Multiple Sparse Sources Separation Based on Multichannel Frequency Domain Adaptive Filtering

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Abstract—Underdetermined sparse sources separation is a challenge problem especially in adverse environment, where there are often some non-sparse interferences or more than one sparse interferences located closely to the target sources. While in some applications, such as in-car or hands-free environments, references of the interferences (\(P \geq 2\)) coming from loudspeakers are available. Common sparse source separation approaches have not yet used these reference information, we call them traditional approaches in this paper. We propose a FD-MENUET (Frequency domain aDaptive filtering based Multiple sENsor degene-rate Umixing Es-timation Technique) approach, in which we get full use of those reference information to help to separate the target sources. Even if no reference is available, the approach would only degenerate to the traditional approaches. The experimental results show that the proposed approach is more general and could achieves better separation performance than the traditional one.

I. INTRODUCTION

Underdetermined Sparse Source Separation is more interesting in recent years for its capability of handling the problem that the number of sources \(N\) could be bigger than the number of sensors \(M\) (\(N > M\)). For the advantage of being implemented in real time, the binary mask approaches such as Degenerate Unmixing Estimation Technique (DUET) [1] and Multiple Sensor DUET (MENUET) [2] are more attractive in all approaches. It assumes that the sources are efficiently sparse, and assigns the mask with 1 if the energy of the source is over the threshold, and 0 otherwise. While the ideal binary mask (IBM) approach sets the mask 1 if the source's energy exceeds other sources and 0 otherwise [3]. However, in practice, there are often interferences that may be non-sparse or locate closely to the target source, and the references are usually available when these interferences come from loudspeakers. In such situation, we could no longer assume that at most one source is dominant at each T-F unit. Then, the traditional approach without using these references would be difficult or even impossible to separate the target sources. So for the problems, our motivation is that using adaptive filtering technique to cancel these interferences’ components from the T-F units in frequency domain before features extraction.

Adaptive filtering techniques have been developed from time domain to frequency domain and single channel to multi-channel. Most adaptive algorithms in time domain could be classified into Least Mean Square (LMS) family, Recursive Least Square (RLS) family and Affine Affine Projection (AP) family. In recent years frequency domain adaptive filter (FDAF) become more and more attractive [4–6], because comparing with algorithms in time-domain it converges faster, could be effectively applied in multi-channel (MC) case (MC-FDAF) while requiring poor cross correlation conditions among channels, and update step size of the filter coefficients could be independent to each other at different frequencies [7]. For advantages mentioned above, we apply MC-FDAF into our system.

This paper is organized as follows. In Section II, we introduce the proposed flow model, definitions and problems description. Section III describes the procedures of the proposed approach. Experiments setup and discussions are presented in Section IV. The last section concludes this paper.

II. BACKGROUND DESCRIPTION

In the paper, lowercase and uppercase bold font represent vector and matrix quantities, respectively; all vectors are column vector; and \([\cdot]^T\) stands for matrix or vector transposition. Underlined quantities denote DFT-domain variables and \(k\) is a discrete time index.

A. Model and Definition

We consider the integration of all sources and sensors as a MIMO system with \(M\) input channels and \(Q\) output channels. The received signal from the \(q\)th sensor at time \(k\) is given by

\[
x_q(k) = \sum_{m=1}^{M} u_m^T(k) h_{mq}(k) = u^T(k) h_q(k)
\]

where \(M = N + P\) and

\[
u_m(k) = [u_m(k), u_m(k-1), \cdots, u_m(k-L+1)]^T
\]

\[
h_{mq}(k) = [h_{mq,0}(k), h_{mq,1}(k), \cdots, h_{mq,L-1}(k)]^T
\]

are a vector containing the latest \(L\) samples captured from the \(m\)th input channel and the current adaptive filter for the path from source \(m\) to sensor \(q\), respectively. There are \(N\) non-referenced sources and \(P\) referenced sources. For convenient, we integrate each of them for all \(m\) in one vector

