Packet Loss Rate Estimation with Active and Passive Measurements

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***Abstract*—Network tomography is a problem of estimating network properties such as the packet loss rates of links using available packets. There are two kinds of methods to measure packets: active and passive. An active measurement specifies link information (paths) of packets a priori while a passive measurement gets only the origins and destinations of packets. The conventional methods for estimating the packet loss rate of each link, one of the network tomography problems, utilize only active measurements because passive measurements have no link information. We propose a method to utilize passive measurements also. The method regards the link information in the passive measurements as latent variables and estimates the variables and the loss rates of links simultaneously in the framework of Bayesian inference. We show through numerical experiments that our method outperforms the conventional algorithm with only active measurements in the estimation accuracy.**

I. Introduction

Monitoring internal properties of a communication network such as delay, congestion and packet loss rates is crucial for maintaining it [1]–[5]. However, some properties are impossible or costly to measure directly. The problems of estimating such properties from indirect measurements are called Network tomography [6]–[10]. In this paper, we treat the problem of estimating the packet loss rates of links [11], [12].

Consider success rates instead of loss rates. A success rate of a path of a packet is expressed as the product of the success rates of links therein if the paths are known and the losses occur independently. Hence, estimating the success rates of links from the rates of paths results in a linear inverse problem. The conventional methods tackle this problem from two approaches. One is to introduce additional assumptions such as sparsity, that is, few links have low success rates. Sparse solutions are given by compressed sensing [13] or its variations [14]. The other is to increase the packets whose paths are specified a priori [11], [12], [15]. This is called active measurements [16] and requires additional costs.

In this paper, we take another approach and propose a method utilizing passive measurements instead of increasing active measurements. Passive measurements require no additional cost to send packets because they observe the origins and the destinations of ordinary packets. However, passive measurements can give the success rate between an origin and a destination using the packet capture technique [17]. In other words, passive measurements have information on the success rate of links although they do not specify which links the packets passed.

The difficulty of using passive measurements lies in their unknown paths, which are required to connect the success rates of links with the success rate between two nodes. In our method, the unknown paths are treated as missing values (latent variables in the Bayesian framework) and are estimated simultaneously using the expectation-maximization (EM) algorithm based on a probabilistic model [18].

The rest of the paper is organized as follows. In Sec. II, we mathematically formulate our problem, that is, active and passive measurements as well as unobserved paths. We derive our algorithm according to a standard way in Bayesian framework in Section III. Section IV is devoted to numerical experiments to confirm the superiority of our method. We conclude our study in Section V.

II. Problem Formulation

In this section, we mathematically formulate active and passive measurements as well as unobserved paths. Under the assumption that packet losses occur independently, active/passive measurements lead to linear equations. Furthermore, passive measurements introduce latent variables due to its unobserved paths.

In the following, we express a network as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a path as a set of links $\mathcal{W} \subset \mathcal{E}$, where $\mathcal{V}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ are the sets of nodes and links. We denote the ith path and the jth link by $\mathcal{W}_i$, $i = 1, 2, ..., \mathrm{M}$ and e_j, $j = 1, 2, ..., \mathrm{L}$, respectively.

A. Active Measurements

An active measurement sends packets along a specific path and calculates the packet loss rate based on the received packets at the destination [6], [11], [12], [16]. Let the packet loss rates of the path $\mathcal{W}_i$ and the link e_j be $\boldsymbol{Q}_i$ and $\boldsymbol{q}_j$, respectively.

Under the assumption of independent packet losses, they satisfy

$$1 - \boldsymbol{Q}_i = \prod_{e_j \in \mathcal{W}_i} (1 - \boldsymbol{q}_j). \tag{1}$$

By taking logarithm, (1) is expressed in a matrix form as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}, \tag{2}$$

where the ith elements of $\boldsymbol{x}$ and $\boldsymbol{y}$ and the i,jth element of $\boldsymbol{A}$ are respectively

$$\boldsymbol{x}_j = -\log(1-\boldsymbol{q}_j), \tag{3}$$

$$\boldsymbol{y}_i = -\log(1-\boldsymbol{Q}_i), \tag{4}$$

$$\boldsymbol{A}_{i,j} = \begin{cases} 1 & \text{if } e_j \in \mathcal{W}_i, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

B. Passive Measurements

A passive measurement counts the packets sent from an origin and that received at a destination and calculates the packet loss rate [17]. In this case, some packets go through a path and others do another path, depending on the traffic condition [19], [20]. Hence, we need to consider multiple paths.

