Statistical Delay QoS Driven Power and Rate Allocation for Cognitive Multi-Relay DF Networks

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***Abstract*—In this paper, we propose a statistical delay Quality-of-Service (QoS) driven power and rate allocation scheme for cognitive multi-relay Decode-and-Forward (DF) networks, which aims at maximizing the relay network throughput under the given delay QoS constraint. Specifically, by integrating the information theory and the concept of effective capacity, our derived optimal power and rate allocation scheme can maximize the effective capacity of the cognitive relay network subject to the given delay QoS constraint, which can be characterized by the QoS exponent θ, and a serize of power constraints, such as the average total transmit power constraint and the average and peak interference power constraints imposed by the Primary User (PU). Simulation results show that the relay network throughput decreases when the delay QoS constraint becomes stringent and the relay network can achieve better throughput performance when the number of relays increases. Moreover, we also find that while utilizing more relays, the average interference power perceived by the PU will reduce and the primary network can achieve better performance.**

I. Introduction

Cognitive radio (CR) is a promising yet challenging technology to solve wireless-spectrum underutilization problem caused by the traditional static spectrum allocation strategy [1] [2]. Dynamic spectrum sharing (underlay) and spectrum access (overlay) are two available methods for the secondary users (SUs) to dynamically utilize the spectrum which belongs to the primary users (PUs). Because the former method allows the SU to use the spectrum occupied by the PUs subject to a interference constraint, which can increase the spectrum utilization more obviously than the latter one, it has attracted a great deal of research attention. On the other hand, relay communication, which was initially investigated by Cover [3], has emerged as a powerful approach to improve the reliability, coverage, spatial diversity, and capacity of the wireless system and has been vastly researched during the past several years. Because of the advantages of the CR technology and relay communication, cognitive relay communication, which applies both the CR and relay techniques simultaneously, becomes an emerging research focus in recent years.

In the above systems, power allocation and capacity analysis are two important problems and a large number of works have been investigated. In the dynamic spectrum sharing networks, [4] and [5] are two fundamental researches, in which the optimal power allocation policies aimed at maximizing the ergodic capacity of the secondary link over additive white Gaussian noise (AWGN) and fading channels, respectively, are developed. In [6] and [7], the authors analyzed the ergodic, outage, and delay-constrained capacities of the secondary link under the block Rayleigh fading channel and obtained the corresponding power allocation schemes. In [8], the ergodic sum capacities of fading cognitive multiple-access and broadcast channels are investigated and the corresponding TDMA structures are derived. In the relay systems, the capacity of relay channels were initially studied in [3]. In [11], the effects of two different relay techniques, which are amplify-and-forward (AF) and decode-and-forward (DF), respectively, on the capacity of relay channel are researched. The capacity analysis, power allocation, and relay selection are also studied in [12] and [13]. Moreover, the automatic repeat request (ARQ) mechanism is also investigated in the relay system, e.g., [14] and [15]. In recent two years, the researches on the power and rate allocation for the cognitive relay networks have been arisen, e.g., [16]- [19]. In particular, the authors in [16] derived the optimal power and rate allocation scheme, which can maximize the end-to-end throughput of the three-node cognitive radio networks, and in [17], the joint relay selection and power allocation policy are studied to maximize system throughput with limited interference constraint.

In the wireless communication systems, providing delay Quality-of-Service (QoS) guarantees for different applications is a very challenging task due to the time-variant channel quality. Because the well-known Shannon theory cannot place any restrictions on delay and the outage as well as delay-constrained capacities can only deal with coding delay, which can be viewed as a constant time during the transmission, it is necessary to use the theory, which can take the QoS guarantee into consideration, to guide our resource allocation. Fortunately, effective capacity, which is first proposed by Wu [21], is an efficient tool to provide the statistical delay QoS guarantee for the wireless communication system. Tang and Zhang first derived the optimal power and rate allocation scheme for the fading wireless link based on the effective capacity [22], and then investigated the resource allocation for the AF and DF relay networks [23]. Based on these works, the authors in [24] and [25] analyzed the effective capacity of the interference-constrained cognitive wireless link and cognitive relay link under DF protocol, respectively. In the cognitive system, the transmission of the primary link is an important factor that will affect the performance and QoS guarantee of the cognitive link, but is ignored in the previous works. In [26], we analyzed the effective capacity of the point-to-point cognitive link over Rayleigh fading channel and derived the optimal power and rate allocation policy while considering the impact from the transmission of the primary link to the cognitive link.

In this paper, we consider the scenario that one cognitive wireless relay network coexists with one primary network by sharing particular portion of the spectrum. The cognitive relay network contains one transmitter, multiple relays, and one receiver. The primary network contains one pair of transmitter and receiver. All relays use the Decode-and-Forward (DF) protocol for packet forwarding. Under such scenario, we propose a statistical delay Quality-of-Service (QoS) driven power and rate allocation scheme, which aims at maximizing the effective capacity of the cognitive relay network. In this work, not only the average total transmit power constraint is considered, but also the average and peak interference power constraints are imposed on the cognitive transmitter and relays. Moreover, the impact of the interference from the transmission of the primary link to the cognitive relay link is taken into consideration. Through solving the convex optimization problem, we obtain the optimal power allocation scheme, which can satisfies the given statistical delay QoS constraint. Simulation results show that the relay network throughput decreases when the delay QoS constraint becomes stringent and the relay network can achieve better throughput performance when the number of relays increases. Moreover, we also find that while utilizing more relays, the average interference power perceived by the PU will reduce and the primary network can achieve better performance.

The rest of this paper is organized as follows. Section II presents the system model. Section III introduces the concepts of the statistical delay QoS guarantees and effective capacity. Section IV develops the optimal power allocation scheme based on the effective capacity introduced in Section III. Simulation results are given in Section **??**. The paper concludes with Section VI.

