A Low Complexity QR-based Selection Criterion for MIMO Precoding

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***Abstract*—This paper proposes a simplified QR-based selection criterion for the efficient optimization of multiple-input multiple-output (MIMO) precoding in a limited feedback system. The proposed selection criterion is developed which determines the precoder yielding the maximum of minimum diagonal entries of the upper triangular matrix of the QR-decomposition on the MIMO channel. Performance evaluation via computer simulations shows that the proposed QR-based precoding scheme optimizes detection performance with low-complexity compared to existing schemes.**

I. INTRODUCTION

In multiple-input multiple-output (MIMO) systems [1]-[3], spatial multiplexing (SM) [4] is capable of achieving high data transmission rates between wireless communication links. Unfortunately, the rank deficiency of a MIMO channel matrix degrades the detection performance of the SM receiver [4]. To overcome this problem, linear precoding was developed to enhance the reliability of SM transmission [4]. However, linear precoding is impractical because the receiver sends all the channel state information (CSI) to the transmitter via a limited feedback channel. Without the entire CSI, MIMO precoding was developed [2]-[3], in which the receiver only provides the index of the optimal precoder in a selected codebook via the limited feedback channel. The feedback related to this index contains fewer bits because codebooks are stored in both the transmitter and receiver. In [2], the minimum mean square error (MMSE) detector with a singular value (SV) based selection criterion was proposed to select the optimal precoder that maximizes the minimum squared singular value of the effective channel. At the receiver, this SV-based selection criterion may involve high computational complexity due to the need of singular value decomposition (SVD) and matrix inversion.

A QR-based detection scheme with reduced complexity was devised using the QR-decomposition (QRD) on the MIMO channel (i.e., $\mathbf{H} = \mathbf{QR}$) [5]-[6], to perform successive interference cancellation (SIC) detection [7]-[8] to achieve the maximum likelihood (ML) detection performance [7]-[8]. In QR detection, the minimum absolute value of the diagonal entries in the upper triangular matrix $\mathbf{R}$ degrades detection performance. Motivated to maximize the minimum absolute value of diagonal entries of $\mathbf{R}$, we here propose a low-complexity QR-based criterion for the selection of optimal precoder to optimize the effective channel. The effective channel matrix, which is the product of the channel matrix and precoder matrix of the codebook, varies with the precoder. As a result, detection performance could be improved by selecting the optimal precoder from the codebook for which the minimum absolute value of diagonal entries is large. This is because that the selected precoder will produce an effective channel matrix with near equal-diagonal QRD [5]-[6]. With this approach, the proposed QR-based selection criterion has better selection performance than the SV-based selection criterion [4]. Simulation results show that the proposed scheme is capable of achieving the optimal performance with a very low level of complexity.

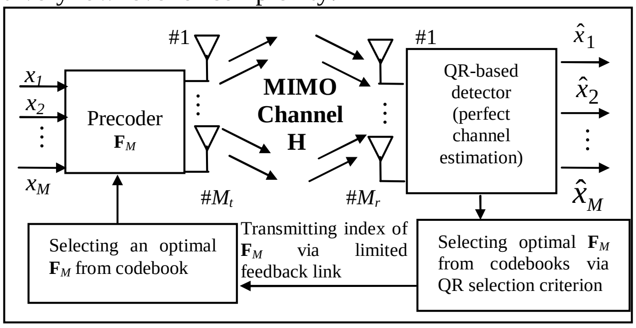


Fig. 1. A limited feedback MIMO system in which the transmitter selects the precoder via the limited feedback link from the receiver.

II. SYSTEM MODEL

We here consider MIMO precoding with M_t transmit antennas and M_r receive antennas in a limited feedback system, where W bits are mapped to M different data streams to use the spatial subchannels. An equivalent system model is given by [2]-[3]

$$\mathbf{y} = \mathbf{H}\mathbf{F}_M\mathbf{x} + \mathbf{v}, \tag{1}$$

where $\mathbf{Y} \in C^{Mr \times 1}$ is the received signal vector, $\mathbf{x} \in C^{Mt \times 1}$ is the transmitted signal vector with the correlation matrix $E_{\mathbf{x}}[\mathbf{x}\mathbf{x}^H] = (\mathcal{E}_{\mathbf{x}}/M)\mathbf{I}_M$, $\mathbf{H} \in C^{Mr \times Mt}$ is the MIMO channel with complex Gaussian entries, $\mathbf{F}_M \in C^{Mt \times M}$ with $\mathbf{F}_M^H\mathbf{F}_M = \mathbf{I}_M$ is the quantized precoder matrix with $E_{\mathbf{x}}[(\mathbf{F}_M\mathbf{x})^H\mathbf{F}_M\mathbf{x}] \le \mathcal{E}_{\mathbf{x}}$ regardless of the modulation scheme or the value of M, $\mathbf{v} \in C^{Mr \times 1}$ has *i.i.d.* complex Gaussian entries with noise power σ_v^2 and $E(\cdot)$ is the expectation operator.

