Improved Fast DOA Estimation Based on Propagator Method

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***Abstract*—Direction of arrival (DOA) estimation by array is an important research branch in the field of modern signal processing. It has a great variety of applications in many fields, such as communications, radar and sonar, etc. This document focuses on fast subspace calculation for the data received by array, because the implementation of subspace methods in detection application with real-time operation usually experiences a bottleneck in the calculation of the signal or noise subspace. A new concept 'propagator' for array signal processing is recently introduced, the properties of which have been proven useful for DOA estimation. This attractive concept is the propagator which can easily be extracted from the cross spectral matrix of the received signals for DOA estimation without Eigen-decomposition. Based on Propagator Method (PM), 2D fast algorithms of DOA parameter estimation have designed on L-shape of array structure. This array structure decrease the computation load, meanwhile its detection accuracy becomes less than that of MUSIC or ESPRIT. In this paper we proposed two improved DOA estimation based on PM for L-shape array. One brings more accuracy without increasing computation complex, and another has faster speed of signal processing. The simulation shows effectiveness and high resolution of proposed methods**

I. INTRODUCTION

Estimation of DOA has received a significant amount of attention. It is also important branch in array signal processing, which widely used in radar, sonar and mobile communication systems, since the deployment of multiple antenna, can significantly increase the system capacity. The general techniques for the DOA estimation of array signal processing are the subspace method such as MUSIC and ESPRIT [1-3]. But these algorithms employ either eigen-value decomposition (EVD) of cross-correlation matrix or singular value decomposition (SVD) of the received data matrix. Using these techniques, the computational complexity is costly and high so that eignstructure-based algorithms suffer from limited application in real-time signal processing environment.

Marcos [4] has shown the 'propagator' method (PM) for array signal processing to estimate DOA of incident signals without eigen-value decomposition of cross-correlation matrix of the received data or the singular value decomposition of the received data matrix. Propagator is a linear operator which can easily be extracted from the received data matrix based on the partition of the steering matrix. The properties of this operator have been proven useful for source bearing estimation problems in difficult environment. Papers about PM techniques discussed DOA estimation with L-shape array [4-5].Though the computation load decreased, the DOA estimation accuracy decayed because of PM is a suboptimal which does not require any modeling of the background noise.

The purpose of this paper proposed two improved methods based on PM of L-shape array for DOA estimation. One method use aperture compensating to improve DOA estimation accuracy. Another method firstly transform the received complex data to real field, then use PM to obtain DOA estimation in order to decrease the computation load deeply The proposed algorithms have low computational complexity. The simulation results show that the proposed methods can outperform lower computational cost or more accuracy. Section 2 presents the system model; Section 3 illuminates the proposed algorithm. Section 4 is shows the simulation results.

II. DATA MODEL AND PROPAGATOR METHOD

Consider the two L-shape uniform Linear array(ULA) in *x-z* and *y-z* plane shown in Fig. 1. Each linear array consists of M elements. Suppose that there are D narrow band sources, $s(t)$, with same wavelength λ impinging on the array, such that i-th source has an elevation angle θ_i and an azimuth angle φ_i. We divide the cells on *z*-axis into two sub-array X,Y. We put the complex base-band representation of the signal received by the m-th element of one sub-array as $\boldsymbol{x}_m(k)$ and $y_m(k)$ $(m=1:M-1)$, the signal sources are far apart from the sub-array. The sub-arrays output vectors at the snapshot k is then given by

$$x(k) = [x_1(k), x_2(k), \cdots, x_{M-1}(k)]^T$$
$$y(k) = [y_1(k), y_2(k), \cdots, y_{M-1}(k)]^T \quad (1)$$