II. System Model

We consider the scenario that one cognitive (secondary) relay network coexists with one primary network by sharing a particular portions of the spectrum, as shown in Fig. 1. The primary network contains one PU transmitter and one PU receiver. The cognitive relay network contains one SU transmitter, N SU relays, and one SU receiver. All SU relays use Decode-and-Forward (DF) method and the direct channel between the SU transmitter and SU receiver exists. The total spectrum bandwidth is denoted by B. The additive white Gaussian noise (AWGN) at the PU receiver, SU relays, and SU receiver are modeled as independent zero-mean Gaussian random variables with *unit* variance.

The channel power gains between the PU transmitter and PU receiver, the SU transmitter and SU receiver, the SU transmitter and the ith SU relay, the ith SU relay and SU receiver, the SU transmitter and PU receiver, the ith SU relay and PU receiver, the PU transmitter and the ith SU relay, as well as the PU transmitter and SU receiver are denoted by $g_{sd}^{\rm P}$, $g_{sd}^{\rm S}$, $g_{sr_i}^{\rm S}$, $g_{r_id}^{\rm S}$, $g_{sd}^{\rm SP}$, $g_{r_id}^{\rm SP}$, $g_{sr_i}^{\rm PS}$, and $g_{sd}^{\rm PS}$, respectively, where $i \in \{1, \cdots, N\}$. All these channel power gains follow the Nakagami-m distribution with the probability density functions (PDF)

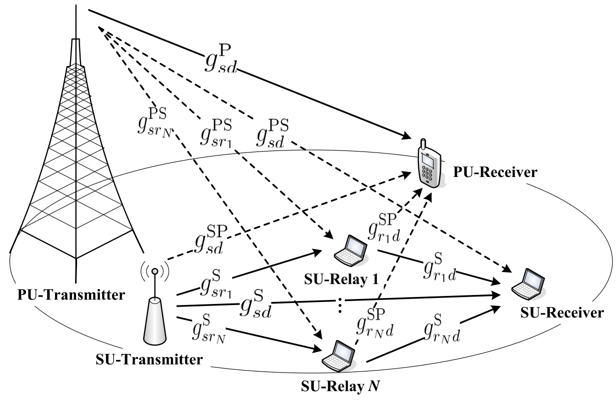

Fig. 1. The scenario that the secondary relay network coexists with the primary network.

$$f(x) = \frac{m^m x^{m-1}}{\Gamma(m)} e^{-mx}, \ x \geq 0 \qquad (1)$$

where $x \in \left\{g_{sd}^{\rm P}, g_{sd}^{\rm S}, g_{sr_i}^{\rm S}, g_{r_id}^{\rm S}, g_{sd}^{\rm SP}, g_{r_id}^{\rm SP}, g_{sr_i}^{\rm PS}, g_{sd}^{\rm PS}\right\}$ and m is the fading parameter of Nakagami-m distribution.

We assume that the upper-protocol-layer packets of the SU transmitter are divided into frames, which have the same time duration denoted by T_f, at the datalink layer, as shown in Fig. 2. The frames are stored at the transmit buffer and split into bit-streams at the physical layer. The SU transmitter employs the adaptive modulation and power control based on the statistical QoS constraint and the channel state information (CSI). We assume that the channel gains are stationary, ergodic, independent, and block fading processes, which represent that the gains are invariant within a frame, but independently vary from one frame to another. Moreover, we define the system channel gain vector as

$$\mathbf{G} \triangleq \left[g_{sd}^{\rm P}, g_{sd}^{\rm S}, g_{sd}^{\rm PS}, g_{sd}^{\rm SP}, \left\{g_{sr_i}^{\rm S}, g_{r_id}^{\rm S}, g_{sr_i}^{\rm PS}, g_{r_id}^{\rm SP}\right\}_{i=1}^{N}\right], \qquad (2)$$

which can be perfectly estimated at the corresponding receivers (SU receiver and SU relays) and reliably fed back to the SU transmitter.

III. Statistical Delay QoS Guarantees

Delay QoS guarantee plays a critically important role in the wireless communication systems. However, the deterministic delay QoS guarantee will most likely result in extremely conservative guarantee. For example, in a Rayleigh fading channel, the lower bound of the capacity that can be deterministically guaranteed is zero, which implies that the delay QoS guarantee is infinite. This conservative guarantee is clearly useless. Therefore, we need to use the statistical version to satisfy the delay constraint of the system.

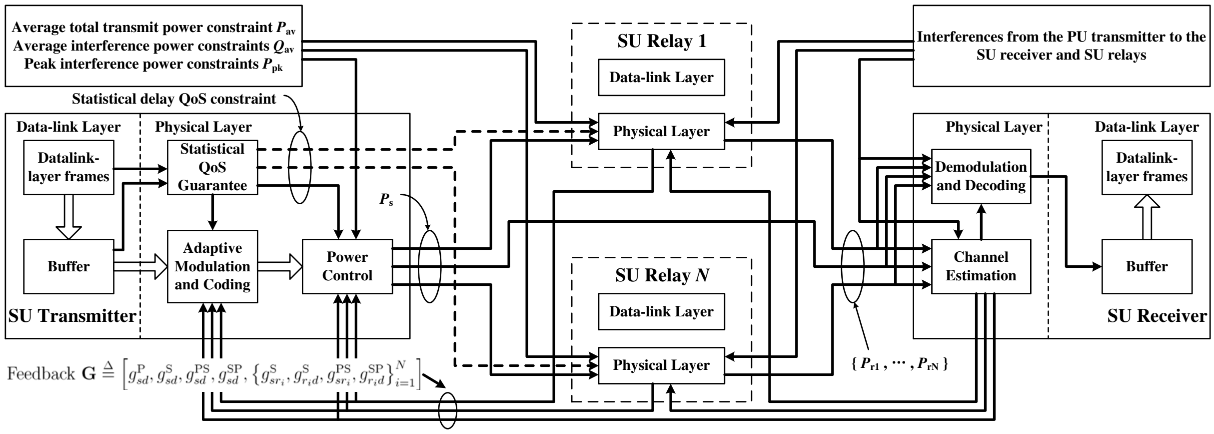

Fig. 2. The system model with statistical QoS guarantee.