Assuming that $\mathbf{H}$ changes at each transmission, the precoder matrix $\mathbf{F}_M$ is adapted using the condition of the current channel based on feedback parameters received from an error-free feedback channel, as depicted in Fig. 1. This precoder matrix $\mathbf{F}_M$ is selected from N_M different $M_t \times M$ precoder matrices denoted as

$$\mathcal{F}_M = \{\mathbf{F}_{M,1}, \mathbf{F}_{M,2}, ..., \mathbf{F}_{M,N_M}\}, \quad (2)$$

where $M \in \mathcal{M}$ and $\mathcal{M}$ is the set of supported mode values. Then, considering the codebooks of supported modes, the receiver transmits the total feedback bits to inform the transmitter of the selection of an optimum precoder. The number of total feedback bits is given by

$$B = \lceil \log_2(N_M) \rceil, \quad (3)$$

where $\lceil a \rceil$ denotes an integer larger than or equal to a.

III. PROPOSED QR BASED MIMO PRECODING SCHEME

A scheme based on QR decomposition (QRD) is proposed to perform MIMO precoding in limited feedback MIMO systems. The proposed scheme is illustrated in A) QR detection, B) QR-based MIMO precoding and C) low complexity QR-based selection criterion as follows.

A. QR detection in precoded systems

In QR-based detection [4], the QRD of the effective channel $\mathbf{HF}_M$ can be expressed as

$$\mathbf{HF}_M = [\mathbf{Q}_1 \ \ \mathbf{Q}_2] \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} = \mathbf{QR}, \quad (4)$$

where $\mathbf{Q} = [\mathbf{Q}_1 \in C^{Mr \times Mt} \ \mathbf{Q}_2 \in C^{Mr \times (Mr - Mt)}]$ is an $M_r \times M_r$ unitary matrix (therefore, $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_{Mr}$) and $\mathbf{R}$ represents an $M_t \times M_t$ upper triangular matrix. The QR-based detection can be described as follows:

$$\tilde{\mathbf{y}} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_{M_r} \end{bmatrix} = \mathbf{Q}^H\mathbf{y} = \underbrace{\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,M_t} \\ 0 & r_{2,2} & \cdots & r_{2,M_t} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & r_{M_t,M_t} \end{bmatrix}}_{\mathbf{R}} \mathbf{x} + \tilde{\mathbf{v}} = \mathbf{Rx} + \tilde{\mathbf{v}}, \quad (5)$$

where $\mathbf{R} = \mathbf{Q}^H\mathbf{H}$, $\tilde{\mathbf{v}} = \mathbf{Q}^H\mathbf{v}$, and $r_{i,j}$, $1 \le i, j \le M_t$, is the $(i, j)^{\text{th}}$ entry of $\mathbf{R}$. The i^{th} entry of the modified received signal is detected as

$$\hat{y}_i = \tilde{y}_i - \sum_{j=i+1}^{M_t} r_{i,j}\hat{x}_j = r_{i,i}x_i + \sum_{j=i+1}^{M_t} r_{i,j}(x_j - \hat{x}_j) + \tilde{v}_i, \quad (6)$$

where $\hat{x}_j$ is the j^{th} entry of the detected transmitted signal. Assuming that there is no error in the previous symbol detection, we can obtain

$$\hat{x}_i = Decision\left(\frac{\hat{y}_i}{r_{i,i}}\right), \text{ where } \hat{y}_i = r_{i,i}x_i + \tilde{v}_i \cdot \quad (7)$$