So the received data model of both of the sub-arrays is give by

$$x(k) = A_{M-1} s(k) + n(k)$$
$$y(k) = A_{M-1} \Phi_1 s(k) + n(k) \quad (2)$$

Where $n(k)$ an (M-1)-dimensional complex white noise vector with mean zero and covariance $\sigma^2 I$; I is the identity matrix ;superscript T denotes transpose of a matrix; $A_{M-1} = [a(\theta_1), \cdots, a(\theta_D)]$ is the $(M-1) \times D$ steering matrix, it is the function of the propagation parameters which are complex gains of the sensors, DOA and sensor locations. $a(\theta_i)$ is the steering vector, which is the array response of associated to the i-th source:
$a_i = [1, e^{j\tau(\theta_i)}, e^{j2\tau(\theta_i)}, \cdots, e^{j(M-2)\tau(\theta_i)}]^T$ is steering vector,
$\tau(\theta_i) = d \cos\theta_i / \lambda$;
$\Phi_1 = \mathrm{diag}\left[\tau(\theta_1), \tau(\theta_2), \cdots, \tau(\theta_D)\right]$.
We divide the steering matrix A_{M-1} into two sub-matrix as

$$A_{M-1} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \quad (3)$$

Where A_1 and A_2 are sub-matrix with dimension $D \times D$ and $(M-1-D) \times D$, respectively.. Define a new matrix

$$C = \begin{bmatrix} A_{M-1} \\ A_{M-1}\Phi_1 \end{bmatrix} \quad (4)$$

After partitioning C in the similar way, it can be written as

$$C = \begin{bmatrix} A_1 \\ C_1 \end{bmatrix} \quad (5)$$

where $C_1 = \left[A_2^T, (A_1\Phi_1)^T, (A_2\Phi_1)^T\right]^T$. Under the hypothesis that A_1 is a non-singular matrix, the propagator P is a unique linear operator which can be give by

$$P^H A_1 = C_1 \quad (5)$$

Where $(*)^H$ denotes the Hermitian transposition. Let the matrix $b(k) = \left[x(k)^T \; y(k)^T\right]^T$, then N snapshots data matrix can be written as $B = \left[b(1), \cdots, b(N)\right]$. The cross-correlation matrix (CSM) of received data can be defined as

$$\hat{R} = \frac{1}{N} BB^H = \frac{1}{N}\sum_{k=1}^{N} b(k) b^H(k) \quad (6)$$

The partitions cross-correlation matrix

$$\hat{R} = \left[\hat{R}_1 \; \hat{R}_2\right] \quad (7)$$

$\hat{R}_1$, $\hat{R}_2$ are sub-matrixes with dimension $2(M-1) \times D$, $2(M-1) \times (2(M-1) - D)$, respectively. We can get propagator by computation

$$\hat{P}_{CSM} = (R_1 R_1^H)^{-1} R_1 R_2^H \quad (8)$$

Partition $\hat{P}_{CSM}$ as following

$$\hat{P}_{CSM}^H = [P_1^T \; P_2^T \; P_3^T]^T \quad (9)$$

Where the dimension of P_1 ,P_2 ,P_3 is same to the dimension of $A_2, A_1\Phi_1, A_2\Phi_1$. According to (5) ,(4) and (9), the following equations can be obtained

$$\hat{P}_1 A_1 = A_2 \quad (10)$$
$$\hat{P}_2 A_1 = A_1 \Phi_1 \quad (11)$$
$$\hat{P}_3 A_1 = A_2 \Phi_1 \quad (12)$$

Using equations (10), (12) we can write

$$\hat{P}_3 \hat{P}_1^{\dagger} A_2 = A_2 \Phi_1 \quad (13)$$

where † donates the pseudo-inverse. This means that the estimation of the diagonal elements of matrix Φ_1 can be obtained by finding the K eigen-values of $\hat{P}_3 \hat{P}_1^{\dagger}$ .Using the EVD, we can write

$$\hat{P}_3 \hat{P}_1^{\dagger} = V_1 \Phi_1 V_1^{-1} \quad (14)$$

Then elevation angle θ_i for each source can easily found as

$$\theta_k = \cos^{-1}\left[\frac{\arg([\Phi_1]_{kk})}{2\pi d/\lambda}\right] \quad (15)$$

Note that we can use Schur Decomposition instead of EVD because the eigen vectors of V are not the parameters, which the method need. So if we use Schur Decomposition to transform upper triangular matrix and obtain the diagonal elements of Φ_1, which correspond to diagonal elements of the triangular matrix.