\[
h_q(k) = [h_{1q}^T(k), h_{2q}^T(k), \cdots, h_{Mq}^T(k)]^T
\]
\[\mathbf{u}(k) = [\mathbf{u}^T_1(k), \mathbf{u}^T_2(k), \cdots, \mathbf{u}^T_{N}(k)]^T \]
\[= [s^T_{1}(k), \cdots, s^T_{N}(k), r^T_1(k), \cdots, r^T_{P}(k)]^T \]

where \( s^T_{n}(k) \) and \( r^T_{p}(k) \) are the \( n \)-th non-referenced source and the \( p \)-th referenced source, respectively. We mainly concern about the situation that non-referenced sources number \( N \) is greater than sensors number \( Q \) \((N > Q)\), namely underdetermined situation. There is no restriction to the number of reference sources \((P \geq 0)\,\text{but we focus on the situation that the number of reference sources} \, P \geq 2\). The proposed approach aims to separate out each source \( y_{nq}(k) \) from every mixed signal \( x_{nq}(k) \), and make good use of all valuable information to obtain more pure \( y_{nq}(k) \).

**B. Problems Description**

1) Problem of traditional approach: According to the sparse theory \([1, 2]\), we know each source sparsely distributes in the time-frequency domain, and it is a small probability event that multiple sources appear in the same T-F unit of the mixture. Accordingly, it can be approximated that the T-F unit likely belongs to the source whose energy is major here

\[\mathbf{X}_{L,q}(f_i) \approx \mathbf{u}^T_{L,m}(f_i)\mathbf{H}_{L,mq}(f_i)\]

where \( i \) is a block time index, \( L \) is DFT length. However, in practice, some sources are not sparse enough or even non-sparse, not only the prominent-energy source but also other considerable-energy sources exist in the same T-F unit. Due to the greatly increased probability of multiple sources’ energy overlap in one T-F unit, we can not approximate it as (6).

2) Problem of sensor pair: Interaural Level Difference (ILD) and Interaural Time Difference (ITD) \([8]\) are two important and popular features to estimate the binary mask. When the locations of source and sensor pair follow

\[\left|SF_1 \right| - \left|SF_2 \right| = \pm 2a \quad (0 \leq |2a| \leq |F_1F_2|)\]

where \( a \) is a constant scalar, and \( |SF_1|, |SF_2|, |F_1F_2| \) and \( |2a| \) represent the length from source \( S \) to sensor \( F_1 \) and sensor \( F_2 \), the length between \( F_1 \) and \( F_2 \), the distance difference between \( S \) to \( F_1 \) and \( F_2 \), respectively. The ITD features will be exactly the same, then for omnidirectional sensor pair, it would be difficult to separate the sources. Particularly, when \( a = 0 \), even directional sensor pair could not separate them.

3) Problem of multiple sources nearby: In practice, there are often some sources located closely to each other, noticed that the positions of these sources and sensor pair would not satisfy (7). No matter for omnidirectional or directional sensor pair, the ILDs and ITDs of these sources are extremely similar, especially in reverberate environment. It is difficult to separate these sources efficiently. In Fig.2, there are 5 sparse sources in (a) and (c), in which 2 sources are close to each other. Results shows that there are only 4 peaks, the 2 sources are mixed into one peak. For detail discussion, see IV-B.

**III. PROPOSED METHOD**

In order to improve the separation performance, especially in the challenging situations mentioned above, we propose to make full use of the possible references. The main procedures of our proposed approach are present in Fig.1.

A. Adaptive filtering Procedures

Before feature extraction, we process adaptive filtering first whenever only referenced sources are present. Here we can use double talk detector (DTD) \([9, 10]\) as a controller to determine when the adaptive filter should work. And we must take different DFT transforms before and after every adaptive
filtering iteration. In adaptive filtering procedure, 2Lth DFT is employed, for referenced source $r_p$

$$R_{2L\times2LP}(i) = [\mathbf{R}_{2L,1}(i), \mathbf{R}_{2L,2}(i), ..., \mathbf{R}_{2L,P}(i)]$$  \hspace{1cm} (8)

$$r_p(iL - L)$$

$$\mathbf{R}_{2L,p}(i) = \text{diag} \left\{ \begin{bmatrix} \mathbf{F}_{2L\times2L} & \mathbf{0}_{L\times Q} \\ \vdots & \vdots \\ \mathbf{0}_{L\times P} & \mathbf{F}_{2L\times2L} \end{bmatrix} \right\}$$  \hspace{1cm} (9)

