Adaptive Detection of Narrowband Signals Using Frequency-Transformation-Based Variable Digital Filters

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***Abstract*—This paper presents an application of variable infinite impulse response (IIR) digital filters based on the Constantinides frequency transformation to adaptive filtering. We propose a new unified framework for adaptive detection of narrowband signals using variable band-stop IIR digital filters with arbitrary transfer functions. It is also shown that our proposed method includes the all-pass-based adaptive notch filters as a special case. A simulation example demonstrates the utility of our proposed method from the viewpoint of the signal-to-noise ratio (SNR) improvement. This result shows that the Constantinides frequency transformation plays significant roles in not only the filter design, but also the adaptive signal processing.**

I. Introduction

In many practical applications of digital signal processing, there is a great need for real-time tuning of the characteristics of frequency selective digital filters such as low-pass, high-pass, band-pass, and band-stop filters. Such tunable digital filters are called variable digital filters, and many methods of design of variable digital filters have been proposed [1].

In this paper we focus on infinite impulse response (IIR) variable digital filters. Design of variable IIR digital filters is classified into two major methods: one is based on the Constantinides frequency transformation [2], and the other is based on the spectral parameters [3], [4], [5]. The method based on the Constantinides frequency transformation is known to be very simple, but this method can tune only the cutoff frequencies of filters. On the other hand, the spectral-parameter-based method is claimed to be more general than the frequency-transformation-based method because the spectral-parameter-based method can deal with much more flexible characteristics such as the independent tuning of various band edges.

However, it is not true that the frequency-transformation-based method is less useful than the spectral-parameter-based method. For example, the frequency-transformation-based method offers many results available to design and synthesize high-accuracy variable digital filters with respect to the quantization effects (see [1], [6], [7] and the references therein). In addition, the frequency-transformation-based method also plays important roles in adaptive signal processing. This can be mostly seen in the field of adaptive notch filtering. In [8], [9] the all-pass-based notch filters are obtained through the Constantinides frequency transformation and they are applied to the adaptive filtering. Also, our previous results [10], [11] apply the Constantinides frequency transformation to construction of second-order adaptive band-pass filters. Furthermore, in [12] we presented a new adaptive band-pass filter that can deal with high-order transfer functions: it was shown that the filter order can be set to be larger than two, and that the use of higher-order filters results in better signal-to-noise-ratio (SNR) improvement.

In this paper, we further extend the aforementioned existing methods of adaptive signal processing based on the Constantinides frequency transformation. We propose a unified framework for adaptive detection and suppression of narrowband signals immersed in broadband signals, by using variable band-stop filters (VBSFs) based on the Constantinides frequency transformation. While the existing methods [8], [9] make use of second-order transfer functions for construction of adaptive band-stop (notch) filters, our proposed method can be applied to arbitrary band-stop transfer functions. We achieve this goal by deriving a simple filtering algorithm based on the block diagram of VBSFs. This approach is similar to our previous approach to derivation of high-order adaptive band-pass filters [12]. It is shown that our proposed method given in this way includes the conventional all-pass-based adaptive notch filters [8], [9] as a special case. Furthermore, we give a simulation example to demonstrate another significance of our proposed method. It is shown that, by choosing a high-order VBSF we can obtain higher SNR at the filter output than choosing low-order VBSFs.

II. VBSFs Using Constantinides Frequency Transformation

This section introduces the method of design of VBSFs using Constantinides frequency transformation.

The first step is to prepare a low-pass prototype digital filter with the transfer function $H_{\rm p}(z)$ given by

$$H_{\rm p}(z) = \frac{\sum_{j=0}^{N} b_j z^{-j}}{1 + \sum_{i=1}^{N} a_i z^{-i}} \qquad (1)$$

where N is the order of the filter and a_i's and b_j's are the coefficients. In this paper, it is assumed that $H_{\rm p}(z)$ is the Butterworth low-pass filter of which cutoff frequency is denoted by $\omega_{\rm p}$.