The effective capacity, which is a dual concept of effective bandwidth [20] and first proposed by Wu [21], is an efficient tool to statistically guarantee the delay QoS of the system. In the effective bandwidth theory, the distribution of queue length process $Q(t)$ converges to a random variable $Q(\infty)$, which can be written as

$$-\lim_{x\to\infty}\frac{\log\left(\Pr\left\{Q\left(\infty\right)>y\right\}\right)}{y}=\theta, \tag{3}$$

where y is the queue length threshold and θ is the QoS exponent. From the above equation, we can find that the probability that the queue length exceeds the certain threshold decays as fast as the threshold increases. Therefore, the probability of the queue length exceeding the certain threshold can be calculated by

$$\Pr\{Q(t)>y\} \approx e^{-\theta y}, \quad \text{for large } y \tag{4}$$
$$\Pr\{Q(t)>y\} \approx \varepsilon e^{-\theta y}, \quad \text{for small } y \tag{5}$$

where ε is the probability of the buffer being no empty. Furthermore, if we mainly consider the delay QoS metric, the delay-bound violation probability can be obtained from Eq. (5), which can be written as

$$\Pr\{\tau>\tau_{max}\}\approx\varepsilon e^{-\theta\delta\tau_{max}}, \tag{6}$$

where τ_{max} is the delay threshold and δ is determined by both the arrival and service processes. From the analysis above, we can observe that the QoS exponent θ plays a critically important role for statistical delay QoS guarantee. The smaller θ is, the looser the QoS guarantee is, and the larger θ, the more stringent the QoS guarantee is. Specifically, when $\theta\to 0$, the system can tolerate the arbitrarily long delay. On the other hand, when $\theta\to\infty$, the system cannot tolerate any delay.

Inspired by the effective bandwidth, the effective capacity can be defined as the maximum constant arrival rate that a given service service process can support in order to guarantee a QoS requirement specified by θ. Assume that the sequence $\{R\left[i\right],\ i=1,2,\cdots\}$ is a discrete-time stationary and ergodic stochastic service process and the partial sum of the service process is $S\left[t\right]\triangleq\sum_{i=1}^{t}R[i]$. Assume that the Gartner-Ellis limit of $S[t]$, which can be expressed as

$$\Lambda_{\mathrm{C}}\left(\theta\right)=\lim_{t\to\infty}\frac{1}{t}\log\left(\mathbb{E}\left\{e^{\theta S[t]}\right\}\right), \tag{7}$$

exists and is a convex function which is differentiable for all real θ. Therefore, the effective capacity of the system, denoted by $\mathrm{E_C}(\theta)$, is

$$\mathrm{E_C}\left(\theta\right)\triangleq-\frac{\Lambda_{\mathrm{C}}\left(-\theta\right)}{\theta}=-\lim_{t\to\infty}\frac{1}{\theta t}\log\left(\mathbb{E}\left\{e^{-\theta S[t]}\right\}\right), \tag{8}$$

where $\theta>0$. If the sequence $\{R\left[i\right],\ i=1,2,\cdots\}$ is an uncorrelated process, the effective capacity can be reduced to

$$\mathrm{E_C}\left(\theta\right)=-\frac{1}{\theta}\log\left(\mathbb{E}\left\{e^{-\theta R[i]}\right\}\right). \tag{9}$$

In this paper, we aim at maximizing the effective capacity of the secondary relay network when the SU transmitter and all SU relays are subject to a series of power constraints.

IV. Optimal Power Allocation

In the cognitive relay networks, there are one SU transmitter, N SU relays, and one SU receiver. Therefore, each frame is divided into $(N+1)$ slots and each slot has the same duration $T_f/(N+1)$. In the first slot of each frame, the SU source transmits the data packets to the SU receiver and all SU relays. In the $(i+1)$th $(i=1,\cdots,N)$ slot of each frame, the ith SU relay decodes the received data packets and then forwards them to the SU receiver. Suppose the adaptive modulation and coding scheme can achieve the Shannon capacity, therefore, the service rates of the ith SU relay and the SU receiver, which are denoted by R_i and R_D, respectively, can be expressed as

$$R_i=\frac{T_fB}{N+1}\log_2\left(1+\frac{(N+1)P_s\left(\theta,\mathbf{G}\right)g_{sr_i}^{\mathrm{S}}}{P_pg_{sr_i}^{\mathrm{PS}}+1}\right),\ i=1,\cdots N \tag{10}$$

and

$$R_D=\frac{T_fB}{N+1}\log_2\left(1+\frac{(N+1)P_s\left(\theta,\mathbf{G}\right)g_{sd}^{\mathrm{S}}}{P_pg_{sd}^{\mathrm{PS}}+1}+\sum_{i=1}^{N}\frac{(N+1)P_{r_i}\left(\theta,\mathbf{G}\right)g_{r_id}^{\mathrm{S}}}{P_pg_{sd}^{\mathrm{PS}}+1}\right), \tag{11}$$

respectively, where P_p is the transmit power of PU transmitter and $P_s(\theta, \mathbf{G})$ and $P_{r_i}(\theta, \mathbf{G})$ are the average transmit power assigned to the source and the ith relay. Since the source and each relay send data packets only for one $(N+1)$th of the frame duration, the source transmits during the first slot with the power $(N+1)P_s(\theta, \mathbf{G})$ and the ith relay uses power $(N+1)P_{r_i}(\theta, \mathbf{G})$ during the $(i+1)$th slot. Obviously, the average transmit power $P_s(\theta, \mathbf{G})$ and $P_{r_i}(\theta, \mathbf{G})$ are functions of QoS exponent θ and system channel gain vector $\mathbf{G}$. Therefore, the instantaneous service rate of the cognitive relay network, denoted by $R(\theta, \mathbf{G})$, can be written as

$$R(\theta, \mathbf{G}) = \min\{R_1, \cdots, R_N, R_D\}. \tag{12}$$

Therefore, the effective capacity of the cognitive relay network can be expressed as

$$\mathrm{E_C}(\theta) = -\frac{1}{\theta}\log\left(\mathbb{E}_{\mathrm{G}}\left[\exp\left(-\theta R(\theta, \mathbf{G})\right)\right]\right). \tag{13}$$