Then to estimate the azimuth angle φ_i, collect the received signal vectors from the elements array in the x -axis sub-array Z and sub-array W, see fig.1.

$$z(t) = [z_1(k), z_2(k), \cdots, z_{M-1}(k)]^T$$
$$w(t) = [w_2(k), w_3(k), \cdots, w_M(k)]^T \quad (16)$$

We have

$$z = A_{M-1}(\theta, \varphi) s + n$$
$$w = A_{M-1}(\theta, \varphi) \Phi_2 s + n \quad (17)$$

$A_{M-1} = [a(\theta_1, \varphi_1), \cdots, a(\theta_D, \varphi_D)]$;
$a(\theta_i, \varphi_i) = [1, e^{j\tau(\theta_i, \varphi_i))}, e^{j2\tau(\theta_i, \varphi_i)}, \cdots, e^{j(M-2)\tau(\theta_i, \varphi_i)}]^T$;
$\tau(\theta_i, \varphi_i) = d \sin\theta_i \cos\varphi_i / \lambda$;
$\Phi_2 = \mathrm{diag}\left[\tau(\theta_1, \varphi_1), \tau(\theta_2, \varphi_2), \cdots, \tau(\theta_D, \varphi_D)\right]$.
With the same PM procedure used for estimation of Φ_1 , we can estimate Φ_2 which contains information of (θ, φ) . After estimating Φ_2 , we can eventually commutate the azimuth

angle φ by applying the PM on the information from the elements in the x axis with the elevation angle estimate $\hat{\theta}$ which has already obtained above. Thus the azimuth angle DOA estimation is

$$\hat{\varphi}_k = \cos^{-1}\left[\frac{\arg[\boldsymbol{\Phi}_2]_{kk}}{2\pi d \sin\hat{\theta}_k / \lambda}\right] \tag{18}$$

We briefly discuss the performance of the method in this section. L Shape Array in Figure 1, is the combination of two mutually perpendicular ULA, So ESPRIT can be introduce to estimate θ, φ by two mutually perpendicular ULA of the L Shape Array , respectively(We call it 2D ESPRIT here). For 2D DOA estimation of L-shape array, 2D ESPRIT needs two EVDs and two Schur decompositions for elevation angle and azimuth angle estimation[2]. Compared to 2D ESPRIT, the method in this section, only needs two Schur Decompositions, so the computation load is greatly less than that of 2D ESPRIT. Because of the PM characteristic of LS process[3], the accuracy of DOA estimation is not better than that of 2D ESPRIT. Fig.2 shows the performance of Root Mean Square Error (RMSE) of DOA estimation by two methods. RMSE is defined as

$$\text{RMSE}(\theta,\varphi) = \sqrt{\frac{1}{N}\sum_{l=1}^{N}\left[\left|\hat{\theta}_l - \theta\right|^2 + \left|\hat{\varphi}_l - \hat{\varphi}\right|^2\right]} \tag{19}$$

In the simulation, half wavelength of the incoming signals is used for the spacing between the adjacent elements in each uniform linear array. We assume two single sources impinge on array with M=8 cells, with DOAs of $(\theta = 80^\circ, \varphi = 30^\circ)$ and $(\theta = 60^\circ, \varphi = 10^\circ)$. The snapshot is N =100 and 100 independent trials in total are used.

III. IMPROVED METHOD BASED ON PM

A. aperture compensating method

In the above section, we use (14) to obtain the eigen-values of $\hat{\boldsymbol{P}}_3\,\hat{\boldsymbol{P}}_1^{\dagger}$ for DOA estimation, in which we only use the data of $M-D-1$ cells of data ($\hat{\boldsymbol{P}}_3\,\hat{\boldsymbol{P}}_1^{\dagger}$ is a square matrix with $M-D-1$ dimension). So there is great aperture loss in above method. In order to improve the accuracy of DOA estimation, we modify the method as following to compensate aperture loss, let

$$\tilde{\boldsymbol{P}} = \begin{bmatrix} \boldsymbol{I}_D \\ \boldsymbol{P}_{\text{CSM}}^{\text{H}} \end{bmatrix}$$

Where $\boldsymbol{I}_D$ is a square matrix with D dimension,.From (5) and (6), we know that

$$\begin{bmatrix} \boldsymbol{I}_D \\ \boldsymbol{P}^{\text{H}} \end{bmatrix} \boldsymbol{A}_1 = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{C}_1 \end{bmatrix} = C$$