where $\mathbf{F}_{2L\times2L}$ is a $2L \times 2L$ DFT matrix with elements $e^{-j2\pi vu/2L}$, where $v, l = 0, ..., 2L - 1$; for $q$th sensor received signal $x_q$

$$\mathbf{X}_{2L\times Q}(i) = \mathbf{F}_{2L\times2L} \begin{bmatrix} x_1(i), ..., x_q(i), ..., x_Q(i) \end{bmatrix}$$  \hspace{1cm} (10)

$$x_q(i) = [x_q(iL), x_q(iL + 1), ..., x_q(iL + L - 1)]$$  \hspace{1cm} (11)

and for frequency domain adaptive filter

$$\mathbf{H}_{2L\times Q}(i) = \begin{bmatrix} \mathbf{h}_{2L,1}(i), ..., \mathbf{h}_{2L,P}(i), ..., \mathbf{h}_{2L,2LQ}(i) \end{bmatrix}$$  \hspace{1cm} (12)

$$\mathbf{h}_{2L,P}(i) = \mathbf{F}_{2L\times2L} \begin{bmatrix} h_1(i), ..., h_pq(i), ..., h_{2LQ}(i) \end{bmatrix}$$  \hspace{1cm} (13)

The MC-FDAF is used [5, 7] to update the adaptive filter until non-referenced sources are present.

$$\mathbf{E}_{2L\times Q}(i) = \mathbf{X}_{2L\times Q}(i) - \mathbf{G}^{(1)}_{2L\times2L} \mathbf{R}_{2L\times2LP}(i) \mathbf{H}_{2L\times Q}(i)$$  \hspace{1cm} (14)

$$\mathbf{H}_{2L\times Q}(i) = \mathbf{H}_{2L\times Q}(i-1) + \mathbf{G}^{(1)}_{2L\times2LP} \mathbf{K}(i) \mathbf{E}_{2L\times Q}(i)$$  \hspace{1cm} (15)

where $\mathbf{G}^{(1)}_{2L\times2L}$, $\mathbf{G}^{(1)}_{2L\times2LP}$ and the kalman gain $\mathbf{K}(i)$ are derived in [5, 11]. We can also calculate the kalman gain by using [12]. Then all sources are present and it is only needed to process $L$th DFT with $\mathbf{F}_{L\times L}$ similar to $\mathbf{F}_{2L\times2L}$, adding a frame window is optional here. The interferences with references can be eliminated from the mixture by

$$\mathbf{X}'_{L\times Q}(i) = \mathbf{X}_{L\times Q}(i) - \sum_{p=1}^{P} \sum_{l=1}^{L} \mathbf{E}_{L,p}(i) \cdot \mathbf{H}_{L,p}(i)$$  \hspace{1cm} (16)

$$\mathbf{X}_{L\times Q}(i) = [\mathbf{x}_{L,1}(i), ..., \mathbf{x}_{L,q}(i), ..., \mathbf{x}_{L,Q}(i)]$$  \hspace{1cm} (17)

$$\mathbf{H}_{L,p}(i) = [\mathbf{h}_{L,p1}(i), ..., \mathbf{h}_{L,pq}(i), ..., \mathbf{h}_{L,pQ}(i)]$$  \hspace{1cm} (18)

and $\mathbf{h}_{L,pq}(i)$ is $L$th DFT transform of the current filter for the path from source $p$ to sensor $q$. After the adaptive elimination, then we could make separation on $\mathbf{X}_{L\times Q}(i)$.

### B. Separation Procedures

**Step 1. Clustering:** Because an individual cluster in the histogram corresponds to an individual source [1], we can separate each source by selecting the observation signal at T-F units in each cluster with a binary mask [2]. Suppose we know the number of non-referenced sparse sources is $N$, then by using the popular clustering methods such as $K$-means or GMM to cluster the features $\mathbf{X}_i(f, i)$ extracted from $\mathbf{X}_{L\times Q}(i)$, and we can obtain $N$ current clustered centers $\mathbf{C}_n(i)$, where $1 \leq n \leq N$. Kinds of features are discussed in [2].