The cognitive relay network should subject to the average total transmit power constraint, which can be expressed as

$$\mathbb{E}_{\mathbf{G}}\left[P_s(\theta, \mathbf{G}) + \sum_{i=1}^{N} P_{r_i}(\theta, \mathbf{G})\right] \le P_{\mathrm{av}}, \tag{14}$$

where P_{av} is the maximal allowed average total transmit power of the system. Moreover, the SU source and all SU relays should satisfy the average interference power constraints, which can be written as

$$\mathbb{E}_{\mathbf{G}}\left[g_{sd}^{\mathrm{SP}} P_s(\theta, \mathbf{G})\right] \le Q_{\mathrm{av}} \tag{15}$$

and

$$\mathbb{E}_{\mathbf{G}}\left[g_{r_i d}^{\mathrm{SP}} P_{r_i}(\theta, \mathbf{G})\right] \le Q_{\mathrm{av}},\ \forall\, i = 1, \cdots, N \tag{16}$$

respectively, where Q_{av} is the maximal average interference power that the PU receiver can tolerate. In order to further protect the PU network, the cognitive relay network should also subject to the peak interference power constraints, which can be expressed as

$$(N+1) g_{sd}^{\mathrm{SP}} P_s(\theta, \mathbf{G}) \le Q_{\mathrm{pk}},\ \forall\ \mathbf{G} \tag{17}$$

and

$$(N+1) g_{r_i d}^{\mathrm{SP}} P_{r_i}(\theta, \mathbf{G}) \le Q_{\mathrm{pk}},\ \forall\ \mathbf{G}, \forall\, i = 1, \cdots, N \tag{18}$$

respectively, where Q_{pk} is the maximal peak interference power that the PU receiver can tolerate. Therefore, our optimization problem can be formulated as

$$\text{(P1)} \quad \max_{\{P_s, P_{r_i}, \cdots, P_{r_N}\}} -\frac{1}{\theta}\log\left(\mathbb{E}_{\mathrm{G}}\left[\exp\left(-\theta R(\theta, \mathbf{G})\right)\right]\right) \tag{19}$$
$$\text{s.t. } (14),\ (15),\ (16),\ (17),\ (18)$$

Because the function $\log(x)$ is a monotonically increasing function of x, the solution of the maximization problem (P1) is the same with the following minimization problem:

$$\text{(P2)} \quad \min_{\{P_s, P_{r_i}, \cdots, P_{r_N}\}} \mathbb{E}_{\mathrm{G}}\left[\max\{F_0, F_1, \cdots, F_N\}\right] \tag{20}$$
$$\text{s.t. } (14),\ (15),\ (16),\ (17),\ (18)$$

where

$$F_0 = \left[1 + (N+1)\gamma_1 P_s(\theta, \mathbf{G}) + \sum_{i=1}^{N}(N+1)\gamma_3^i P_{r_i}(\theta, \mathbf{G})\right]^{-\frac{\beta}{N+1}} \tag{21}$$

$$F_i = \left[1 + (N+1)\gamma_2^i P_s(\theta, \mathbf{G})\right]^{-\frac{\beta}{N+1}},\ \forall\, i = 1, \cdots, N \tag{22}$$

$$\gamma_1 = \frac{g_{sd}^{\mathrm{S}}}{P_p g_{sd}^{\mathrm{PS}} + 1}, \tag{23}$$

$$\gamma_2^i = \frac{g_{sr_i}^{\mathrm{S}}}{P_p g_{sr_i}^{\mathrm{PS}} + 1},\ \forall\, i = 1, \cdots, N \tag{24}$$

$$\gamma_3^i = \frac{g_{r_i d}^{\mathrm{S}}}{P_p g_{sd}^{\mathrm{PS}} + 1},\ \forall\, i = 1, \cdots, N \tag{25}$$

and $\beta = \theta T_f B / \ln 2$ is the normalized QoS exponent. It is easy to prove that the minimization problem (P2) is the strictly convex optimization problem, thus has the unique optimal solution. We can solve the optimal solution to (P2) under the following two scenarios.

A. Scenario 1: $\gamma_1 \ge \min\left\{\gamma_2^1, \cdots, \gamma_2^N\right\}$

Define

$$a = \arg\min_i \left\{\gamma_2^1, \cdots, \gamma_2^N\right\}. \tag{26}$$

Then, $\gamma_1 \ge \min\left\{\gamma_2^1, \cdots, \gamma_2^N\right\}$ is equivalent to $\gamma_1 \ge \gamma_2^a$. In this scenario, we have $F_a \ge F_0$ for arbitrary relay power allocation $\{P_{r_1}(\theta, \mathbf{G}), \cdots, P_{r_N}(\theta, \mathbf{G})\}$. Therefore, in order to save constrained power resource and reduce the interference to the PU receiver, the optimal power allocation scheme must satisfy

$$P_{r_1}^*(\theta, \mathbf{G}) = \cdots = P_{r_N}^*(\theta, \mathbf{G}) = 0, \tag{27}$$

and the optimization problem (P2) becomes

$$\text{(P3)} \quad \min_{P_s(\theta, \mathbf{G})} \mathbb{E}_{\mathbf{G}}\left\{\left[1 + (N+1)\gamma_2^a P_s(\theta, \mathbf{G})\right]^{-\frac{\beta}{N+1}}\right\} \tag{28}$$
$$\text{s.t. } \mathbb{E}_{\mathbf{G}}\left[P_s(\theta, \mathbf{G})\right] \le P_{\mathrm{av}}, \tag{29}$$
$$\mathbb{E}_{\mathbf{G}}\left[g_{sd}^{\mathrm{SP}} P_s(\theta, \mathbf{G})\right] \le Q_{\mathrm{av}}, \tag{30}$$
$$(N+1) g_{sd}^{\mathrm{SP}} P_s(\theta, \mathbf{G}) \le Q_{\mathrm{pk}},\ \forall\ \mathbf{G} \tag{31}$$