So $\tilde{\boldsymbol{P}}$ is the estimation of $\left[\boldsymbol{I}_D\ (\boldsymbol{P}^{\text{H}})^T\right]^T$, we have the conclusion

$$\tilde{\boldsymbol{P}}\boldsymbol{A}_1 = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \boldsymbol{A}_1\boldsymbol{\Phi}_1 \\ \boldsymbol{A}_2\boldsymbol{\Phi}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{M-1} \\ \boldsymbol{A}_{M-1}\boldsymbol{\Phi}_1 \end{bmatrix} \tag{20}$$

Divide $\tilde{\boldsymbol{P}}$ into two sub-matrixes $\tilde{\boldsymbol{P}}_1$ and $\tilde{\boldsymbol{P}}_2$ with same $(M-1)\times D$ dimension. According to the (20), the relations can be obtained

$$\tilde{\boldsymbol{P}}_1\boldsymbol{A}_1 = \boldsymbol{A}_{M-1} \qquad \tilde{\boldsymbol{P}}_2\boldsymbol{A}_1 = \boldsymbol{A}_{M-1}\boldsymbol{\Phi}_1 \tag{21}$$

So

$$(\tilde{\boldsymbol{P}}_2\tilde{\boldsymbol{P}}_1^{\dagger})\boldsymbol{A}_{M-1} = \boldsymbol{A}_{M-1}\boldsymbol{\Phi}_1 \tag{22}$$

If exploring the eigen-value of $\tilde{\boldsymbol{P}}_2\tilde{\boldsymbol{P}}_1^{\dagger}$, we also can get the estimation of $\boldsymbol{\Phi}_1$. Further computation of the elevation angle estimate $\hat{\theta}$ and the azimuth angle estimation of $\hat{\varphi}$ for signals are same to (15) and (18). . Since $\tilde{\boldsymbol{P}}_2\tilde{\boldsymbol{P}}_1^{\dagger}$ is a matrix with $M-1$ dimension, it can take the place of $\hat{\boldsymbol{P}}_3\,\hat{\boldsymbol{P}}_1^{\dagger}$ to DOA estimation to improve estimation accuracy of θ and φ .

B. Unitary transformation method

In section Ⅱ, we know ESPRIT can be use to estimation of DOAs of signal. So idea of the combination of ESPRIT and PM is designed to dwell with the DOA estimation. Papers [6-7] introduce Unitary-ESPRIT, which can decrease the computation load of ESPRIT. Essence of this method can be summarized as: the received complex data by array is pre-processed to data of real field, then use ESPRIT of the real data. In this way, the computation load can be decreased deeply, because real number computation is much simply than complex number computation. Reference of Unitary-ESPRIT, we can induce the ideal to PM.

Let $\boldsymbol{X}_z$ denote the data received by sub-array of z-axis(Note there are M cells include the cell on original point). We can get $\boldsymbol{Y}_z$ after unitary transformation

$$\begin{aligned} \boldsymbol{Y}_z &= \boldsymbol{U}_M^{\text{H}}\boldsymbol{X}_z \\ &= \boldsymbol{U}_M^{\text{H}}\boldsymbol{A}\,\boldsymbol{S} + \boldsymbol{U}_M^{\text{H}}\boldsymbol{N} \\ &= \boldsymbol{A}_R\boldsymbol{S} + \boldsymbol{U}_M^{\text{H}}\boldsymbol{N} \end{aligned} \tag{23}$$

where $\boldsymbol{U}_M$ is unitary matrix; if M is even , then

$$\boldsymbol{U}_M = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_{M/2} & j\boldsymbol{I}_{M/2} \\ \boldsymbol{J}_{M/2} & -j\boldsymbol{J}_{M/2} \end{bmatrix} \tag{24}$$

if M is odd , then

$$\boldsymbol{U}_M = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{I}_{(M-1)/2} & \boldsymbol{0} & j\boldsymbol{I}_{(M-1)/2} \\ \boldsymbol{0}^{\text{T}} & \sqrt{2} & \boldsymbol{0}^{\text{T}} \\ \boldsymbol{J}_{(M-1)/2} & \boldsymbol{0} & -j\boldsymbol{J}_{(M-1)/2} \end{bmatrix} \tag{25}$$