**Step 2. Pattern recognition:** By extracting new feature $\Theta_{q1, q2}(f, i)$ from $\mathbf{X}_{L, q1}(i)$ and $\mathbf{X}_{L, q2}(i)$, we can use distance or probability as the criterion to determine which clustered center, generated in step 1, the new feature belongs to. Then we could obtain the binary mask accordingly by

$$M_{n, q1, q2}(f, i) = \begin{cases} 1 & \Theta_{q1, q2}(f, i) \in \mathbf{C}_{n, q1, q2}(i) \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (19)

By applying the binary mask to $\mathbf{X}_{L, q1}$ and $\mathbf{X}_{L, q2}$, we can obtain the $n$th source’s components from sensor $q_1$ and $q_2$, respectively.

$$\mathbf{X}'_{n, q1}(f, i) = M_{n, q1, q2}(f, i) \cdot \mathbf{X}_{L, q1}(f, i)$$

$$\mathbf{X}'_{n, q2}(f, i) = M_{n, q1, q2}(f, i) \cdot \mathbf{X}_{L, q2}(f, i)$$  \hspace{1cm} (20)

We can finally obtain the separated signal $\mathbf{y}_{n, q}(i)$ from the $q$th source’s mixed-signal $x_q(i)$ by using overlap-add method [2, 13],

$$\mathbf{y}_{n, q}(i) = \frac{1}{\lambda} \sum_{\beta=0}^{\lambda-1} \mathbf{y}_{n, q}^\beta(i - \beta)$$  \hspace{1cm} (21)

where $\lambda = L/S$ is overlap factor, $S$ is frame shift length, $\mathbf{y}_{n, q}(i)$ is a vector with length $S$,

$$\mathbf{y}_{n, q}^\beta(j) = [y_{n, q}^\beta((j + \beta)S + 1), y_{n, q}^\beta((j + \beta)S + 2), ..., y_{n, q}^\beta((j + \beta + 1)S)]^T$$  \hspace{1cm} (22)

where $y_{n, q}^\beta(r)$ is the $r$th element of $\mathbf{y}_{n, q}^\beta(j)$, and $\mathbf{y}_{n, q}^\beta(j)$ is the inverse DFT transform of $\mathbf{X}'_{n, q}^\beta(j)$.

A common problem in using binary mask is that there are inevitable errors in estimating the masks, this would lead to musical noise. The problem can be prevented by using spectral smoothing technique [14].

### IV. Experiments

In this section, we show the performance of the proposed approach and compare it with the traditional [2] and the IBM approaches.

**A. Experiment Environment**

The experiments are carried out in a typical-sized room with $T60 = 200ms$ and there are one sensor pair. We obtain the sensor received signals by convolving the source signals and the related measured impulse responses. The inter-distance of the sensor pair is $4cm$ and sampling frequency is $8KHz$. The sources include Chinese speech and white noise. Instead of
DTD, we update the adaptive filter first until non-referenced sources are present.

The frame size of STFT is $L = 512$, the frame shift length is $S = 128$ and the chosen feature [2] is

$$\Theta_{q_1q_2}(f, i) = \left[ \frac{|x_{q_1}(f, i)|}{A_{q_1q_2}(f, i)} \right] \left[ \frac{|x_{q_2}(f, i)|}{A_{q_1q_2}(f, i)} \right] \frac{1}{\alpha_{q_1q_2}} \arg \left[ \frac{x_{q_1}(f, i)}{x_{q_2}(f, i)} \right]$$

where $A_{q_1q_2}(f, i) = \sqrt{|x_{q_1}(f, i)|^2 + |x_{q_2}(f, i)|^2}$, $\alpha_{q_1q_2} = 4c^{-1}d_{q_1q_2}$ is the normalization factor, $c = 343 \text{ m/s}$ is the speed of sound propagating in the air and $d_{q_1q_2}$ is the distance between the $q_1$th sensor and the $q_2$th sensor. We cluster the extracted features by using $k$-means method. The adaptive filter length is $L$, the update rate is $\omega = (1 - 1/(3L))^2$ and the frame overlap factor is $\lambda = L/S$.