The Lagrangian of problem (P2) is

$$\begin{aligned}\mathcal{L}_1\left(P_s(\theta, \mathbf{G}), \lambda, \mu\right) = {} & \mathbb{E}_{\mathbf{G}}\left\{\left[1 + (N+1)\gamma_2^a P_s(\theta, \mathbf{G})\right]^{-\frac{\beta}{N+1}}\right\} \\ & + \lambda\left[\mathbb{E}_{\mathbf{G}}\left\{P_s(\theta, \mathbf{G})\right\} - P_{\mathrm{av}}\right] \\ & + \mu\left\{\mathbb{E}_{\mathbf{G}}\left[g_{sd}^{\mathrm{SP}} P_s(\theta, \mathbf{G})\right] - Q_{\mathrm{av}}\right\}\end{aligned} \tag{32}$$

where λ and μ are the nonnegative Lagrangian multipliers associated with corresponding power constraints. Then, the Lagrangian dual function can be written as

$$\mathcal{G}_1(\lambda, \mu) = \min_{0 \le (N+1) g_{sd}^{\mathrm{SP}} P_s(\theta, \mathbf{G}) \le Q_{\mathrm{pk}}} \mathcal{L}_1\left(P_s(\theta, \mathbf{G}), \lambda, \mu\right), \tag{33}$$

and the dual problem can be defined as

$$\min_{\lambda\geq 0,\mu\geq 0} \mathcal{G}_1\left(\lambda,\mu\right) \tag{34}$$

Therefore, problem (P3) is equivalent to the following optimization problem:

$$\text{(P4)} \min_{P_s(\theta,\mathbf{G})} \left[1+(N+1)\gamma_2^a P_s\left(\theta,\mathbf{G}\right)\right]^{-\frac{\beta}{N+1}}+\left(\lambda+\mu g_{sd}^{\mathrm{SP}}\right) \tag{35}$$

$$\text{s.t.}\quad 0\leq P_s\left(\theta,\mathbf{G}\right)\leq Q_{\mathrm{pk}}/(N+1)g_{sd}^{\mathrm{SP}},\ \forall\ \mathbf{G}$$

Through solving the optimization problem (P4), we can derive the optimal solution of problem (P3), which is given by the following theorem.

Theorem 1: When $\gamma_1 \geq \min\left\{\gamma_2^1,\cdots,\gamma_2^N\right\}$, all SU relays do not forward the data packets to the SU receiver. Only the SU source will transmit with nonzero power and the corresponding optimal power allocation is determined by Eq. (36), which is shown at the bottom of this page.

B. Scenario 2: $\gamma_1 < \min\left\{\gamma_2^1,\cdots,\gamma_2^N\right\}$

In this scenario, we first select relay a that satisfies Eq. (26). If $\gamma_1 < \min\left\{\gamma_2^1,\cdots,\gamma_2^N\right\}$, we can find appropriate power allocation$\left\{P_s\left(\theta,\mathbf{G}\right),P_{r_1}\left(\theta,\mathbf{G}\right),\cdots,P_{r_N}\left(\theta,\mathbf{G}\right)\right\}$, which satisfies

$$F_0 = F_a \tag{37}$$

Then, we can obtain the relationship between $P_s\left(\theta,\mathbf{G}\right)$ and $P_{r_i}\left(\theta,\mathbf{G}\right)$ $(i=1,\cdots,N)$, which can be expressed as

$$P_s\left(\theta,\mathbf{G}\right)=\frac{1}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}\left(\theta,\mathbf{G}\right). \tag{38}$$

Substitute Eq. (38) into problem (P2), we can find the optimal power allocation scheme$\left\{P_{r_1}^*\left(\theta,\mathbf{G}\right),\cdots,P_{r_N}^*\left(\theta,\mathbf{G}\right)\right\}$ by solving the following optimization problem:

$$\text{(P5)} \min_{P_{r_1},\cdots,P_{r_N}} \mathbb{E}_{\mathbf{G}}\left\{\left[1+(N+1)\frac{\gamma_2^a}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}\left(\theta,\mathbf{G}\right)\right]^{-\frac{\beta}{N+1}}\right\}$$

$$\text{s.t.}\quad \mathbb{E}_{\mathbf{G}}\left[\sum_{i=1}^{N}\left(1+\frac{\gamma_2^a}{\gamma_2^a-\gamma_1}\right)P_{r_i}\left(\theta,\mathbf{G}\right)\right]\leq P_{\mathrm{av}}, \tag{39}$$

$$\mathbb{E}_{\mathbf{G}}\left[\frac{g_{sd}^{\mathrm{SP}}}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}\left(\theta,\mathbf{G}\right)\right]\leq Q_{\mathrm{av}}, \tag{40}$$

$$(N+1)\frac{g_{sd}^{\mathrm{SP}}}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}\leq Q_{\mathrm{pk}},\ \forall\ \mathbf{G} \tag{41}$$

(16) and (18)