The second step is to apply the Constantinides frequency transformation [2] to (1) and obtain the desired VBSFs. In this paper we use the following type of Constantinides frequency transformation

$$H(z,\xi) = H_{\rm p}(z)|_{z^{-1}\leftarrow T(z,\xi)} \quad (2)$$

$$T(z,\xi) = z^{-1}\frac{z^{-1}-\xi}{1-\xi z^{-1}} \quad (3)$$

where $H(z,\xi)$ is the $2N$-th order transfer function of the desired VBSFs. The frequency transformation function $T(z,\xi)$ is the specific second-order all-pass function and the tuning parameter ξ satisfies the following relationship

$$\xi = \cos\left(\frac{\omega_2+\omega_1}{2}\right) / \cos\left(\frac{\omega_2-\omega_1}{2}\right) \quad (4)$$

where ω_1 and ω_2 are the cutoff frequencies of the VBSFs ($\omega_1 < \omega_2$). It is important to note that ω_1 and ω_2 are related to $\omega_{\rm p}$ by

$$\tan\left(\frac{\omega_2-\omega_1}{2}\right)\tan\frac{\omega_{\rm p}}{2} = 1 \quad (5)$$

from which we find

$$\omega_2 - \omega_1 = \pi - \omega_{\rm p}. \quad (6)$$

This relationship shows that the VBSFs given by (2) and (3) have the same stop-bandwidth as that of the prototype low-pass filter, regardless of the value of ξ. Therefore, the VBSFs considered in this paper have the fixed stop-bandwidth and thus only the location of the stop-band is tunable. An example of such VBSFs is shown in Fig. 1.

Finally, for narrowband VBSFs the following relationship holds:

$$\xi \simeq \cos\left(\frac{\omega_2+\omega_1}{2}\right) = \cos\omega_0. \quad (7)$$

In this case, ω_0 can be considered as the center frequency of the stopband.

III. Proposed Method

In this section we propose a new system for adaptive detection and suppression of an unknown narrowband noise that is immersed in a broadband signal. The block diagram of the proposed system is shown in Fig. 2, where a VBSF $H(z,\xi)$ designed by (2) and (3) is used for adaptive filtering. The filter input $u(n)$ is of the form

$$u(n) = u_{\rm n}(n) + u_{\rm b}(n) \quad (8)$$

where $u_{\rm n}(n)$ and $u_{\rm b}(n)$ are a narrowband noise and a broadband signal, respectively, and $u_{\rm b}(n)$ is assumed to be a white sequence with zero mean and uncorrelated with $u_{\rm n}(n)$. The adaptive algorithm controls the tuning parameter ξ in such a manner that the location of the stop-band coincides with the location of the narrowband noise. This can be achieved by minimizing the mean square output $E[y^2(n)]$ with respect to ξ, and this strategy is the same as the conventional adaptive notch filtering.

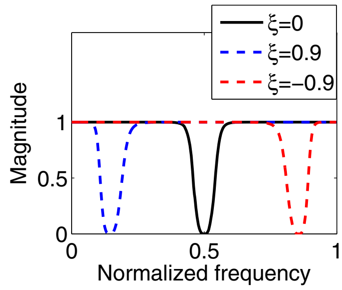

Fig. 1. Example of a VBSF.

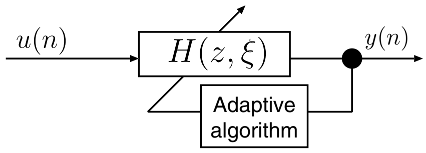

Fig. 2. Proposed system based on a VBSF as the adaptive filter.

