# Human Motion Description Based on GPDM and 3D Curve Moment Invariants in Latent Spaces

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*Abstract*— **Motion description and analysis is important for automatically generating new realistic motions in computer animation and virtual environments. In this paper, we propose a new motion analysis method in low-dimensional latent spaces by using Gaussian Process Dynamical Model (GPDM) and 3D curve moment invariants. GPDM is used to mapping the high-dimensional motion data into low-dimensional latent space. 3D curve moment invariants are used to describe the features of motion curves learned by GPDM. This method can be used to describe the characteristic of different motion. We verify our method using CMU motion capture database.**

## I. INTRODUCTION

3D human animations are now wildly used in movies, sports analysis, virtual reality et al. The animation motion data is easier to get than before using motion capture techniques. But the captured motion is fixed and it is still hard to generate high credibility new motion data from existed motion data at present. This is partly because human are greatly lack of motion understanding while they are very familiar with their own motions. Even a tiny flaw will make the motion looks unnatural. Related research has shown that people are very sensitive to the detail of human motions (Kozlowski and Cutting, 1977).

The degree-of-freedom (DOF) number of a natural human is exceeded 250 (Watt and Policarpo, 2005), which makes human motion state space very huge. At present, a new motion data is often produced by manual or through constraints optimizing in high-dimensional state space on motion capture data. The effectiveness and efficiency of these methods cannot meet people's requirement. For improving the quality of automatically motion generation, people need to get a more precision understanding of motion rules and motion characteristics.

The movement of human beings contains many rigid bodies and many DOFs. It is difficult to be analyzed directly. It can be noticed that human motions have certain cooperative relationships. For example, in the process of walking, the movement relation between upper limbs and lower limbs follows certain cooperation rules. These kinds of cooperation are not only in spatial but also in temporal. These cooperative constraints are also true for complex motions. It might be more local in time. By now, the coordination relationship is not well described both in biomechanics and computer animation. The study of such regularity can set up the analysis model and statistics model of human beings. These models can be used to reduce motion states and improve motion generating efficiency of computer animation.

Motion capture has now become the mainstream technology to get the real human motion data. With about ten year's development, researchers have accumulated a large amount of captured data, which provides the basis for human motion analysis.

This paper is mainly about our works of human motion analysis in low-dimensional latent spaces using GPDM and 3D curve movement invariants.

## II. RELATED WORKS

Though there is not a systemic report about human motion analysis in low-dimensional spaces, many research works have used the cooperative characteristic in human motions. In SIGGRPAH2004, Safonova et al. (Safonova et al., 2004) proposed a method of synthesizing human motions in low-dimensional spaces. They observed there are two basic characteristics of human motions: 1. Many human motions can be represented using five to ten DOFs. 2. Closely related motions could be used to construct low-dimensional spaces to express other motions of same behavior.

Using the cooperative relationship of joints, Pullen and Bregler (Pullen and Bregler, 2002) suggested that the incomplete motion data could be predicted by using previous motion data. They proposed a method of generating completed human motion using sketch map containing a little key frames. The correlation of human motion in space could also be used to add physical constraints during motion synthesization (Kovar et al., 2002). Chai and Hodgins (Chai and Hodgins, 2005) proposed their work on using low-

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dimensional control signals to build human animation in SIGGRAPH2005. In their work, they build the animation using few retro-markers. The missing data can be makeup using the captured database.

The motion model learned in low-dimensional spaces can be used to help motion generation and makeup missing motion data. Taylor et. al. (Taylor et al., 2007) proposed the method of using two latent invariants to express human motion model, he demonstrated the effective of the method for using existed data to makeup the missing motion capture data. Wang et al. (Wang et al., 2006) proved that the Gaussian Process Dynamical Model (GPDM) could be used to describe the spatial characteristic of motion data.

The main methods used for high-dimensional reduction include LLE (Roweis and Saul, 2000), ISOMAP (Tenenbaum et al., 2000), GPLVM (Lawrence, 2006) and GPDM (Wang et al., 2006) et al. But LLE cannot confirm the dimension after dimension reducing. ISOMAP is difficult to select the neighborhood points. GPLVM cannot reflect the spatial continuity of motion data. We use GPDM to reduce the dimension of human motion data in our work.