The Lagrangian and dual function of problem (P5) can be expressed as

$$\begin{aligned}&\mathcal{L}_2\left(\left\{P_{r_i}(\theta,\mathbf{G})\right\},\lambda,\mu_0,\left\{\mu_i\right\}\right)\\ =\ & \mathbb{E}_{\mathbf{G}}\left\{\left[1+\frac{(N+1)\gamma_2^a}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}(\theta,\mathbf{G})\right]^{-\frac{\beta}{N+1}}\right\}\\ &+\lambda\left\{\mathbb{E}_{\mathbf{G}}\left[\sum_{i=1}^{N}\left(1+\frac{\gamma_3^i}{\gamma_2^a-\gamma_1}\right)P_{r_i}\right]-P_{\mathrm{av}}\right\}\\ &+\mu_0\left\{\mathbb{E}_{\mathbf{G}}\left[\frac{g_{sd}^{\mathrm{SP}}}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}\right]-Q_{\mathrm{av}}\right\}\\ &+\sum_{i=1}^{N}\mu_i\left\{\mathbb{E}_{\mathbf{G}}\left[g_{r_id}^{\mathrm{SP}}P_{r_i}\right]-Q_{\mathrm{av}}\right\},\end{aligned} \tag{42}$$

and

$$\mathcal{G}_2\left(\lambda,\mu_0,\left\{\mu_i\right\}\right)=\min_{\left\{P_{r_i}\right\}}\mathcal{L}_2\left(\left\{P_{r_i}(\theta,\mathbf{G})\right\},\lambda,\mu_0,\left\{\mu_i\right\}\right), \tag{43}$$

respectively. Therefore, problem (P5) can be converted to the following problem:

$$\text{(P6)} \min_{\left\{P_{r_i}\right\}} \left[1+(N+1)\frac{\gamma_2^a}{\gamma_2^a-\gamma_1}\sum_{i=1}^{N}\gamma_3^i P_{r_i}\left(\theta,\mathbf{G}\right)\right]^{-\frac{\beta}{N+1}} +\sum_{i=1}^{N}x_i P_{r_i}\left(\theta,\mathbf{G}\right)$$

$$\text{s.t.}\quad \sum_{i=1}^{N}\gamma_3^i P_{r_i}\left(\theta,\mathbf{G}\right)\leq \tilde{Q}_{\mathrm{pk}},\ \forall\ \mathbf{G} \tag{44}$$

$$0\leq P_{r_i}\left(\theta,\mathbf{G}\right)\leq \tilde{Q}_{\mathrm{pk}}^i,\ \forall\ i,\ \mathbf{G} \tag{45}$$

where

$$x_i=\lambda\left(1+\frac{\gamma_3^i}{\gamma_2^a-\gamma_1}\right)+\mu_0\frac{g_{sd}^{\mathrm{SP}}}{\gamma_2^a-\gamma_1}+\mu_i g_{r_id}^{\mathrm{SP}},\ \forall\ i \tag{46}$$

$$\tilde{Q}_{\mathrm{pk}}=\frac{\gamma_2^a-\gamma_1}{(N+1)g_{sd}^{\mathrm{SP}}}Q_{\mathrm{pk}}, \tag{47}$$

and

$$\tilde{Q}_{\mathrm{pk}}^i=\frac{Q_{\mathrm{pk}}}{(N+1)g_{r_id}^{\mathrm{SP}}},\ \forall\ i \tag{48}$$

In order to solve above problem, we first consider optimization problem (P7), which is same with problem (P6) only without constraint (44). If the optimal solution of (P7) satisfies constraint (44), it is the optimal solution of (P6).

$$P_s^*\left(\theta,\mathbf{G}\right)=\begin{cases}\frac{Q_{\mathrm{pk}}}{(N+1)g_{sd}^{\mathrm{SP}}}, & \frac{\lambda+\mu g_{sd}^{\mathrm{SP}}}{\beta}<\gamma_2^a\left[1+\frac{\gamma_2^a}{g_{sd}^{\mathrm{SP}}}Q_{\mathrm{pk}}\right]^{-\frac{\beta+N+1}{N+1}}\\ \frac{1}{N+1}\left[\left(\frac{\lambda+\mu g_{sd}^{\mathrm{SP}}}{\beta}\right)^{-\frac{N+1}{\beta+N+1}}\left(\gamma_2^a\right)^{-\frac{\beta}{\beta+N+1}}-\left(\gamma_2^a\right)^{-1}\right], & \gamma_2^a\left[1+\frac{\gamma_2^a}{g_{sd}^{\mathrm{SP}}}Q_{\mathrm{pk}}\right]^{-\frac{\beta+N+1}{N+1}}\leq\frac{\lambda+\mu g_{sd}^{\mathrm{SP}}}{\beta}\leq\beta\gamma_2^a\\ 0, & \frac{\lambda+\mu g_{sd}^{\mathrm{SP}}}{\beta}>\beta\gamma_2^a\end{cases} \tag{36}$$

Theorem 2: Let $\{\pi(1), \cdots, \pi(N)\}$ denote a permutation of the SU relays such that

$$\frac{\gamma_3^{\pi(1)}}{x_{\pi(1)}} \geq \cdots \geq \frac{\gamma_3^{\pi(N)}}{x_{\pi(N)}}. \tag{49}$$

Then, there must exist k such that the optimal solution satisfies: $P^*_{r_{\pi(1)}} = \tilde{Q}_{\rm pk}^{\pi(i)}$, $1 \leq i \leq (k-1)$, $0 < P^*_{r_{\pi(k)}} \leq \tilde{Q}_{\rm pk}^{\pi(i)}$, and $P^*_{r_{\pi(i)}} = 0$, $(k+1) \leq i \leq N$.