Where $\boldsymbol{J}$ commuting matrices. Use $\boldsymbol{U}_M$ and $\boldsymbol{U}_{M-1}$ .We can design the transformation matrixes $\boldsymbol{K}_1(\boldsymbol{U}_M, \boldsymbol{U}_{M-1})$ and $\boldsymbol{K}_2(\boldsymbol{U}_M, \boldsymbol{U}_{M-1})$,which are the functions $\boldsymbol{U}_M$ and $\boldsymbol{U}_{M-1}$ and can be found in [6-7]. $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ multiply $\boldsymbol{Y}_z$, respectively. We can get two real matrixes

$$\begin{aligned} \boldsymbol{K}_1\boldsymbol{Y}_z &= \boldsymbol{K}_1\boldsymbol{A}_R\boldsymbol{S} + \boldsymbol{K}_1\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N} \\ \boldsymbol{K}_2\boldsymbol{Y}_z &= \boldsymbol{K}_2\boldsymbol{A}_R\boldsymbol{S} + \boldsymbol{K}_2\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N} \end{aligned} \tag{26}$$

Where $\boldsymbol{U}_M^{\mathrm{H}}\boldsymbol{A}(\theta) = \boldsymbol{A}_R(\theta)$ .Then we have following relation

$$\boldsymbol{K}_2\boldsymbol{A}_R = \boldsymbol{K}_1\boldsymbol{A}_R\boldsymbol{\Omega}$$

where $\boldsymbol{\Omega} = \mathrm{diag}\left[\tan(\frac{\theta_1}{2}), \tan(\frac{\theta_2}{2}), \cdots, \tan(\frac{\theta_D}{2})\right]$. So

$$\begin{aligned} \boldsymbol{K}_1\boldsymbol{Y}_z &= \boldsymbol{K}_1\boldsymbol{A}_R\boldsymbol{S} + \boldsymbol{K}_1\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N} \\ \boldsymbol{K}_2\boldsymbol{Y}_z &= \boldsymbol{K}_1\boldsymbol{A}_R\boldsymbol{\Omega}\boldsymbol{S} + \boldsymbol{K}_2\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N} \end{aligned} \tag{27}$$

From (27) , we can make the conclusion that (27) and (2) have same matrix structure. Then we can use PM to estimate the diagonal value of $\boldsymbol{\Omega}$ to obtain θ . Steps of algorithm are following

1) Combine $\boldsymbol{K}_1\boldsymbol{Y}_z$ and $\boldsymbol{K}_2\boldsymbol{Y}_z$ as a new matrix

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{K}_1\boldsymbol{Y}_z \\ \boldsymbol{K}_2\boldsymbol{Y}_z \end{bmatrix} \tag{28}$$

2) Choose the real part of CSM of $\boldsymbol{B}$

$$\boldsymbol{R}_r = \mathrm{Re}(\boldsymbol{R}) = \mathrm{Re}(E[\boldsymbol{B}\boldsymbol{B}^{\mathrm{H}}]) \tag{29}$$

3) Use PM method on $\boldsymbol{R}_r$ as (7) and (8) to get the propagator $\hat{\boldsymbol{P}}_{r\,\mathrm{CSM}}$ of real matrix $\boldsymbol{R}_r$

4) Partition $\hat{\boldsymbol{P}}_{r\,\mathrm{CSM}}$ as $\hat{\boldsymbol{P}}_{r\,\mathrm{CSM}}^{\mathrm{H}} = [\boldsymbol{P}_{r1}^{\mathrm{T}}\ \boldsymbol{P}_{r2}^{\mathrm{T}}\ \boldsymbol{P}_{r3}^{\mathrm{T}}]^{\mathrm{T}}$, then we have the equation

$$\hat{\boldsymbol{P}}_{r3}\hat{\boldsymbol{P}}_{r1}^{\dagger} = \boldsymbol{V}_1\boldsymbol{\Omega}\boldsymbol{V}_1^{-1}$$

5) Compute the diagonal values of $\boldsymbol{\Omega}$ to obtain the elevation angle θ Use the data received by the cells on x-axis to estimate the azimuth angle estimation φ. The processes is same.