Suppose that N_i packets are sent from an origin through the path $\mathcal{W}_i$. Then, $N_i(1-\boldsymbol{Q}_i)$ packets among them are received at the destination. Summarizing such packets for all paths, the total numbers N_S and N_R of packets sent and received satisfy

$$N_S = \sum_{i}^{|\mathcal{W}|} N_i, \tag{6}$$

$$N_R = \sum_{i}^{|\mathcal{W}|} N_i(1-\boldsymbol{Q}_i). \tag{7}$$

Equation (7) is written as

$$\alpha = \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{Q}, \tag{8}$$

where $\alpha = 1 - N_R/N_S$ is the measured packet loss rate and $\boldsymbol{\beta}_i = N_i/N_s$ is the ratio at which the ith path is chosen.

Note that Eq. (8) is linear in $\boldsymbol{Q}$ while Eq. (2) is linear in $\boldsymbol{x}$. Because they have a nonlinear relationship $\boldsymbol{Q} = \boldsymbol{1} - \exp(-\boldsymbol{A}\boldsymbol{x})$, estimation with active and passive measurements is a nonlinear problem.

C. Unobserved Paths

In our problem, some paths are observed and others are not. Hence, we divide vectors and matrices, $\boldsymbol{y}$, $\boldsymbol{A}$, $\boldsymbol{\beta}$, $\boldsymbol{Q}$, into two parts as

$$\boldsymbol{y}^{\mathrm{T}} = (\boldsymbol{y}_o^{\mathrm{T}}, \boldsymbol{y}_m^{\mathrm{T}}), \qquad \boldsymbol{A}^{\mathrm{T}} = (\boldsymbol{A}_o^{\mathrm{T}}, \boldsymbol{A}_m^{\mathrm{T}}), \tag{9}$$

$$\boldsymbol{\beta}^{\mathrm{T}} = (\boldsymbol{\beta}_o^{\mathrm{T}}, \boldsymbol{\beta}_m^{\mathrm{T}}), \qquad \boldsymbol{Q}^{\mathrm{T}} = (\boldsymbol{Q}_o^{\mathrm{T}}, \boldsymbol{Q}_m^{\mathrm{T}}), \tag{10}$$

where o means observed and m means missing. Using the above notation, Eqs. (2) and (8) are expressed as

$$\boldsymbol{y}_o = \boldsymbol{A}_o\boldsymbol{x}, \tag{11}$$

$$\boldsymbol{y}_m = \boldsymbol{A}_m\boldsymbol{x}, \tag{12}$$

$$\alpha = \boldsymbol{\beta}_o^{\mathrm{T}}\boldsymbol{Q}_o + \boldsymbol{\beta}_m^{\mathrm{T}}\boldsymbol{Q}_m. \tag{13}$$

The missing values are treated as latent variables to be estimated in our method.

III. Derivation of Proposed Method

Our method is based on the expectation-maximization (EM) algorithm [18]. The EM algorithm estimates latent variables and the parameters of a probabilistic model simultaneously by maximizing the posterior distribution iteratively. In our problem, the latent variables are $\boldsymbol{Q}_m$ and the parameters are $\boldsymbol{x}$. Hence, the EM algorithm maximizes the logarithm of the posterior distribution of $\boldsymbol{x}$ given $\boldsymbol{y}_o$ and α,

$$\log p(\boldsymbol{x} \mid \boldsymbol{y}_o, \alpha) = \log \int p(\boldsymbol{x}, \boldsymbol{Q}_m \mid \boldsymbol{y}_o, \alpha) d\boldsymbol{Q}_m, \tag{14}$$

through the two steps, the expectation-step (E-step) and the maximization-step (M-step). In the following, we derive the concrete form of the EM algorithm.