If $P^*_{r_{\pi(k)}} < \tilde{Q}_{\rm pk}^{\pi(i)}$, based on the K. K. T conditions, the inequation

$$\frac{(N+1)\gamma_2^a}{\gamma_2^a - \gamma_1} \sum_{i=1}^{k-1} \gamma_3^{\pi(i)} \tilde{Q}_{\rm pk}^{\pi(i)} < \left[\frac{x_{\pi(k)}(\gamma_2^a - \gamma_1)}{\beta \gamma_3^{\pi(k)} \gamma_2^a}\right]^{-\frac{N+1}{\beta+N+1}} - 1 < \frac{(N+1)\gamma_2^a}{\gamma_2^a - \gamma_1} \sum_{i=1}^{k} \gamma_3^{\pi(i)} \tilde{Q}_{\rm pk}^{\pi(i)} \tag{50}$$

must be satisfied. On the other hand, if $P^*_{r_{\pi(k)}} = \tilde{Q}_{\rm pk}^{\pi(i)}$, we have

$$\left[\frac{x_{\pi(k+1)}(\gamma_2^a - \gamma_1)}{\beta \gamma_3^{\pi(k+1)} \gamma_2^a}\right]^{-\frac{N+1}{\beta+N+1}} - 1 \leq \frac{(N+1)\gamma_2^a}{\gamma_2^a - \gamma_1} \sum_{i=1}^{k} \gamma_3^{\pi(i)} \tilde{Q}_{\rm pk}^{\pi(i)} \leq \left[\frac{x_{\pi(k)}(\gamma_2^a - \gamma_1)}{\beta \gamma_3^{\pi(k)} \gamma_2^a}\right]^{-\frac{N+1}{\beta+N+1}} - 1 \tag{51}$$

Based on the above analysis, we can find the unique k such that the optimal solution can be derived and the optimal solution is determined by the following theorem.

Theorem 3: If Eq. (50) is satisfied, the optimal power allocation for problem (P7) is determined by Eq. (52) shown at the bottom of this page. On the other hand, if Eq. (51) is satisfied, the optimal power allocation becomes

$$P^*_{r_{\pi(i)}}(\theta, \mathbf{G}) = \begin{cases} \frac{Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(i)}d}}, & 1 \leq i \leq k \\ 0, & k+1 \leq i \leq N \end{cases} \tag{53}$$

If the optimal solution of (P7) satisfies the constraint (44), Eq. (52) or (53) is the optimal power allocation scheme for problem (P6) and the optimal power of SU source can be obtained by Eq. (38). On the other hand, if the constraint (44) cannot be satisfied, the optimal solution of (P6) must satisfy

$$\sum_{i=1}^{N} \gamma_3^i P_{r_i}(\theta, \mathbf{G}) = \tilde{Q}_{\rm pk}, \ \forall\, \mathbf{G} \tag{54}$$

Substituting the above equation into problem (P6), we have

$$\begin{aligned} \text{(P8)} \ & \min \sum_{i=1}^{N} x_i P_{r_i}(\theta, \mathbf{G}) \\ \text{s.t.} \ & \sum_{i=1}^{N} \gamma_3^i P_{r_i}(\theta, \mathbf{G}) = \tilde{Q}_{\rm pk}, \ \forall\, \mathbf{G} \\ & 0 \leq P_{r_i}(\theta, \mathbf{G}) \leq \tilde{Q}_{\rm pk}^i, \ \forall\, i, \ \mathbf{G} \end{aligned} \tag{55}$$

The optimal solution of above problem is determined by the following theorem.

Theorem 4: Let $\{\pi(1), \cdots, \pi(N)\}$ denote a permutation of the SU relays such that

$$\frac{x_{\pi(1)}}{\gamma_3^{\pi(1)}} \leq \cdots \leq \frac{x_{\pi(N)}}{\gamma_3^{\pi(N)}}. \tag{56}$$

Then, there exists k such that the optimal power allocation is

$$P^*_{r_{\pi(i)}}(\theta, \mathbf{G}) = \begin{cases} \frac{Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(i)}d}}, & 1 \leq i \leq k-1 \\ \min\left\{\frac{Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(k)}d}}, \frac{(\gamma_2^a - \gamma_1) Q_{\rm pk}}{(N+1) g^{\rm SP}_{sd}} - \sum_{i=1}^{k-1} \gamma_3^{\pi(i)} \frac{Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(i)}d}}\right\}, & i = k \\ 0, & k+1 \leq i \leq N \end{cases} \tag{57}$$

and the optimal power allocation for the SU source is

$$P_s^*(\theta, \mathbf{G}) = \frac{1}{\gamma_2^a - \gamma_1} \min\left\{\sum_{i=1}^{k} \frac{\gamma_3^{\pi(i)} Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(i)}d}}, \left(1 - \gamma_3^{\pi(k)}\right) \sum_{i=1}^{k-1} \frac{\gamma_3^{\pi(i)} Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(i)}d}} + \gamma_3^{\pi(k)} \frac{(\gamma_2^a - \gamma_1) Q_{\rm pk}}{(N+1) g^{\rm SP}_{sd}}\right\} \tag{58}$$

V. Simulation Results

In this section, we evaluate the performance of proposed statistical delay QoS driven power and rate allocation scheme over cognitive multi-relay DF networks by simulations. In our simulation, the transmit power P_p of PU network is set as 20 dB. The average total transmit power $P_{\rm av}$ of the cognitive relay network is set as 10 dB. The average interference power constraint is set as $Q_{\rm av} = 5$ dB. The peak interference power constraint is set as $Q_{\rm pk} = 10$ dB. Moreover, we set the product of the frame duration and bandwidth $T_f B / \log(2) = 1$, thus the normalized QoS exponent $\beta = \theta$.

Fig. 3 shows the normalized effective capacity of the cognitive relay network with different QoS exponents θ and

$$P^*_{r_{\pi(i)}}(\theta, \mathbf{G}) = \begin{cases} \frac{Q_{\rm pk}}{(N+1) g^{\rm SP}_{r_{\pi(i)}d}}, & 1 \leq i \leq k-1 \\ \frac{\gamma_2^a - \gamma_1}{(N+1)\gamma_2^a \gamma_3^{\pi(k)}} \left\{\left[\frac{x_{\pi(k)}(\gamma_2^a - \gamma_1)}{\beta \gamma_3^{\pi(k)} \gamma_2^a}\right]^{-\frac{N+1}{\beta+N+1}} - 1\right\} - \frac{Q_{\rm pk}}{(N+1)\gamma_3^{\pi(k)}} \sum_{i=1}^{k-1} \frac{\gamma_3^{\pi(i)}}{g^{\rm SP}_{r_{\pi(i)}d}}, & i = k \\ 0, & k+1 \leq i \leq N \end{cases} \tag{52}$$

