Efficient Algorithm with Lognormal Distributions for Overloaded MIMO Wireless System

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***Abstract*—Due to outstanding search strength and well organized steps, genetic algorithm (GA) has gained high interest in the field of overloaded multiple-input/multiple-output (MIMO) wireless communications system. For overloaded MIMO system employing spatial multiplexing transmission we evaluate the performance and complexity of genetic algorithm (GA)-based detection, against the maximum-likelihood (ML) approach. We consider transmit-correlated fading channels with realistic Laplacian power azimuth spectrum. The values of the azimuth spread (AS) and Rician K-factor are set by the means of the lognormal distributions obtained from WINNER II channel models. First, we confirm that for constant complexity, GA performance is same for different combinations of GA parameters. Then, we compare the GA performance with ML in several WINNER II scenarios and channel matrix means. Finally, we compare the complexity of GA with ML. We find that GA perform similarly with ML throughout the SNR points for different scenarios and different deterministic rank. We also find that for achieving performance, GA complexity is much less than ML and thus, is an advantage in field programmable gate array (FPGA) design.**

***Index Terms*—AS, fading, FPGA, genetic algorithm, K-factor, maximum likelihood, overloaded MIMO, WINNER II.**

I. Introduction

Signal processing for wireless communication is one of the most dynamic areas of technology development. MIMO system is called overloaded when transmit antennas are more than receive antennas, i.e., $N_\mathrm{T} > N_\mathrm{R}$ [1] [3]. Spatial multiplexing (SM) techniques transmit information sequence independently over multiple transmit antennas without requiring additional bandwidth or transmission power. In some cases, in wireless communications, e.g., airborne cellular, overloaded MIMO system plays an important role. Since, the number and weight requirements also impose limitations on the antenna used, so, overloaded system makes practical sense for achieving the required performance. Not only in airborne base station but also in terrestrial base station, overloaded system may be used, and this minimum required antennas at the receiver can reduce overall network costs. Overloaded system can support more mobile users without decreasing the signal quality [2].

Signal detection for overloaded system is a complex problem [1] [3]. Due to co-channel interference, antenna at the base station faces high interference from numerous interfering mobile subscribers as well as from other nearby radio or TV signals. MIMO system can provide available diversity, but conventional linear detectors fail to exploit these available diversity. It is because of the increases traffic in a fixed bandwidth creates more interference and then corrupt the signal quality. Furthermore, ML detection can bring optimum performance by exploiting all of the available diversities but computationally complex and thus, require substantial hardware and power usage [4] [5].

In quality communications link for the desired users, the receiver must be able to reliably extract signals from the interference environment. Efficient detection algorithm deals effectively with overloaded MIMO system [1] and thus, enable the recovery of user signals with a few antennas at the receiver. Since, GAs implement evolutionary concepts and have been shown feasible in finding solutions to optimization problems in MIMO system [6] [7]. Thus, GA-based MIMO detection of spatial multiplexing transmission promises to achieve performance with much lower complexity [7] than the optimum, but high-complexity, ML detection [8].

In our previous work [9] [10], we evaluated genetic algorithm parameter requirements for detection in MIMO fading channels. We have shown in [11], by employing meta GA that, GA parameters are the function of channel parameters. We have also shown in [12] the importance of initialization (when GA has been initialized using the linear detector output, incest prevention is no longer required) for GA performance and complexity in WINNER II lognormal distributions.

However, previous work on GAs for MIMO [1] [7] detection has not considered the performance and complexities of overloaded MIMO system in WINNER II lognormal distributions and in correlated low rank Rician fading channel. Therefore, herein, we have filled this gap by numerical simulation.

The rest of the paper is organized as follows. Section II introduces the system model. Section III describes the conventional ML detection. Sections IV describes the GA-based MIMO detection. Finally, section V shows simulation results.