In order to derive an adaptation mechanism for tuning ξ, we first derive a time-domain filtering algorithm for the VBSFs in a simple form. As is well-known, the transformation given by (2) and (3) implies that we can obtain the desired VBSFs $H(z,\xi)$ by replacing each delay element of the prototype filter $H_{\rm p}(z)$ with the all-pass filter $T(z,\xi)$. Note that this strategy is indeed feasible because $T(z,\xi)$ does not have a direct feedthrough path and thus the aforementioned replacement does not yield delay-free loops in $H(z,\xi)$. Figure 3 shows the design and realization of VBSFs from a given prototype filter. Now, by introducing the internal variables $v_0(n), v_1(n), \cdots v_N(n)$ and $w_1(n), \cdots w_N(n)$ as in Fig. 3(b), we can derive a simple time-domain filtering algorithm for the VBSF as follows:

$$\begin{aligned} v_i(n) &= -\xi w_i(n-1) + w_i(n-2) \qquad (1 \le i \le N) \\ v_0(n) &= -\sum_{i=1}^{N} a_i v_i(n) + u(n) \\ w_i(n) &= \xi w_i(n-1) + v_{i-1}(n) \qquad (1 \le i \le N) \\ y(n) &= \sum_{j=0}^{N} b_j v_j(n). \end{aligned} \quad (9)$$

The significance of the above algorithm is that the location of the stop-band can be easily tuned: only the parameter ξ must be updated and thus we do not have to change the coefficients a_i's and b_j's in the tuning process. Moreover, the above algorithm can be applied to any kind of transfer function. Therefore, by choosing high-order transfer function for the prototype filter, we can obtain a VBSF of better magnitude response (i.e. sharper cutoff characteristic and higher stopband attenuation) than using low-order prototype filters. This fact means that the use of high-order VBSFs gives better SNR at the filter output than low-order VBSFs, as will be demonstrated in the next section.

Next we derive the adaptive algorithm based on the nor-

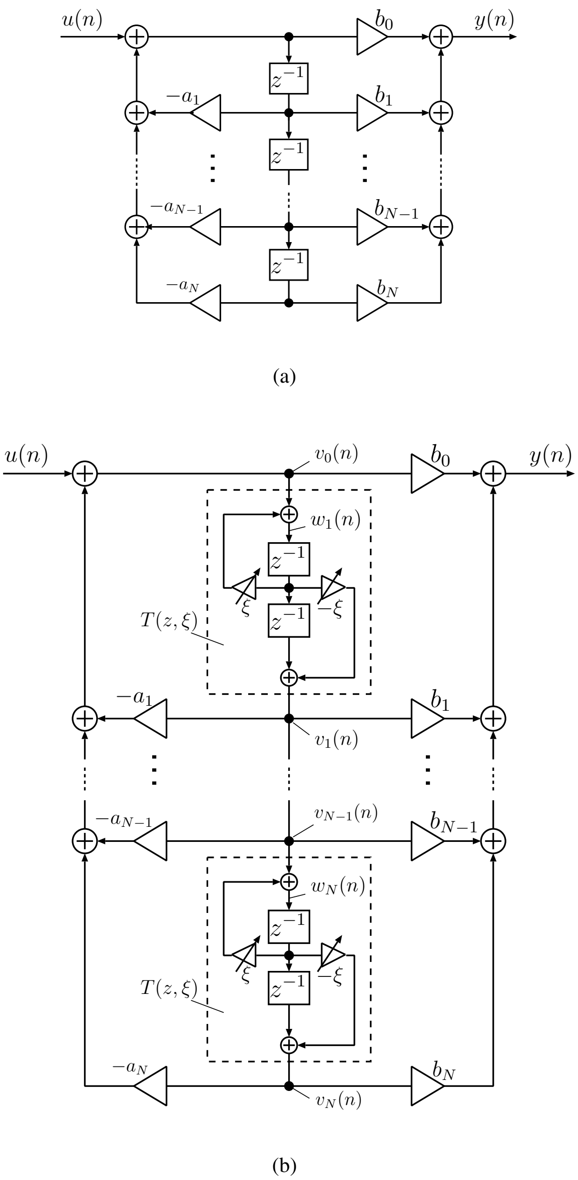

Fig. 3. Design and realization of VBSF: (a) block diagram of prototype filter $H_{\mathrm{p}}(z)$, and (b) block diagram of VBSF $H(z,\xi)$.