## III. MOTION ANALYSIS IN LOW-DIMENSIONAL LATENT SPACES

### A. Gaussian Process Dynamical Model (GPDM)

The Gaussian Process Dynamical Model (GPDM) reflects the spatial continuity of motion data and the low-dimensional dynamic characteristic of motion data (Wang et al., 2006). GPDM comprises a mapping from a latent space to the data space, and a dynamical model in the latent space (Fig.1). These mappings are typically nonlinear. GPDM is obtained by marginalizing out the parameters of the two mappings, and optimizing the latent coordinates of training data.

GPDM is to model the probability density of a sequence of vector-valued states $y_1 ..., y_t, ..., y_N$ with discrete-time index $t$ and $y_t \in R^D$. Consider a latent-variable mapping with first-order Markov dynamics.

$$x_t = f(x_{t-1}; A) + n_{x,t} \quad (1)$$

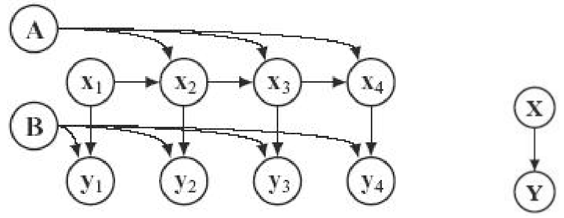
$$y_t = g(x_t; B) + n_{y,t} \quad (2)$$

Here, $x_t \in R^d$ denotes the $d$ dimensional latent coordinates at time $t$. $n_{x,t}$ and $n_{y,t}$ are zero-mean white Gaussian noise processes, $f$ and $g$ are (nonlinear) mappings parameterized by $A$ and $B$ respectively. Fig.1 depicts the graphical model. $f$ and $g$ are linear combinations of basis functions $\phi_i$ and $\varphi_i$.

$$f(x; A) = \sum_i a_i \phi_i(x) \quad (3)$$

$$g(x; B) = \sum_j b_j \varphi_j(x) \quad (4)$$

Here $A = [a_1, a_2, ...]$, $B = [b_1, b_2, ...]$




(a) (b)

(a)Nonlinear latent-variable model for time series (b) GPDM model

Fig.1. Time-series graphics models

### B. 3D Curve Moment Invariants

3D curve moment invariant could be used to describe the shape characteristics of 3D curves. Xu et al. (Xu and Li, 2008) proposed an intuitionistic method to deduce the 3D movement invariants. And the invariants could be used as shape descriptors for the representation of parametric curves. They are independent with rotation, scaling and translation.

The curve moment in the 3D space can be defined as follow: suppose $P(t) = (x(t), y(t), z(t))$ is a parametric curve in $R^3$, $T$ is definition domain of parameter $t$, and $\rho(x, y, z)$ is the density distribution function of the curve. 3D curve movements of order $l + m + n$ are defined by the path integrals defined in the path $L$ of $P(t)$:

$$M_{lmn} = \int_L x^l y^m z^n \rho(x,y,z) ds$$
$$= \int_T x(t)^l y(t)^m z(t)^n \sqrt{(\frac{dx(t)}{dt})^2 + (\frac{dy(t)}{dt})^2 + (\frac{dz(t)}{dt})^2} \rho(x(t), y(t), z(t)) dt \quad (5)$$

Here $L$ is the integral path.

The centroid of the 3D curve can be determined from the zero-order and the first-order moments:

$$\bar{x} = \frac{M_{100}}{M_{000}}, \quad \bar{y} = \frac{M_{010}}{M_{000}}, \quad \bar{z} = \frac{M_{001}}{M_{000}} \quad (6)$$

Then the central moments are defined, they are invariants under translation:

$$\mu_{lmn} = \int_L (x - \bar{x})^l (y - \bar{y})^m (z - \bar{z})^n \rho(x, y, z) ds \quad (7)$$

Based on the central moments, the moments which are independent of the scaling can be getting as follow:

$$\eta_{lmn} = \frac{\mu_{lmn}}{\mu_{000}^{1+l+m+n}} \quad (8)$$

3D curve could be rotated in the 3D space, to get the moments invariants independent of rotation. Here we defined 4 basic geometric elements.

1. The distance of every point in the curve to the coordinate origin.
2. The area of the triangle which is constructed by the two points in the curve and the coordinate origin.
3. The dot product of the two vectors which were constructed by the two points in the curve and the coordinate origin.

4. The orientation volume of the tetrahedron which was constructed by three points in the curve and the coordinate origin.