II. System Model

A. Received signal model

Fig. 1 shows the principle of overloaded MIMO spatial multiplexing system. We consider a frequency-flat fading channel with N_T transmit antennas and N_R receive antennas. Herein, $N_\mathrm{T} > N_\mathrm{R}$. For the numerical results shown later in this paper, each component $\mathbf{x}$ is drawn from an M–PSK modulation constellation. The base station transmits an $N_\mathrm{T} \times 1$ complex-valued signal vector $\mathbf{x}$. Therefore, the $N_\mathrm{R} \times 1$ signal vector received at the mobile station can be written as [8]

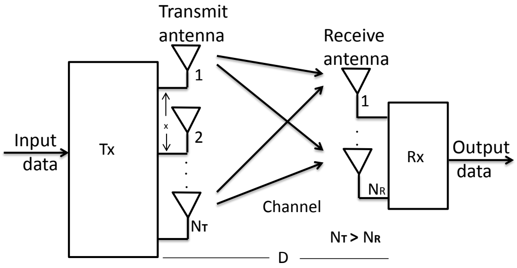

Fig. 1. Overloaded MIMO system.

$$\mathbf{y} \;=\; \sqrt{\frac{E_{\text{s}}}{N_{\text{T}}}}\mathbf{H}\mathbf{x}+\mathbf{n}, \qquad (1)$$

where $\frac{E_{\text{s}}}{N_{\text{T}}}$ is the energy transmitted per antenna, $\mathbf{H}$ is the $N_{\text{R}}\times N_{\text{T}}$ channel matrix with the channel fading gains $|[\mathbf{H}]_{i,j}|$, hereafter assumed perfectly known and $\mathbf{n}$ is the noise vector with variance N_0.

The European project, WINNER II has shown in [13] that measured $\mathbf{H}$ is a combination of a deterministic component due to specular propagation and a random component due to diffuse propagation i.e., $\mathbf{H}=\mathbf{H}_{\text{d}}+\mathbf{H}_{\text{r}}$. In terms of normalized components the channel matrix can be written as [8]

$$\mathbf{H}=\sqrt{\frac{K}{K+1}}\mathbf{H}_{\text{d,n}}+\sqrt{\frac{1}{K+1}}\mathbf{H}_{\text{r,n}}, \qquad (2)$$

where K is the power ratio of the deterministic and random components of the channel gain matrix. $\mathbf{H}_{\text{d,n}}$ and $\mathbf{H}_{\text{r,n}}$ are the further normalized versions of $\mathbf{H}_{\text{d}}$ and $\mathbf{H}_{\text{r}}$ respectively, i.e., $\mathbb{E}|[\mathbf{H}_{\text{r,n}}]_{i,j}|^2=1, \forall_{i,j}$ and $\|\mathbf{H}_{\text{d,n}}\|_F^2=N_{\text{T}}N_{\text{R}}$ and, then $\mathbb{E}\|\mathbf{H}_{\text{r,n}}\|_F^2=N_{\text{T}}N_{\text{R}}$. Therefore, $\mathbf{H}$ is properly normalized. Thus, the value,

$$\frac{\|\mathbf{H}_{\text{d}}\|_F^2}{\mathbb{E}||\mathbf{H}_{\text{r}}||_F^2}=\frac{\frac{K}{K+1}\|\mathbf{H}_{\text{d,n}}\|_F^2}{\frac{1}{K+1}\mathbb{E}\|\mathbf{H}_{\text{r,n}}\|_F^2}=K. \qquad (3)$$

Then, for $K=0$ in (2) the term $\mathbf{H}_{\text{d}}=0$ therefore, fading gains $|[\mathbf{H}]_{i,j}|$ are purely Rayleigh distributed [8]. For $K\neq 0$, the elements of $\mathbf{H}_{\text{r}}$ are complex-valued Gaussian random variables, $[\mathbf{H}]_{i,j}$ are Rice distributed and can model most measured scenario [8].

Antenna geometry and channel determine $(\mathbf{H}_{\text{d,n}})$ rank. Our simulation results in section V, shown for Rayleigh fading and for Rician fading. For Rician fading hereafter we assume r = 1, in practice r is typically low. For Rician r = 1 the transmitter and receiver separation, D, is much greater than antenna elements distance, x, (i.e., $D\gg x$) [8].

B. Statistical models for AS and K- factor

AS and K affects system performance [8]. AS is the root mean square of the power azimuth spectrum (PAS), and determines the fading correlation. For smaller AS signals are highly correlated and then performance is low, but for higher AS performance is high due to low antenna correlation [10] [13]. Using thorough measurements, the WINNER II project has modeled AS and K as lognormal distributions with scenario-dependent mean, variance, cross correlation, as shown in Table I, where $\chi,\psi\sim N(0,1)$ and have correlation ρ [13]. This table depicts the WINNER II scenario A1 (indoor or residential), B1 (urban microcell) and C2 (urban macrocell), these scenarios are illustrative for our purposes, e.g., A1 has very wide AS, B1 very narrow AS, and C2 moderate AS. Therefore, we only consider scenarios A1, B1, and C2.

