections, namely

$$\mathbf{y}_{nk} = \mathbf{E}^{\mathbf{w}} \mathbf{y}_k, 1 \le k \le K \qquad (25)$$

$$x_{0k}(n) = < \mathbf{R}^n, \mathbf{y}_{nk} > \qquad (26)$$

where $\mathbf{y}_k$ is the RSS values collected online at the location of the kth target to be located , with $\mathbf{y}_{nk}$ of its minor components projected by $\mathbf{E}^{\mathbf{w}}$. Then $\mathbf{x}_{\mathbf{0k}}$ is the initial point utilized in the BP algorithm.

## IV. Simulation Results and Analysis

In order to compare the performance of the proposed CS-based method with MCA-based pre-processing with that of the CS-based scheme with orthogolization pre-processing, simulation evaluations are performed under the same scenario assumption. The RSS values in the measurements matrix is obtained by the indoor propagation model defined by the IEEE 802.15.4 standard [21]:

$$P_r(d) = \begin{cases} P_t - 40.2 - 20\log d, d \le 8 \\ P_t - 58.5 - 33\log d, d > 8 \end{cases} \qquad (27)$$

where $P_t$ is the transmission power for each target. $d$ is the real transmitter-receiver distance.

The average localization error versus the number of measurements $M$ is presented in Fig.2. $M$ varies from 2 to

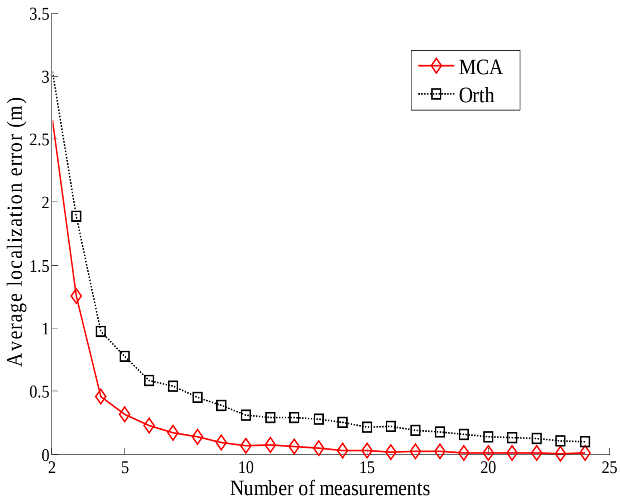


Fig2. The localization error versus the number of measurements.

24, while the number of targets is fixed at 20 and SNR is fixed at 5dB. As shown in Fig.2, the average localization error declines very rapidly with the increasing of the number of measurements for two methods. The MCA-based algorithm achieves much less location error than the orthogonalization-based algorithm under every fixed number of RPs. When $M$ is larger than 15, the MCA-based algorithm can perfectly recover the location of targets.

The cumulative average localization error versus the number of targets is given in Fig.3. The number of measurements is fixed at 20, with the number of targets varying from 2 to 30 and SNR equal to 5dB. The value of parameter $\alpha$ is the same as the first simulation. In Fig.3, with the increasing of the number of the targets, the accumulated average localization error of the MCA-based algorithm increases very slowly with some small fluctuation. However, the accumulated positioning error increases very fast with the increasing of the number of the targets for the orthogonalization-based algorithm. For a fixed number of targets, the localization error of the MCA-based algorithm is also smaller.

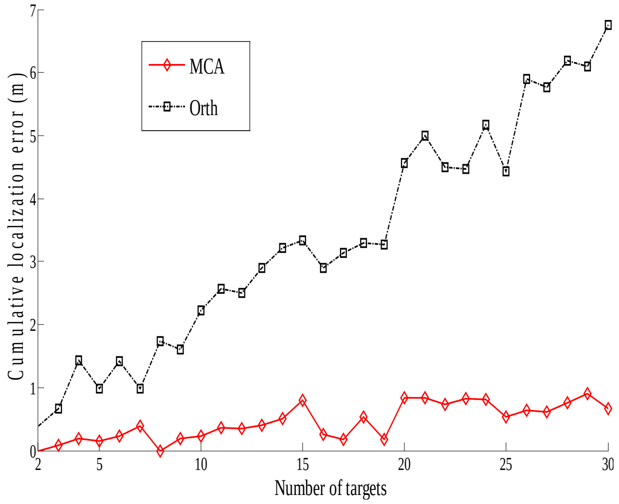


Fig.3. The cumulative localization error versus number of targets.

## V. Conclusions

In this paper, a novel CS-based fingerprinter positioning algorithm with MCA-based pre-processing is proposed. This algorithm uses MCA theory to rotate the RSS variables in the direction of dimensions with smaller variance and lower correlation to reduce the influence of measurements noise and space correlation. Simulation and analysis illustrate that the proposed approach achieves the much higher localization accuracy than the previous algorithm with the orthogolization pre-processing, and also reduces the number of measurements during the online phase in some extent.

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