Assume the probabilistic model,

$$p(\boldsymbol{y}_o \mid \boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}_o \mid \boldsymbol{A}_o\boldsymbol{x}, \sigma_y^2\boldsymbol{I}), \tag{15}$$

$$p(\boldsymbol{x}) = \frac{1}{2\sigma_x}\exp\left(-\frac{\|\boldsymbol{x}\|_1}{\sigma_x}\right), \tag{16}$$

$$p(\alpha \mid \boldsymbol{Q}_m) = \mathcal{N}\left(\alpha \mid \boldsymbol{\beta}_o^{\mathrm{T}}\boldsymbol{Q}_o + \boldsymbol{\beta}_m^{\mathrm{T}}\boldsymbol{Q}_m, \sigma_\alpha^2\boldsymbol{I}\right), \tag{17}$$

$$p(\boldsymbol{Q}_m \mid \boldsymbol{x}) = \mathcal{N}(\boldsymbol{Q}_m \mid \tilde{\boldsymbol{Q}}_m, \sigma_Q^2\boldsymbol{I}), \tag{18}$$

$$\tilde{\boldsymbol{Q}}_m = \boldsymbol{1} - \exp(-\boldsymbol{A}_m\boldsymbol{x}), \tag{19}$$

where $\mathcal{N}(\cdot)$ denotes the density function of the Gaussian distribution. Maximum a posteriori estimation of Eqs. (15) and (16) give the objective function of the sparse estimation of the active measurement of the original network tomography [12], [14]. Then, the E-step calculates the expectation of the log likelihood function,

$$U(\boldsymbol{x}, \boldsymbol{x}^{(old)}) = \boldsymbol{E}[\log p(\boldsymbol{x}, \boldsymbol{Q}_m, \boldsymbol{y}_o, \alpha)] \tag{20}$$

with respect to the conditional distribution,

$$p(\boldsymbol{Q}_m \mid \boldsymbol{x}^{(old)}, \boldsymbol{y}_o, \alpha) = \mathcal{N}(\boldsymbol{Q}_m \mid \boldsymbol{\mu}^{(old)}, \boldsymbol{\Sigma}), \tag{21}$$

where

$$\boldsymbol{\mu}^{(old)} = \boldsymbol{\Sigma}\left(\sigma_\alpha^{-2}\boldsymbol{\beta}_m(\alpha - \boldsymbol{\beta}_o^{\mathrm{T}}\boldsymbol{Q}_o) + \sigma_Q^{-2}\tilde{\boldsymbol{Q}}_m^{(old)}\right), \tag{22}$$

$$\boldsymbol{\Sigma}^{-1} = \sigma_\alpha^{-2}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\mathrm{T}} + \sigma_Q^{-2}\boldsymbol{I}, \tag{23}$$

$$\tilde{\boldsymbol{Q}}_m^{(old)} = \boldsymbol{1} - \exp(-\boldsymbol{A}_m\boldsymbol{x}^{(old)}). \tag{24}$$

This is rewritten as

$$U(\boldsymbol{x}, \boldsymbol{x}^{(old)}) = \boldsymbol{E}[\log p(\alpha, \boldsymbol{Q}_m \mid \boldsymbol{x})] + \log p(\boldsymbol{y}_o \mid \boldsymbol{x})p(\boldsymbol{x}) \tag{25}$$

$$= -\frac{1}{2\sigma_Q^2}\|\boldsymbol{Q}_m - \tilde{\boldsymbol{Q}}_m^{(old)}\|^2 - \frac{1}{2\sigma_y^2}\|\boldsymbol{y}_o - \boldsymbol{A}_o\boldsymbol{x}\|^2 - \sigma_x^{-1}\|\boldsymbol{x}\|_1 + \text{const.} \tag{26}$$

The M-step updates the parameter $\boldsymbol{x}$ so that U is maximized, that is,

$$\boldsymbol{x}^{(new)} = \arg\max_{\boldsymbol{x}} U(\boldsymbol{x}, \boldsymbol{x}^{(old)}). \tag{27}$$

In total, our method is described as follows:

1) $\boldsymbol{y}_o$, $\boldsymbol{Q}_o$, $\boldsymbol{A}_o$, $\boldsymbol{A}_m$, $\boldsymbol{\beta}_o$, $\boldsymbol{\beta}_m$, α are given and $\tilde{\boldsymbol{Q}}_m^{(old)}$ is initialized.
2) $\boldsymbol{x}^{(new)} \leftarrow \arg\max_{\boldsymbol{x}} U(\boldsymbol{x}, \boldsymbol{x}^{(old)})$
3) $\tilde{\boldsymbol{Q}}_m^{(old)} \leftarrow \mathbf{1} - \exp\left(-\boldsymbol{A}_m \boldsymbol{x}^{(new)}\right)$
4) $\boldsymbol{\mu}^{(old)} \leftarrow \boldsymbol{\Sigma}\left(\sigma_\alpha^{-2}\boldsymbol{\beta}_m(\alpha - \boldsymbol{\beta}_o^{\mathrm{T}}\boldsymbol{Q}_o) + \sigma_Q^{-2}\tilde{\boldsymbol{Q}}_m^{(old)}\right)$
5) Compute (20) and $\boldsymbol{x}^{(old)} \leftarrow \boldsymbol{x}^{(new)}$.
6) If it does not converge, return to 2).

In the noiseless case, we can modify the above algorithm to the following:

1) $\boldsymbol{y}_o$, $\boldsymbol{Q}_o$, $\boldsymbol{A}_o$, $\boldsymbol{A}_m$, $\boldsymbol{\beta}_o$, $\boldsymbol{\beta}_m$, α are given and $\boldsymbol{y}_m^{(old)}$ is initialized.
2) $\boldsymbol{x} \leftarrow \arg\min_{\boldsymbol{x}} \|\boldsymbol{x}\|_1$ s.t. $\boldsymbol{y}_o = \boldsymbol{A}_o\boldsymbol{x}$, $\boldsymbol{y}_m^{(old)} = \boldsymbol{A}_m\boldsymbol{x}$
3) $\tilde{\boldsymbol{Q}}_m^{(old)} \leftarrow \mathbf{1} - \exp\left(-\boldsymbol{A}_m\boldsymbol{x}\right)$
4) $\boldsymbol{\mu}^{(old)} \leftarrow \boldsymbol{\Sigma}(\boldsymbol{\beta}_m(\alpha - \boldsymbol{\beta}_o^{\mathrm{T}}\boldsymbol{Q}_o) + \tilde{\boldsymbol{Q}}_m^{(old)})$
5) $\boldsymbol{y}_m^{(old)} \leftarrow -\log\left(\mathbf{1} - \boldsymbol{\mu}^{(old)}\right)$
6) Compute (20) and if it does not converge, return to 2).

This is equivalent to a kind of compressed sensing.

IV. Numerical Experiments

To confirm the superiority of our method, some numerical experiments were done as below.

A. Experiment Settings

We referred to [14] and [19] for the network structure and the experimental setting (Fig. 1). Packets flowed from the origin (S) to the destination (R) through six nodes (A, B, ..., F). The nodes had eleven links (e_j, $j = 1, \ldots, 11$), which produced six paths:

$$\begin{aligned}
\mathcal{W}_1 &= \{e_1, e_5, e_{10}\}, \\
\mathcal{W}_2 &= \{e_1, e_3, e_6, e_8, e_{10}\}, \\
\mathcal{W}_3 &= \{e_1, e_3, e_6, e_9, e_{11}\}, \\
\mathcal{W}_4 &= \{e_2, e_4, e_6, e_8, e_{10}\}, \\
\mathcal{W}_5 &= \{e_2, e_4, e_6, e_9, e_{11}\}, \\
\mathcal{W}_6 &= \{e_2, e_7, e_{11}\}.
\end{aligned}$$

We used the packet loss rates directly without generating packets. The high packet loss rates were set to 0.05 while low ones were set to 0.001. $\boldsymbol{\beta}$ was fixed to the uniform weights, $(1/6, \ldots, 1/6)^{\mathrm{T}}$. No measurement noise was added and hence the modified method was used and compared to the compressed sensing method that uses only active measurements [13]. The performance was measured by the root mean square error (RMSE) between the estimated packet loss rates and the true ones. The methods were implemented in MATLAB and CVX toolbox.