C. Simulation and performance analysis

In the simulation, half wavelength of the incoming signals is used for the spacing between the adjacent elements in each uniform linear array. We assume two single sources impinge on array with M=8 cells, with DOA of $(\theta = 80^\circ, \varphi = 30^\circ)$ and $(\theta = 60^\circ, \varphi = 10^\circ)$. The snapshot is N=100 and 100 independent trials in total are used. Figure 3 show the RMSE of DOA estimation.

From the figure 3, we can see that aperture compensating method has improved DOAs estimation accuracy without computation load increasing. Unitary transformation method has the least performance because the noise is no longer white noise after unitary process. See the noise part of (27),The cross-correlation matrix of noise is following

$$\begin{aligned} (\boldsymbol{K}_1\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N})(\boldsymbol{K}_1\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N})^{\mathrm{H}} &= \sigma^2\boldsymbol{K}_1\boldsymbol{K}_1^{\mathrm{H}} \\ (\boldsymbol{K}_2\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N})(\boldsymbol{K}_2\boldsymbol{U}^{\mathrm{H}}\boldsymbol{N})^{\mathrm{H}} &= \sigma^2\boldsymbol{K}_2\boldsymbol{K}_2^{\mathrm{H}} \end{aligned} \tag{30}$$

When M is a constant, $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are constants, too. $\boldsymbol{K}_1\boldsymbol{K}_1^{\mathrm{H}}$ and $\boldsymbol{K}_2\boldsymbol{K}_2^{\mathrm{H}}$ are not diagonal matrix with same diagonal element，so the noise effect on DOA estimation is extended, which lead to decay the DOA estimation. After all, when SNR>4dB, the RMSEs of 2D DOA estimation by the both methods are less than 0.5° .they both have good performance of DOA estimation. The Unitary transformation method has the least computation load.

IV. CONCLUSION

Two novel algorithms based on PM of L-shape array have been proposed for application of 2D DOA estimation. The two algorithms improve the DOA estimation performance from computation and estimation accuracy aspects, compared to original methods, respectively. Through simulations, we show the applicability and reliability of DOA estimation. So according to application requirement of certain detection system, one of them can be choose to use.

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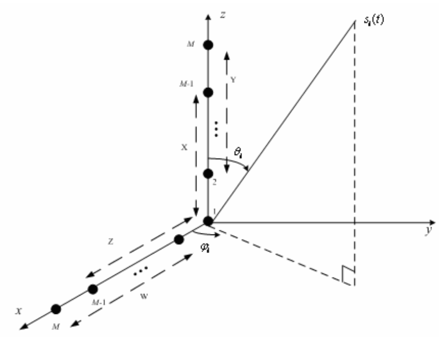

Fig. 1 The L-shape array configuration used for DOA estimation

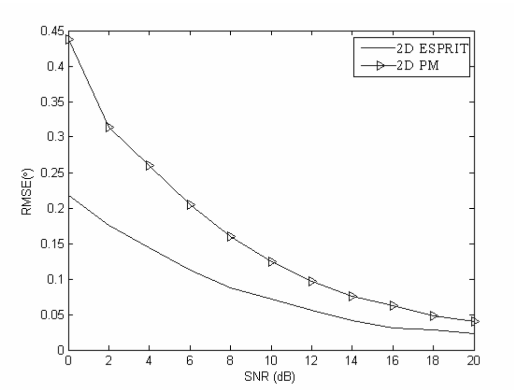

Fig. 2 RMSE of 2D DOA Estimation versus SNR

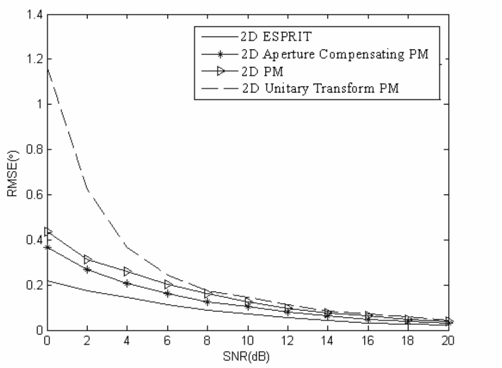

Fig. 3 RMSE of DOA Estimation versus SNR by Modified PM