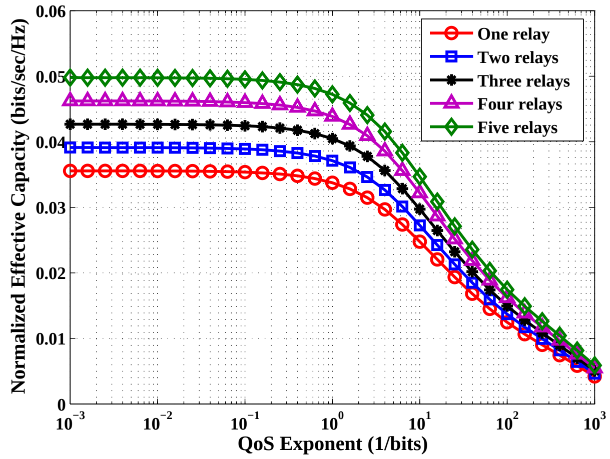

Fig. 3. The effective capacity of the cognitive relay network with different number of relays.

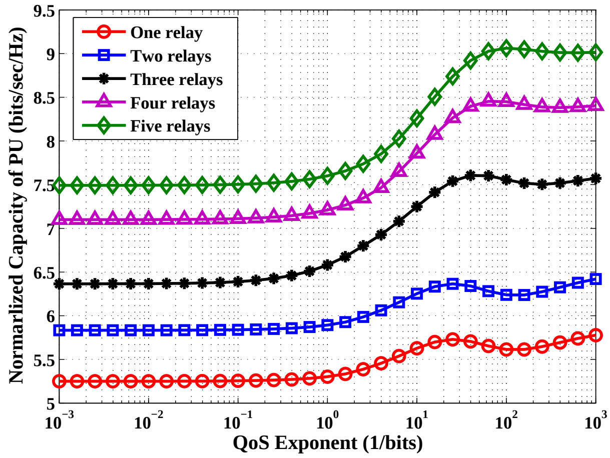

Fig. 4. The capacity of the primary network with different number of relays.

different number of relays. We can observe from Fig. 3 that, under the given average interference power constraint $Q_{\rm av}$ and peak interference power constraint $Q_{\rm pk}$, the normalized effective capacity of the cognitive relay network decreases while increasing the QoS exponent θ. Such observation can be explained as follows. First, when θ is small, the statistical delay QoS constraint is loose, which means the cognitive relay network can tolerate arbitrary long delay. In this case, the cognitive relay network is allowed to stop transmitting data packets when the channel conditions between the SU source and SU relays, the SU source and the SU receiver, as well as the SU relays and the SU source are bad and to restart their transmission when the above channel conditions are good. The effective capacity when $\theta \to 0$ is equivalent to the traditional Shannon capacity and the derived optimal power allocation is the well-know water-filling algorithm. Thus, the traffic of the cognitive relay network can be arrived with higher rate, which brings better throughput performance. However, the statistical delay QoS constraint will get stringent while increasing θ, which represents that the cognitive relay network cannot tolerate any delay at all. In this case, the cognitive relay network are not allowed to stop transmitting and has to keep the transmission rate as a constant even though the channel conditions become bad. Therefore, lots of transmit power are used to overcome the deep fading, which results that the cognitive relay network can only support lower traffic arrival rate and the throughput performance is worse than that when θ is small.

Fig. 3 also shows that the normalized effective capacity of the cognitive relay network increases while increasing the number of SU relays but keeping the maximal average total transmit power $Q_{\rm av}$ unchanged. Such observation can be explained from two aspects. First, as described in Section IV, the SU relays will not participate the data transmission when $\gamma_1 \geq \min\left\{\gamma_2^1, \cdots, \gamma_2^N\right\}$ and will forward the received data packets when $\gamma_1 < \min\left\{\gamma_2^1, \cdots, \gamma_2^N\right\}$. When the number of relays increases, the probability that $\gamma_1 < \min\left\{\gamma_2^1, \cdots, \gamma_2^N\right\}$ will become larger, which means that the SU relay will have more opportunities to forward the data packets to the SU receiver. Thus, the performance of the cognitive relay network will be improved. Second, while utilizing more relays, the diversity of the cognitive relay network can be improved and the constrained power resource can be used more efficiently, which lead to the better capacity performance of the cognitive relay network.

Fig. 4 shows the simulation result of the capacity of PU network with different QoS exponents and different number of relays. From the simulation result, we can observe that the capacity of the PU network increases as the QoS exponent becomes larger. This is because when Qos exponent gets larger, which means the delay QoS constraint becomes stringent, the cognitive relay network will be converted from the transmit power constrained system to the interference power constrained system, i.e., the total transmit power of the cognitive relay network is only bounded by the interference constraints. Therefore, the transmit power will be reduced with larger QoS exponent, which leads to the less interference power to the PU receiver, thus improving the capacity of the PU network. Fig. 4 also shows that the capacity of PU network also increases while increasing the number of SU relays. The reason of such observation is that due to the constrained total transmit power, the transmit power of each relay will be reduced as the number of relays increases, which can efficiently reduce the interference to the PU receiver. Therefore, the capacity of PU network is improved while increasing the number of relays.

VI. Conclusions

In this paper, we proposed a statistical delay Quality-of-Service (QoS) driven power and rate allocation scheme for cognitive multi-relay Decode-and-Forward (DF) networks, which aims at maximizing the relay network throughput under the given delay QoS constraint. We derived the optimal power allocation scheme subject to the average total transmit power constraint, average interference power constraint, and peak interference power constraint. Simulation results show that the relay network throughput decreases when the delay QoS constraint becomes stringent and the relay network can achieve better throughput performance when the number of relays increases. Moreover, interference power perceived by the PU will reduce and the primary network can achieve better performance while utilizing more SU relays.

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