Through multiple the four basic geometric elements, we could get the integral kernel $core(p_1, p_2 ... p_n)$, the integral kernel contains $n$ points. After the multi-integral, we could get the central moments independent of rotation.

$$MI(core(p_1, p_2...p_n)) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} core(p_1, p_2...p_n)\rho(x_1, y_1, z_1)\rho(x_2, y_2, z_2) \cdots\rho(x_n, y_n, z_n)ds_1 ds_2 \cdots ds_n \tag{9}$$

*C. Motion Description*

The method of constructing moments is based on the basic geometry elements such as distance, area and volume etc. The moment invariants have clear geometry meaning.

We use the zero-order moment (the curve length) and three second-order moment invariant to describe the motion curves in 3D latent spaces. The process of computing curve moment invariants is described as follow:

1. For a motion data, mapping the data into 3D latent spaces using GPDM.
2. Compute the distances $D(i)$ of all the sample points in the curve $Data(x, y, z)$ according to the time.
3. Construct the zero-order moment $m_0 = \mathrm{M}_{000} = \sum D(i)$.
4. Compute $\mathrm{M}_{100} = \sum x_i D(i)$, $\mathrm{M}_{010} = \sum y_i D(i)$, $\mathrm{M}_{001} = \sum z_i D(i)$.
5. Translate the curve according to the centroid of the 3D curve:

$$Data_x = Data_x - \frac{\mathrm{M}_{100}}{\mathrm{M}_{000}}$$

$$Data_y = Data_y - \frac{\mathrm{M}_{010}}{\mathrm{M}_{000}}$$

$$Data_z = Data_z - \frac{\mathrm{M}_{001}}{\mathrm{M}_{000}}$$

The coordinate of the points after translation are $Data(x', y'z')$.

6. Each moment is computed by its discrete form

$$\mu_{ijk} = \sum_l (x')^i (y')^j (z')^k D(l) \quad i + j + k < 3$$

Computing $\mu_{ijk}$

7. Construct the three second-order invariants as follow:

$$m_1 = \mu_{200} + \mu_{020} + \mu_{002}$$

$$m_2 = \mu_{200}\mu_{020} + \mu_{200}\mu_{002} + \mu_{002}\mu_{020} - \mu_{101}^2 - \mu_{110}^2 - \mu_{011}^2$$

$$m_3 = \mu_{200}\mu_{020}\mu_{002} + 2\mu_{110}\mu_{101}\mu_{011} - \mu_{200}\mu_{011}^2 - \mu_{020}\mu_{101}^2 - \mu_{002}\mu_{110}^2$$

## IV. EXPERIMENT

We select the “Linear+RBF” kernel to learn the motion models of different motions by using GPDM. Because the learned motion trajectory often intersected in two dimensional latent spaces, this made existing large jump in the learned models. And the models will appear false. We use three dimensional latent spaces while learning motions. In the experiment, we use the data from CMU motion capture database. The joints of hand and toe are slightly related to the poses. Therefore we neglect the DOFs of these joints. The DOFs of the full body is 50, and the sample frequency is 40Hz.

The learning process is divided into three steps.

1. Mean every DOFs of the motion data $Y$, and get the mean motion data $Y'$.
2. Using principal component analysis (PCA) method to initialize the latent coordinate; According to the selected dimension of latent space, using the eigenvalue $v$ and the first three eigenvectors $u$ of PCA to map the motion data into 3D space.

X = Y'*u(:, 1:3)*diag(1/sqrt(v(1:3)))

3. Initialize the hyperparameters of GPDM, and the iteration number of GPDM learning is set to 100 in our experiments.

Fig.2, Fig.3 and Fig.4 are the learned motion models of two walking motions, two kicking ball motions and two playing golf motions respectively. The frame numbers of the motions are shown in table I. The images are the poses in the sample points.

TABLE I. SELECTED MOTION LENGTHS

| Motion | Walk1 | Walk2 | Kick ball1 | Kick ball2 | Golf 1 | Golf 2 |
|---|---|---|---|---|---|---|
| Length | 135 | 156 | 369 | 302 | 313 | 292 |