We performed 4 group experiments, see Fig.3. There are 5 sources with 3 non-referenced ones and 2 referenced ones in every group. In the first group, two sparse sources are both satisfied (7) with $a = 0$; in the second group, two sparse sources are located closely; in the third group, there is a non-sparse source with reference; the last group is the situation that all sparse sources can be separated by traditional approach. We compared the separation performances of the approaches using Signal to Interference Ratio (SIR) and Signal to Distortion Ratio (SDR) as the evaluation measures.

$$\text{SIR}_n = 10\log_{10} \frac{\sum_k |y_{nq}(k)|^2}{\sum_k |\sum_{j \neq n} y_{jq}(k)|^2} - 10\log_{10} \frac{\sum_k |x_{nq}(k)|^2}{\sum_k |\sum_{j \neq n} x_{jq}(k)|^2}$$

$$\text{SDR}_n = 10\log_{10} \frac{\sum_k |x_{nq}(k) - \beta y_{nq}(k - D)|^2}{\sum_k |x_{nq}(k)|^2}$$

where $y_{nq}(k)$ and $x_{nq}(k)$, $(1 \leq v \leq N)$, are the components that belonged to $s_v(k)$, mixed in $y_{nq}(k)$ and $x_{nq}(k)$ respectively; $\beta$ and $D$ are used to compress the amplitude attenuation and the time delay between $y_{nq}(k)$ and $x_{nq}(k)$. $\beta = 1$ and $D = 0$ in our statistics.

**B. Results And Discussion**

Note that we only concern the 3 non-referenced sources and the results are shown in Fig.4 and Fig.5 in which (A)-(D) are relative to (A)-(D) in Fig.3.

Fig.4 is generated in the situation (D) of Fig.3. We can see that the proposed results are more pure than the traditional’s. The intuitive comparison in the situation (B) is shown in Fig.2, in which (a) and (c) are results of traditional approach, there are 5 sources are present but only 4 peaks are distinct; (b) and (d) are generated by the proposed approach, in the 5 sources, the 3 sources are respect to the 3 distinct peaks respectively.

In situation (A), the features of the two sources are exactly the same, the traditional approach can not separate these two sources. The proposed approach cancel the confused referenced source and separate other sources well, see Fig.5.

Results in (B) is similar to (A) and the features of the two nearby sources are similar. In traditional results, these two separated sources interfere each other, in worst case, this situation even cause to large deviation of the other 3 cluster’s regular center. In (C), there is a non-sparse source, white noise with -2dB, in the mixture. The traditional approach could still work with small energy of the non-sparse source, but the separation results are poor. While heavy energy will lead to impossible separation. The proposed approach eliminates the components of the non-sparse source and obtains a good separation result. (D) is a normal situation that the traditional approach generally deal with. All sources are sparse and spatial features are different from each other. The result shows...
that the performance of the proposed approach is also better than the traditional one in this situation.

The results of the IBM approach are present as well. Note that the IBM results are implemented without adaptive filtering and all 5 sources are separated. However, in the proposed approach, only the 3 non-referenced sources are needed to be separated. Hence the proposed results are sometimes better than those of the IBM.

The comparison differences of average SIR and average SDR in situation (A)-(D) among traditional, proposed and IBM approaches are listed in Table I. It also shows the proposed approach is better than the traditional one.

### Table I

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* Proposed approach
* Traditional approach

### V. Conclusions

We proposed a novel FD-MENUET (Frequency domain aDaptive filtering based MENUET) approach. It extends the applicable situations of the traditional approach as follows: 1) sparse sources are lived with referenced non-sparse sources; 2) some referenced interferes are close to target sources or follow the relation (7) with the sensor pair; 3) the situation that traditional approach could achieve and some sources have reference signals. We derived a general MIMO solution, and when there is none referenced sources, the proposed approach will degenerate to the traditional one. Due to the finite length of the adaptive filters, the separation performance is still subject to the condition of reverberation.

There are many issues remain such as source number estimation, precise DTD and reverberation problem, which will be further studied in our future work.

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### References