malized recursive least-mean square (NRLMS) algorithm. Applying the NRLMS algorithm to (9), we obtain the update equations for the tuning parameter ξ as follows:

$$
\begin{aligned}
\xi(n+1) &= \xi(n) - \mu\psi_y(n)y(n)/R(n+1) \\
R(n+1) &= \gamma R(n) + (1-\gamma)\psi_y^2(n) \\
\psi_y(n) &= \sum_{i=0}^{N} b_i\psi_{v_i}(n) \\
\psi_{v_i}(n) &\equiv \frac{\partial v_i(n)}{\partial \xi(n)} \\
&\simeq -w_i(n-1) - \xi(n)\psi_{w_i}(n-1) \\
&\quad +\psi_{w_i}(n-2) \qquad (1 \le i \le N) \\
\psi_{v_0}(n) &\equiv \frac{\partial v_0(n)}{\partial \xi(n)} = -\sum_{i=1}^{N} a_i\psi_{v_i}(n) \\
\psi_{w_i}(n) &\equiv \frac{\partial w_i(n)}{\partial \xi(n)} \\
&\simeq w_i(n-1) + \xi(n)\psi_{w_i}(n-1) \\
&\quad +\psi_{v_i-1}(n) \qquad (1 \le i \le N) \qquad (10)
\end{aligned}
$$

where μ is the adaptation step size and γ is the forgetting factor.

Since the above adaptive algorithm is based on the NRLMS algorithm, the basic concept is similar to our previous methods [10], [11]. However, the above algorithm is of significant importance because it can be easily applied to arbitrary transfer functions, whereas the algorithm of [10] requires more complicated mathematical formulations in the case of higher-order VBSFs and the algorithm of [11] can be applied to only the second-order transfer functions. Therefore we conclude that the proposed adaptive algorithm is a unified framework that is applicable to any kind of VBSF.

Remark 1: The transfer function of the all-pass-based adaptive notch filter [8], which has been widely used in the existing methods, is a special case of the VBSFs given by (2) and (3). This fact can be easily proved as follows. Let the transfer function $H_{\mathrm{p}}(z)$ of a prototype low-pass filter be

$$H_{\mathrm{p}}(z) = \frac{1+\eta}{2}\frac{1+z^{-1}}{1+\eta z^{-1}}. \qquad (11)$$

It readly follows that this transfer function is the first-order Butterworth low-pass filter of which cutoff frequency is determined by the parameter η. Applying the Constantinides frequency transformation given by (2) and (3) to (11) yields the following second-order VBSF:

$$H(z,\xi) = \frac{1+\eta}{2}\frac{1-2\xi z^{-1}+z^{-2}}{1-(1+\eta)\xi z^{-1}+\eta z^{-2}} \qquad (12)$$

which is exactly the same as the transfer function of the all-pass-based notch filter in [8]. Hence our proposed method includes this all-pass-based adaptive notch filter as a special case.

Remark 2: There exists another type of all-pass-based adaptive notch filter derived by the Constantinides frequency transformation. This adaptive notch filter makes use of the following transfer function [9]

$$H(z,\xi) = \frac{1-2\xi z^{-1}+z^{-2}}{1-(1+\eta)\xi z^{-1}+\eta z^{-2}} \qquad (13)$$

which is slightly different from (12). As is explained in [9], the transfer function of (13) is derived by applying the transformation (2) and (3) to the prototype filter of the form

$$H_{\mathrm{p}}(z) = \frac{1+z^{-1}}{1+\eta z^{-1}}. \qquad (14)$$

Therefore it follows that this method is also included in our proposed method as a special case. However, note that this notch filter is not the Butterworth band-stop filter because the prototype filter given by (14) does not have the Butterworth characteristic.