B. QR-based MIMO precoding

Based on (7), the lower bound for the average symbol error rate (SER) [6] given a precoding matrix $\mathbf{F}_M$ under the condition of error-propagation free is given by

$$P_{L,e}(\mathbf{F}_M) = \frac{1}{M}\sum_{i=1}^{M} P_{Le,i}(\mathbf{F}_M) = \frac{1}{M}\sum_{i=1}^{M} Q(\sqrt{\rho} \,|\, r_{i,i} \,|), \quad (8)$$

where ρ is the SNR and the instantaneous SER of the i^{th} sub-channel is $P_{Le,i}(\mathbf{F}_M) = Q(\sqrt{\rho}\,|\, r_{i,i}\,|)$ [6]. The block symbol error rate is thus upper bounded as

$$P_e(\mathbf{F}_M) = P(\mathbf{x} \ne \hat{\mathbf{x}}_i) \le \sum_{i=1}^{M} P_{Le,i}(\mathbf{F}_M) = M \cdot P_{L,e}(\mathbf{F}_M) \cdot \quad (9)$$

With (8) and (9), the block symbol error rate can be bounded as follows

$$\frac{1}{M}\sum_{i=1}^{M} Q(\sqrt{\rho}\,|\, r_{i,i}\,|) \le P_e(\mathbf{F}_M) \le M \cdot Q(\sqrt{\rho}\,|\, r_{i,i}\,|) \cdot \quad (10)$$

Eq. (10) shows that the minimum absolute value of the diagonal entries of $\mathbf{R}$ in the QRD of $\mathbf{HF}_M$ dominates the error probability of QR detection for a high SNR. By taking the minima of both sides of (10), we have

$$Q(\sqrt{\rho} \min_{1\le i\le M} |\, r_{i,i}\,|) \le P_e(\mathbf{F}_M) \le M \cdot Q(\sqrt{\rho} \min_{1\le i\le M} |\, r_{i,i}\,|) \cdot \quad (11)$$

In (11), the block symbol error rate can achieve its minimum by selecting an optimal precoder operating on the effective channel when the diagonal entries of $\mathbf{R}$ are nearly equal. Thus, to minimize the probability of error in the QR detection, we propose a low-complexity QR selection criterion using the maximum of the minimum diagonal entries of upper triangular matrix of QR-decomposition on MIMO channel to select the optimal precoder from the codebook in the MIMO precoding system:

$$\mathbf{F}_M = \arg\max_{\mathbf{F}' \in \mathcal{F}_M} \min_{1\le i\le M} r_{i,i}^2(\mathbf{HF}') \cdot \quad (12)$$

Note that (12) is used to select the precoder that maximizes the minimum absolute value of diagonal entries of $\mathbf{R}$ in M^{th} mode codebook. By conducting an exhaustive search in the optimization of (12), the proposed QR-based selection criterion minimizes the probability of block error in (8). Finally, the receiver sends the index of the selected precoder in the codebook to the transmitter via the limited feedback channel.

C. low complexity QR-based selection criterion

When the codebook size is large in (2), it becomes computationally intensive to perform an exhaustive search in finding the optimum precoder. To remedy this, a simple low-complexity QR-based selection criterion is proposed.

For $M < M_t$, the projection 2-norm distance between two matrices is defined as follows [3]:

$$d_{\text{proj}}(\mathbf{D}, \mathbf{G}) = \| \mathbf{DD}^H - \mathbf{GG}^H \|_2 \,. \quad (13)$$

Given a preliminary reference precoder $\mathbf{E}_M$ for mode M, a reduced set Ω_M can be determined within a spherical region of radius α_M as follows:

$$\Omega_M = \{\mathbf{F}_{M,i} \in \mathcal{F}_M : d_{\text{proj}}(\mathbf{F}_{M,i}, \mathbf{E}_M) \le \alpha_M,\ 1 \le i \le N_M\}, \quad (14)$$

where the number of elements in Ω_M is denoted as P_M. In (14), the search for precoder can then be done over Ω_M instead of $\mathcal{F}_M$ because it is highly possible that the optimal precoder will lie in Ω_M as long as $\mathbf{E}_M$ is close enough to $\mathbf{F}_M$. The following property suggests how to choose a suitable $\mathbf{E}_M$.

Property 1: Given the SVD $\mathbf{H} = \mathbf{U\Lambda V}^H$ with singular values $\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_{Mt} \ge 0$, we have the QRD $\mathbf{HV}_M = \mathbf{U\Lambda}_M$, where $\mathbf{V}_M$ consists of the first M columns of $\mathbf{V}^H$, and $\mathbf{\Lambda}_M$ consists of the first M columns of $\mathbf{\Lambda}$.