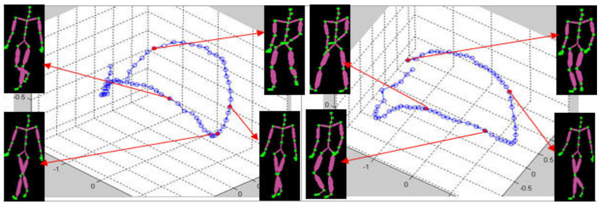


Fig.2. Two walking motion models learned using GPDM

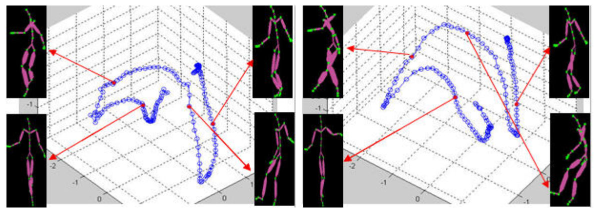


Fig.3. Two kicking ball motion models learned using GPDM

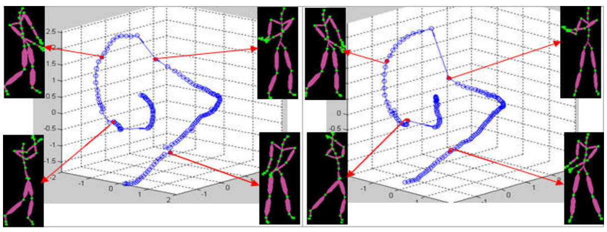


Fig.4. Two playing golf motion models learned using GPDM

Before the learning process, we preprocessed the motion data. The walking motion data selected is about one period. We notice that different motions have different curve shapes in the 3D latent spaces. And similar motions have similar shapes. For example, the walking motion in the latent space is like saddle. 3D curve moment invariants are used to describe the motion model in latent space.

We computed the four invariants of three walking motions (the lengths are 135, 133 and 139 respectively, two kicking ball motions (the lengths are 369 and 302 respectively) and three playing golf motions (the lengths are 313, 314 and 292 respectively).

TABLE II . THE FOUR 3D CURVE MOMENT INVARIANTS OF EIGHT MOTIONS

| Motion | $m_0$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|
| Walk1 | 10.317 | 19.832 | 109.500 | 136.198 |
| Walk2 | 10.728 | 22.619 | 135.738 | 137.800 |
| Walk3 | 9.693 | 21.052 | 120.148 | 108.560 |
| Kick_ball1 | 19.507 | 60.025 | 1138.477 | 6738.425 |
| Kick_ball2 | 19.527 | 56.614 | 1023.861 | 5954.281 |
| Golf1 | 15.262 | 47.914 | 741.376 | 3705.910 |
| Golf2 | 16.295 | 51.833 | 854.212 | 4486.878 |
| Golf3 | 16.718 | 43.669 | 603.590 | 2650.057 |

After normalization the four invariants, we got a four order eigenvector. The Euclidean distances of the eight motions are computed as table III.

TABLE III. DISTANCE MATRIX OF THE MOTION CURVES EIGENVECTORS

| | Walk 1 | Walk 2 | Walk 3 | Kick ball1 | Kick ball2 | Golf 1 | Golf 2 | Golf 3 |
|---|---|---|---|---|---|---|---|---|
| Walk1 | 0 | 0.06 | 0.04 | 1.56 | 1.41 | 0.93 | 1.11 | 0.77 |
| Walk2 | 0.06 | 0 | 0.06 | 1.53 | 1.37 | 0.89 | 1.06 | 0.73 |
| Walk3 | 0.04 | 0.06 | 0 | 1.56 | 1.41 | 0.93 | 1.1 | 0.77 |
| Kick ball1 | 1.56 | 1.53 | 1.56 | 0 | 0.16 | 0.64 | 0.47 | 0.83 |
| Kick ball2 | 1.41 | 1.37 | 1.41 | 0.16 | 0 | 0.49 | 0.32 | 0.67 |
| Golf1 | 0.93 | 0.89 | 0.93 | 0.64 | 0.49 | 0 | 0.17 | 0.22 |
| Golf2 | 1.11 | 1.06 | 1.1 | 0.47 | 0.32 | 0.17 | 0 | 0.38 |
| Golf3 | 0.77 | 0.73 | 0.77 | 0.83 | 0.67 | 0.22 | 0.38 | 0 |

The experiment results indicate that the distance of same kind of motion is clearly smaller than other kind of motions. The zero-order moments and the three second-order curve moment invariants can be regarded as the shape descriptors of the motion curves in the 3D latent spaces. The curve moment invariants can be used to describe and distinguish the motion curves learned using GPDM in 3D latent spaces.

## V. CONCLUSION

GPDM has the good character for describing human motion data in latent space. The motion curve in latent space learned by GPDM is continuity. The 3D curve moment invariants can be used as the shape descriptors of 3D curves in latent space. Experiment certificates the validity of the 3D curve moment invariants in describing the shape of motion curves. The 3D curve movement invariants can describe the character of motion model in latent space well, which help to implement the motion analysis in low-dimensional space. And it could also be used to implement motion identification and classification for motion data.

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