B. Results

When the number of the measured paths increased, RMSEs decreased in both methods (Fig. 2). The proposed method (blue) had lower RMSEs than the conventional method (red). When the number of the high-loss links increased under the condition that two paths are not observed by active measurements, RMSEs increased in both methods (Fig. 3).

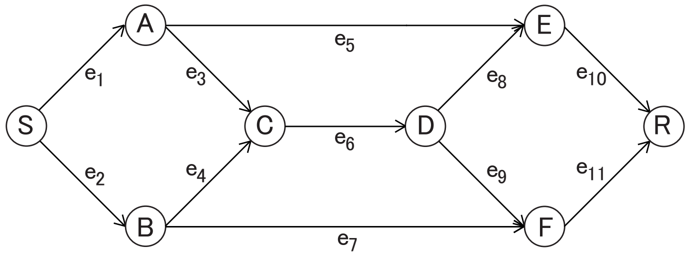

Fig. 1. Network in numerical experiments.

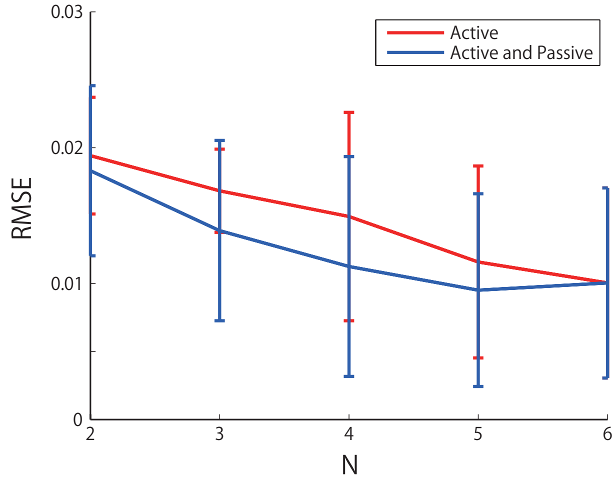

Fig. 2. RMSE by number of paths observed by active measurements.

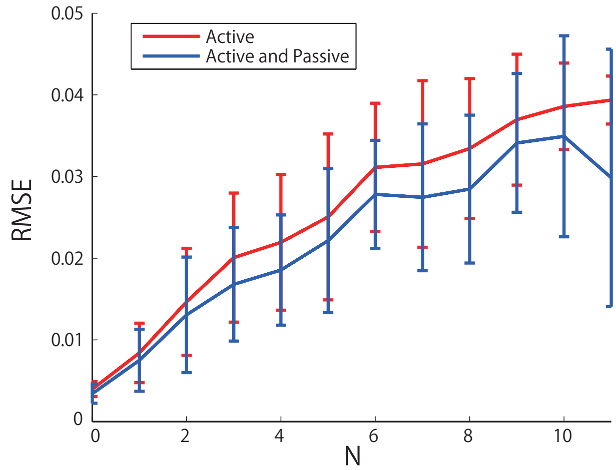

Fig. 3. RMSE by number of links which have high packet loss rates.

The proposed method (blue) had lower RMSEs than the conventional method (red).

C. Discussion

In the numerical experiments above, it is shown that our proposed method usefully estimates the packet loss rates of links in computer network. However, only a simple network is simulated and a few extensions should be considered in the future. Although this paper treated only passive measurements with a specific pair of S and R, it is possible to include multiple pairs for passive measurements. Another restriction is β. It is fixed to the uniform weight here β can be estimated as well or it can depend on other variables. Another issue is scalability. In large-scale network settings, the number of paths exponentially increases by increasing the number of links. This leads to the increase in the number of the estimated parameters of packet loss rates of paths. For this reason, the proposed method does not scale to large-scale networks directly. If we estimate the branching probability of the links, instead of the packet loss rates of the paths, the weights of paths are replaced by the weights of links.

V. Conclusions

We have formulated the packet loss rate estimation problem with the active and passive measurements and proposed an estimation method with the EM algorithm. In the future work, estimation on a large-scale network will be addressed.

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