The proof is straightforward and thus omitted for brevity. It follows that a reasonable choice is $\mathbf{E}_M$=$\mathbf{V}_M$ since it gives an effective channel matrix with M largest diagonal entries in its QRD.

IV. Computational Complexity

Finally, we investigate the computational complexity of SV-based and QR-based selection criteria in MIMO precoding. Table I shows that the proposed QR detector with the QR-based criterion has a lower degree of complexity than the MMSE detector with SV-based selection criterion. This is because the proposed QR scheme can reduce the search set Ω_M to a smaller size of P_M. depicted in (14). It is noteworthy that the computational complexity involved in the symbol detection in the proposed precoding scheme is negligible, because the same QRD for precoder selection can be readily used for symbol detection at the receiver.

TABLE I
COMPLEXITY OF VARIOUS DETECTORS

Detector	Computational complexity
QR detector with QR sel. in (12)	$4M_t^2M_r + 8M_tM_r^2 + 9M_r^3 +$ $P_M(2M^2M_r + M_rM_tM + M + MM_r)$
MMSE detector with SV sel. in (4)	$\frac{3}{2}M_rM_t^2 + \frac{1}{3}M_t^3 + \frac{1}{2}M_rM_t + \frac{1}{2}M_t^2 + \frac{1}{6}M_t +$ $N_M(4M^2M_r + 8M_r^2M + 9M^3 + MM_tM_r)$

V. Simulation Results

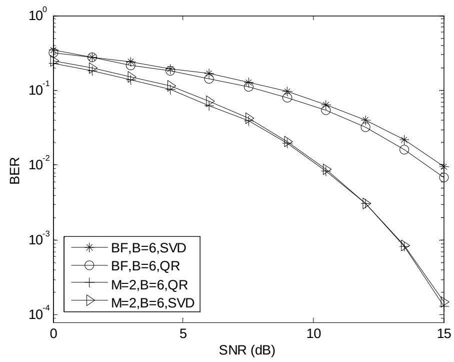
This section uses several numerical examples to illustrate the SER performance of the proposed scheme in a MIMO precoding system. The codebooks are obtained by applying the algorithm in [2]-[3]. It is assumed that the channel is perfectly known at the receiver, but not the transmitter. In the considered Ricean channel, $\mathbf{H}_{\text{sp}}$ denotes the specular component which is spatially deterministic from antenna to antenna. $\mathbf{H}_{\text{sc}}$ denotes the scattered component which varies randomly from antenna to antenna (Rayleigh-distribution). The channel response can be obtained as [9]

$$\mathbf{H} = \sqrt{\frac{\kappa}{\kappa+1}}\mathbf{H}_{\text{sp}} + \sqrt{\frac{1}{\kappa+1}}\mathbf{H}_{\text{sc}}, \quad (15)$$

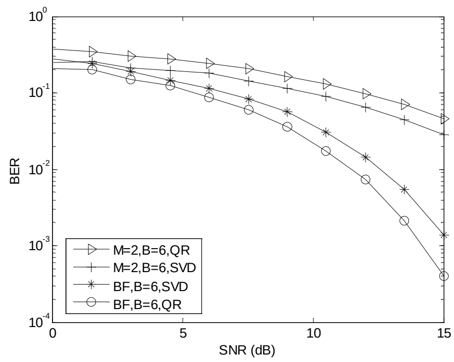
where the Ricean factor κ is defined as the ratio of deterministic-to-scattered power. With $\kappa \to \infty$, the channel is fully correlated; with $\kappa \to 0$, the channel is rich-scattered. A large κ implies strong correlations. The extreme selections $\kappa = 0$ and $\kappa = 10$, respectively, render the channel being independently fading and almost light-of-sight, respectively. For the BER comparison, we consider that SNR is equivalent to Eb/N0 in simulations. Then, we evaluate A) detection performance, B) effect of number of feedback bits and C) computational complexity for the proposed precoded system.