TABLE I
BASE-STATION AS AND K STATISTICS [13, TABLE 4-5]

Scenario	AS [°]	K	$\rho=\mathbb{E}\{\chi\,\psi\}$
A1: indoor	$10^{1.64+0.31\chi}$	$10^{0.1(7+6\psi)}$	-0.6
B1: urban microcell	$10^{0.40+0.37\chi}$	$10^{0.1(9+6\psi)}$	-0.3
C2: urban macrocell	$10^{1.00+0.25\chi}$	$10^{0.1(7+3\psi)}$	$+0.1$

III. ML DETECTION

Maximum Likelihood is a nonlinear optimum interference cancelation algorithm [1] [7]. For given $\mathbf{H}$, ML detection approach is to search over all $M^{N_{\text{T}}}$ candidate vectors [8] for

$$\widehat{\mathbf{x}}_{\text{ML}} \;=\; \arg\min_{\mathbf{x}}\left\|\mathbf{y}-\sqrt{\frac{E_{\text{s}}}{N_{\text{T}}}}\mathbf{H}\mathbf{x}\right\|^2, \qquad (4)$$

where $\|\cdot\|_2$ denotes the norm-2 of the vector. The solution is the signal vector that minimizes the difference between the received signal vector and the linear combination of the channel matrix and the tested signal vector. Due to the exhaustive search ML detection require high complexity [7] and increases exponentially with the number of N_{T} [5]. Our numerical results have shown here for $M\leq 4$. Because, the MATLAB® implementation on our computer took a long time when $M=8$ and $N_{\text{T}}=8$. Furthermore, for $M=16$ and $N_{\text{T}}=8$ was not possible because MATLAB® exceeded the maximum variable size.

IV. GA DETECTION

GAs can quickly search a large solution set and can implement evolutionary concept to solve complex optimization problems. They balance exploitative (i.e., covering the space, by crossover and mutation) and explorative (i.e., choosing the best candidates, by selection) principles to expeditiously optimize functions over wide spaces [7]. Therefore, GAs are feasible competitors in MIMO detection [1] [7].

Fig. 2 shows the component steps for a typical GA for overloaded system. GAs maintain a population P of candidate solutions. In our work, chromosome genes are the symbols possibly sent from the transmit antenna. So, at every GA generation, the current population of candidate solutions is evolved into a new population, as discussed below.

Proper initialization is an important factor for GA convergence [7]. In our case, the population forms a $N_{\text{T}}\times P$ matrices. The columns in this matrix, i.e., the individuals, are candidate transmitted signal vectors, $\mathbf{x}$. The fitness of each individual $\mathbf{x}$ in the population is determined based on the ML criterion, from

$$d=\left\|\mathbf{y}-\sqrt{\frac{E_{\text{s}}}{N_{\text{T}}}}\mathbf{H}\mathbf{x}\right\|, \qquad (5)$$

A smaller value of d indicates a fitter candidate solution.

The parents for the next generations are selected based on their fitness value calculated by (5). Herein, we use the fitness-proportionate method for the selection [7].

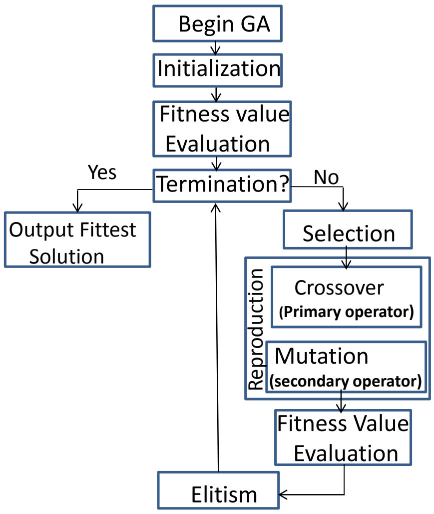

Fig. 2. Genetic algorithm diagram

Crossover and mutation are genetic-like operators employed in obtaining the offspring from the parents. Crossover is working as a primary operator to create new individuals. Herein we use uniform crossover. Uniform crossover allows for gene exchange at any position in the chromosome, i.e., at each of the N_{T} vector positions. Then, offspring individuals are formed by copying symbols from one parent or the other.