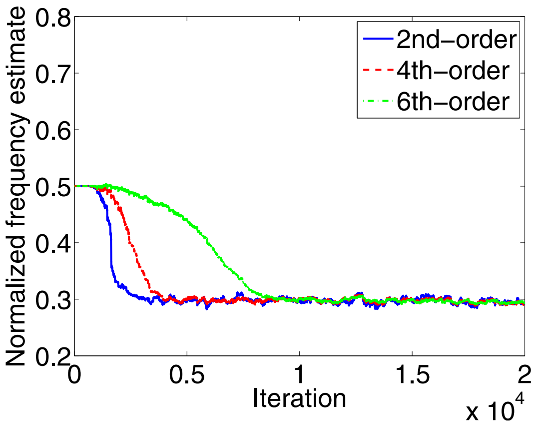

Fig. 4. Estimation of the center frequency of the narrowband noise.

TABLE I
SNR AT THE OUTPUTS OF VBSFS.

	2nd-order	4th-order	6th-order
Output SNR [dB]	8.7788	13.2917	16.2443

IV. Simulation Example

The simulation example makes use of the system of Fig. 2 to suppress a narrowband signal of unknown center frequency. The filter input $u(n)$ is given by (8), where the broadband signal $u_{\rm b}(n)$ is generated by a zero-mean white Gaussian sequence and the narrowband noise $u_{\rm n}(n)$ has the center frequency of 0.3π rad and the bandwidth of 0.1π rad. This narrowband noise is generated by passing a zero-mean white Guassian sequence that is uncorrelated with $u_{\rm b}(n)$ through a band-pass filter with the bandwidth of 0.1π rad. The variances of these two white Gaussian sequences are determined in such a manner that the input SNR becomes 0 dB.

For comparison purpose we use three kinds of VBSFs for the adaptive filtering: 2nd-order, 4th-order, and 6th-order narrowband Butterworth VBSFs are used. The fixed stop-bandwidths of these VBSFs are set to be 0.15π rad and the initial values of the tuning parameter of these VBSFs are specified as $\xi(0) = 0$. This means that the initial value of the center frequency ω_0 in (7) is 0.5π rad. In the adaptive algorithm for all of these VBSFs, we give the following settings: $\mu = 0.005, \gamma = 0.99$ and $R(0) = 10000$.

The simulation results are given in Fig. 4 and Table I. Figure 4 shows the behavior of the center frequency ω_0 of the three VBSFs. This result is obtained by a single experiment. We see from this result that all of the three VBSFs successfully converge to the center frequency of the narrowband noise and thus our proposed method achieves the adaptive detection and suppression of the narrowband noise. However we also see that higher-order VBSFs result in slower convergence speed. Hence the improvement of this problem will be left for a future work.

Table I shows the steady-state SNR at the outputs of the three VBSFs and this result clearly tells us the significance of our proposed method: if we choose a higher-order VBSF, we can obtain higher output SNR. This is because higher-order filters are superior to lower-order ones with respect to the magnitude characteristic, as stated in the previous section.

V. Conclusion

This paper has presented a new method of adaptive filtering for detection of unknown narrowband noises. Our proposed method makes use of a VBSF as the adaptive filter and the VBSF is designed by the well-known Constantinides frequency transformation. Although the Constantinides frequency transformation is also used in the conventional all-pass-based (second-order) adaptive notch filters, our proposed method can be easily applied to adaptation of arbitrary transfer functions. Therefore, our proposed method includes the all-pass-based adaptive notch filters as a special case and offers a unified framework for detection of narrowband signals using adaptive band-stop filters. Furthermore, it is demonstrated that, by choosing a high-order VBSF we can obtain higher SNR at the filter output than choosing low-order VBSFs. These results prove that the variable digital filters given by the Constantinides frequency transformation play significant roles in the adaptive signal processing as well as the filter design.

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