A. Detection performance

We use the SV-based selection criterion in [4] and the QR-based selection criterion according to (12)-(14) to exhibit the MIMO precoding performance, where QR detection is employed to detect the data symbols. We choose M = 1 (beamforming, BF), M = 2, B = 6 (i.e., $N_1 = N_2 = 64$) and W = 4 bits/symbol, with $\kappa = 0$ and $\kappa = 10$ in Fig. 2(a) and Fig. 2(b), respectively. Fig. 2 shows that the proposed QR selection criterion with reduced complexity has a superior performance to the more complicated SV selection criterion [4]. With $\kappa = 0$, Fig. 2(a) shows that M = 2 has a better detection performance than M = 1 due to the effect of receiver diversity [4]. With $\kappa = 10$, Fig. 2(b) shows the opposite trend in that M = 2 has a poorer detection performance than M = 1, due to the effect of ill-conditioned channel [4]. For M = 2 in Fig. 2(b), SV-based selection criterion has a better detection performance than QR-based selection criterion due to the involvement of the noise information in the MMSE detector.



(a)

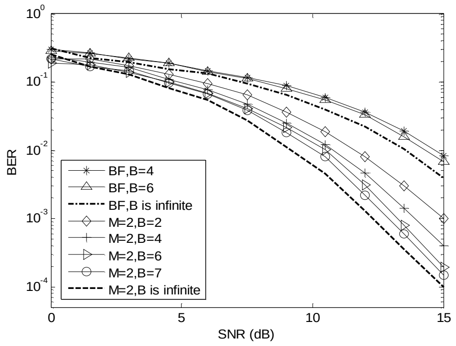


(b)

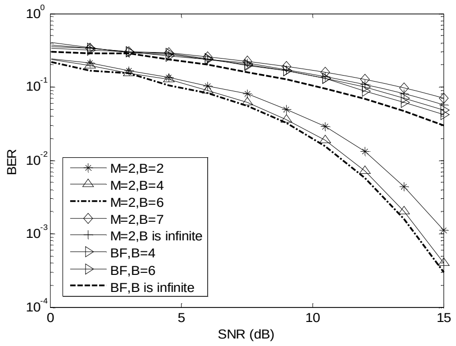
Fig. 2. BER versus SNR for various precoded systems with $M_t = M_r$ = 4, M = 1, 2, B = 6 and W = 4 bits/symbol. (a) $\kappa = 0$; (b) $\kappa = 10$.

B. Effect of number of feedback bits

To address the effect of number of feedback bits in the proposed QR detection scheme, we consider the case with M = 1, 2, and W = 4 bits/symbol, with $\kappa = 0$ and $\kappa = 10$ in Fig. 3(a) and Fig. 3(b), respectively. For each case, the number of feedback bits can be B = 2, 4, 6, 7 and ∞. When B is finite, the reduced search set size P_M in (14) is given as 1, 4, 16 and 32. As expected, the detection performance of the proposed QR selection criterion by using (12)-(14) improves with the number of feedback bits. The improvement is due to that a larger codebook is available for more selection diversity to ease the ill-condition effect.



(a)



(b)

Fig. 3. BER versus SNR for various number of feedback bits, with $M_t = M_r = 4$, M = 1, 2 and W = 4 bits/symbol. (a) $\kappa = 0$; (b) $\kappa = 10$.

C. Computational complexity

Finally, we compare the complexity of the proposed QR-based selection and SVD-based selection criteria working with the QR detector and MMSE detector, respectively. The same simulation setting as in Fig. 3 are adopted. The resulting flop counts obtained with $M_t = M_r = 4$, $M = 2$, $B = \{2, 4, 6\}$, $N_M = 4P_M = 2^B$ and $W = 4$ are shown in Table II. As observed, the proposed QR-based selection criterion can reduce the computational complexity by about 95% over the SV-based selection criterion when B = 6. This reduction in complexity confirms the results depicted previously in Table I.

TABLE II
COMPARISON OF COMPUTATIONAL COMPLEXITY

	Computational complexity (Flops)		
No. of feedback bits	$B = 2$	$B = 4$	$B = 6$
QR-based selection	1556	2192	4736
SV-based selection	5767	22663	90247

VI. CONCLUSIONS

In this paper, a QR-based selection criterion is proposed for MIMO precoding with a limited feedback channel. The proposed precoder exhibits similar performance to the conventional SVD-based precoder, but requires a much lower complexity. Furthermore, it is shown that the proposed precoder can work with a small number of feedback bits, while achieving the performance of the ideal precoder. The proposed precoder can be employed in next generation mobile communications systems, such as 3GPP LTE and IEEE 802.16m, for efficient MIMO precoder selection at the mobile station.

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