Mutation operator acts as a secondary operator. Which helps the algorithm converging to a global optimum instead of local optimum. Random mutations change the components of offspring individuals with probability p_{m}. In our case, we have allowed each symbol of each offspring individual to mutate into any other constellation symbol with mutation probability computed herein with [6]:

$$p_m = \frac{1}{P\sqrt{N_{\mathrm{T}}}}. \tag{6}$$

The strategy of elitism is to avoid discarding the best solutions by replacing the low-fitness offspring with the highest-fitness parent. As a result search space is greatly developed. Finally, in termination stage, the GAs decide whether to stop searching or continue the search for a predetermined number of generations, denoted herein by G.

Then, the GA complexity is proportional to the product of PG i.e., GA complexity increases linearly in both P and G.

V. Numerical Simulation Results

A. Settings

For all the results shown below are from simulations and assuming that $\mathbf{H}$ is perfectly known at the receiver i.e., the receive correlation matrix is set to $\mathbf{I}_{N_{\mathrm{R}}}$. The transmit correlation, is then computed for a given AS value as mentioned in [14]. The GA has been initialized randomly, 'M0' in figure titles, the crossover is uniform approach, elitism is implemented and the mutation probability is computed according to (6).

The figures depict the AER vs. the E_{s}/N_0. The AER plots indicate the performance averaged over the fading as well as over the spatially-multiplexed streams. We have used 2^{N_2} samples of the channel matrix and noise vector (where N_2 appears in the figure titles).

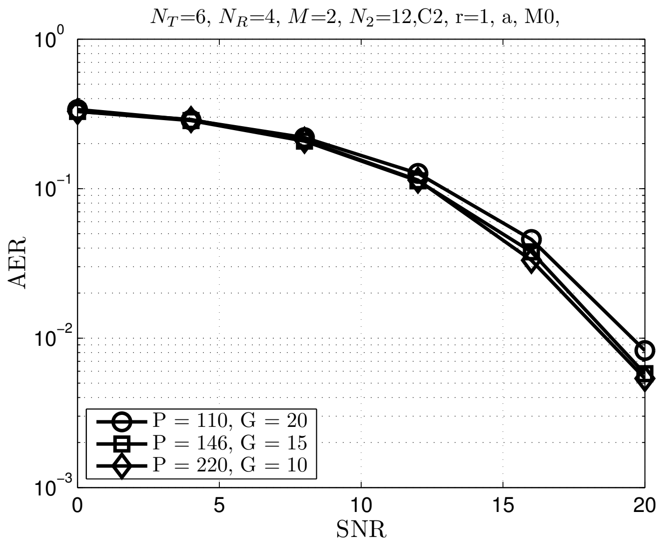

Fig. 3. GA robustness for overloaded MIMO system, scenario C2.

The transmit-side PAS is set to the realistic Laplacian type [13], AS and K set to the WINNER II averages ('a' in the figure titles). The transmit/receive antennas are, $N_{\mathrm{T}} > N_{\mathrm{R}}$ with interelement distance equal to the half of the carrier wavelength. In figure title, r indicates rank$(\mathbf{H}_{\mathrm{d}})$. For Rician fading with rank$(\mathbf{H}_{\mathrm{d,n}}) = 1$ is generated as the outer product of the receive and transmit array steering vectors i.e., $\mathbf{a}_{\mathrm{R}}$ and $\mathbf{a}_{\mathrm{T}}$, respectively. Their elements are given by $e^{-j\pi d_{\mathrm{n}} \sin\theta_{\mathrm{d,R}}(n_{\mathrm{R}}-1)}$, $n_{\mathrm{R}} = 1 : N_{\mathrm{R}}$, and $e^{-j\pi d_{\mathrm{n}} \sin\theta_{\mathrm{d,T}}(n_{\mathrm{T}}-1)}$, $n_{\mathrm{T}} = 1 : N_{\mathrm{T}}$, respectively, where d_{n} is the normalized interelement distance i.e., $d_{\mathrm{n}} = 1$, $\theta_{\mathrm{LOS,R}} = 10°$, and $\theta_{\mathrm{LOS,T}} = 5°$. Then, $\mathbf{H}_{\mathrm{d,n}} = \mathbf{a}_{\mathrm{R}}\mathbf{a}_{\mathrm{T}}^{\mathcal{H}}$. Rayleigh fading is also referred to herein r = 0, which is the only rank-zero matrix.

B. Results and Discussion

1) GA Performance for constant complexity: Fig. 3 depicts the performance of GA for different combinations of GA parameter with constant complexity. Results show similar performance for all sets of GA parameter. This result suggests that GA performance is robust for overloaded MIMO system.

2) Effect of WINNER II scenarios and fading types: Fig. 4 shows GA performance with ML in different WINNER II scenarios. For all the scenarios GA coincide with ML throughout the SNR points. Scenario A1 yield higher performance due to the diversity gain generated by the larger AS in this indoor scenario. B1 yields very poor performance due to the lower AS experienced in this typical urban microcell scenario and C2 generates intermediary performance due to its moderates AS.

Fig. 5 depicts GA performance with ML in Rayleigh and Rician fading channels for scenario C2. For both the channel condition GA performs with ML throughout the SNR points. r = 0 (Rayleigh) and r = 1 (Rician) influences achievable AER performance. As expected, Rician yields the higher AER, whereas Rayleigh yields the lower AER.

3) GA complexity compared to ML, in the scenario, A1: Fig. 6 depicts GA convergence for $N_{\mathrm{T}} = 8, N_{\mathrm{R}} = 5$, $M = 4$, r = 1, and scenarios A1. GA converges with ML after 16

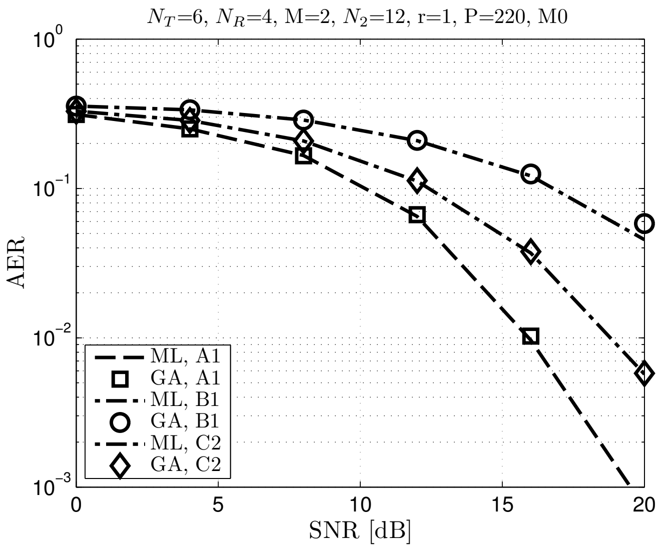

Fig. 4. GA performance for different AS and K values.

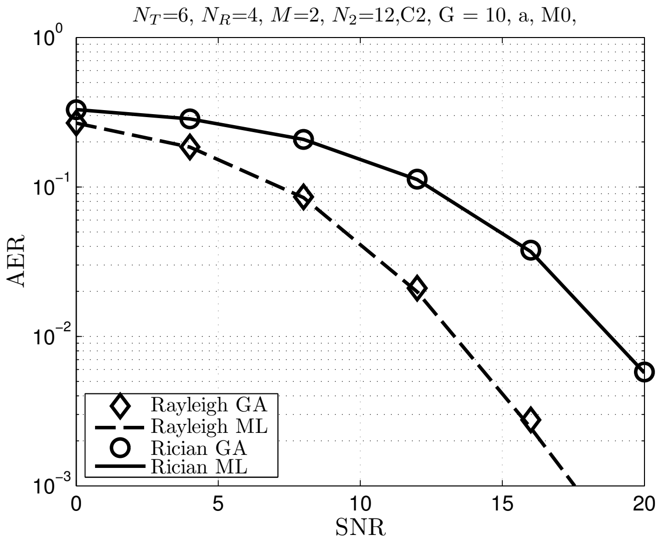

Fig. 5. GA performance in fading channels, scenario C2.

G. Lower AER can be achieved with acceptable increase of GA complexity. Since, the complexity order of GA and ML is P G and $M^{N_{\mathrm{T}}}$ respectively and they are dominated by the likelihood function, which contains of $N_{\mathrm{T}}N_{\mathrm{R}}$ complex multiplication and addition. Thus, the complexity ($PG = 23200$) of GA is much less than the complexity ($M^{N_{\mathrm{T}}} = 65536$) of ML.

VI. Conclusions

In this paper, we have shown GA-based detector performance and complexity of overloaded MIMO system in realistic channel models and correlated fading channels. We found that, at a constant complexity, GA performance is less sensitive to the parameter variable. GA perform with ML for several WINNER II scenarios and in fading channels. GA performance influenced by AS and K values and also r = 0 and r = 1. Fig. 5 shows at AER 10^{-2} Rician fading generates performance about 10dB poorer than that was expected for the Rayleigh fading. GA can achieve ML like performance with lower complexity. In fig. 6 complexity of GA relative to ML is about 35%. Thus, the GA-based detection is able to bridge

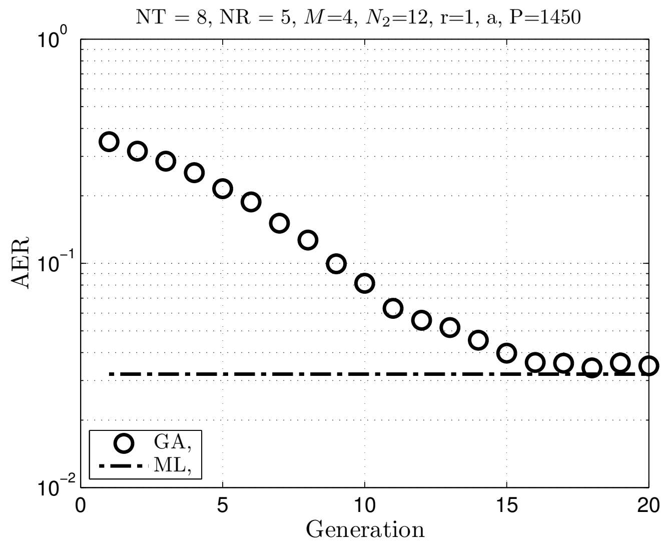

Fig. 6. GA complexity with ML for overloaded system, scenario A1.

the performance and complexity gap between the conventional ML detector. Therefore, much lower complexity is thus, much easier to implement in hardware (FPGA design).

References

[1] G. Colman and T. Willink, "Overloaded array processing using genetic algorithms with soft-biased initialization," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 4, pp. 2123 –2131, July 2008.

[2] W. H. R. J. B. J. Hicks, S. Bayram and J. H. Reed, "Overloaded array processing with spatially reduced search joint detection," *IEEE Journal on Selected areas in Communications*, vol. 19, no. 8.

[3] N. Surajudeen-Bakinde, X. Zhu, J. Gao, and A. Nandi, "Improved signal detection approach using genetic algorithm for overloaded mimo systems," in *Proc. IEEE 4th International Conference on Wireless Communications, Networking and Mobile Computing, WiCOM '08*, Oct 2008, pp. 1 – 4.

[4] A. Burg, "VLSI circuits for MIMO communication systems," Ph.D. dissertation, Swiss Federal Institute of Technology, 2006.

[5] D. S. H. Artes and F. Hlawatsch, "Effcient detection algorithms for MIMO Channels: a Geometrical Approach to Approximate ml Detection," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2808 – 2820, Nov. 2003.

[6] D. A. Coley, *An Introduction to Genetic Algorithms for Scientists and Engineers*. Singapore: World Scientific Publications, 1999.

[7] M. Jiang and L. Hanzo, "Multiuser MIMO-OFDM for next-generation wireless systems," *Proc. IEEE*, vol. 95, no. 7, pp. 1430–1469, July 2007.

[8] A. J. Paulraj, R. U. Nabar, and D. A. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, UK: Cambridge University Press, 2005.

[9] K. Obaidullah, C. Siriteanu, S. Yoshizawa, and Y. Miyanaga, "Evaluation of genetic algorithm-based detection for correlated MIMO fading channels," in *IEEE International Symposium on Communications and Information Technologies (ISCIT11), Hangzhou, China*, October 2011.

[10] K. Obaidullah, C. Siri., S. Yoshizawa, and Y. Miyanaga, "Genetic algorithm parameter requirements for detection in mimo fading channels," *RISP Journal of Signal Processing*, vol. 16, no. 3, May 2012.

[11] K. Obaidullah, C. Sir, S. Yoshizawa, and Y. Miyanaga, "Fading effects on parameter selection in genetic algorithm for MIMO detection," in *IEEE International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS11), Chiang Mai, Thailand*, Dec. 2011.

[12] K. Obaidullah and Y. Miyanaga, "Efficient algorithm with WINNER II lognormal distributions for MIMO systems," in *in IEEE International Symposium on Communications and Information Technologies (ISCIT12), Gold Coast, Australia*, 2-5 October 2012.

[13] P. Kyosti, J. Meinila, L. Hentila, and *et al.*, "WINNER II Channel Models." CEC, Tech. Rep. IST-4-027756, 2008.

[14] C. Siriteanu and S. D. Blostein, "Maximal-ratio eigen-combining for smarter antenna arrays," *IEEE Transactions on Wireless Communications*, vol. 6, no. 3, pp. 917 – 925, March 2007.