Compressed Sensing with Super-resolution in Magnetic Resonance using Quadratic Phase Modulation

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***Abstract*— In recent years, compressed sensing (CS) has attracted considerable attention in areas of rapid MR imaging. Our group and Y. Wiaux have shown independently that the use of quadratic phase modulation prior to data acquisition can greatly improve the accelerating factor of CS. The use of quadratic phase modulation has distinctive features that the extrapolation of signal by post processing calculation is feasible. In this paper, we propose a novel image reconstruction method in which extrapolation of signal is executed in the CS reconstruction algorithm, resulting in the improvement of spatial resolution. Simulation and experimental studies have revealed that the spatial resolution is fairly improved compared to the images obtained in standard CS based on Fourier transform imaging.**

I. INTRODUCTION

Recently, a new theory called compressed sensing (CS) has been applied to MR image reconstruction [1]. The CS theory states that a signal with a sparse representation can be reconstructed from much fewer measurements than previously suggested by the conventional Nyquist sampling theory [2]. In general, MR images do not show sparsity as it defined in mathematical point of view even after they are transformed to sparse domain by sparsifying transform function. Therefore, if CS reconstruction is applied to MR image reconstruction, the imperfectness of sparsity results in degradation of images, such as loss of spatial resolution, change of image contrast.

In this paper, we propose a new CS reconstruction in which spatial resolution is enhanced by extrapolating the acquired signal. We have shown that the signal obtained in phase-scrambling Fourier imaging technique (PSFT) [3,4] allows resolution improvements by restoring the signal outside the sampling signal [5]. The principle of resolution improvement is almost the same as half phase encoding method that is based on the Helmite symmetric property of nmr signal when imaging object is supposed to be a real-value function. It should be mentioned that PSFT has a remarkable feature that the application of CS to PSFT signal provide drastic enhancement in the quality of images and hence allow a large acceleration of the acquisition process[6,7]. The application of resolution enhancement has a possibility to improve the additional acceleration of acquisition process. The numerical procedure of resolution improvement in which PSFT signals are extrapolated by iterative calculation is resembled with CS reconstruction procedure using iterative thresholding algorithm. Therefore, signal extrapolation procedure can be implemented to the CS reconstruction algorithm rather easily.

The improvement of resolution with reference to signal reduction factor was investigated. Finally, the proposed method is applied to the MR signals with phase variation. It was shown that reconstructed images have a higher resolution compared to standard CS.

II. METHOD

A. Phase-Scrambling Fourier Imaging Technique

Phase-Scrambling Fourier Transform (PSFT) imaging is a technique whereby a quadratic field gradient $\Delta B = b(x^2+y^2)$ is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding [3,4]. The signal obtained in PSFT is given by Eq. (1).

$$v(k_x,k_y) = \iint \left[\rho(x,y) e^{-j\gamma b\tau(x^2+y^2)}\right] e^{-j(k_x x + k_y y)} dxdy \quad (1)$$

where $\rho(x,y)$ represents the spin density distribution in the subject, γ is the gyromagnetic ratio, and b and τ are the coefficient and impressing time, respectively, of the quadratic field gradient. Like the standard Fourier Imaging technique, spin density distribution $\rho(x,y)$ can be obtained by taking the inverse Fourier transform of the signal and multiplying the quadratic phase term $\exp[j\gamma b\tau(x^2+y^2)]$.

Since the phase of the object shifts quadratically in the x- and y-directions, the spatio-temporal frequency on the image function increases as the position from the origin of the image space increases, as follows:

$$\rho(x) e^{-j\gamma b\tau(x^2+y^2)} = \rho(x)^{-j[\omega(x)x+\omega(y)y]}, \quad (2)$$

where $\omega(x) = \gamma b\tau x$ and $\omega(y) = \gamma b\tau y$. Consider the localized small-width images as shown in Fig.1(a) A,B, and C, the spatial frequency spectra of each localized images shift in the k-space in accordance with the spatio-temporal frequency $\omega_x(x)$ and $\omega_y(y)$ that is given to the localized images, respectively as shown in Fig.1(b). Spatio-temporal frequency $\omega_x(x)=\gamma b\tau x$, $\omega_y(y)=\gamma b\tau y$ increases as the distance from the center x or y increases in the image domain. When the imaging object is supposed to be a real-value function, then the spectrum can be extrapolated so the spectrum be symmetry with its peak like the Half Fourier encoding method as shown in Fig.1 (c)-(e), which results in the improvement of spatial resolution [5]. The resolution improvement increases in proportion to the distance from the center of the quadratic phase function applied to the subject. The $\gamma b\tau$ at the maximum can be determined as $\gamma b\tau_{max=}\pi/(N\Delta x^2)$ by the relation, $\Delta x\,(\partial\omega_x(x)/\partial x)|_{x=N\Delta x/2} = \pi$.

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B. *Signal Extrapolation with Compressed Sensing*

According to the CS theory, a signal $\mathbf{s}$ with a sparse representation in the basis $\boldsymbol{\Psi}$, can be recovered from the compressed measurements $\mathbf{p}=\boldsymbol{\Phi}\,\mathbf{s}$, where $\boldsymbol{\Phi}$ is an MxN measurement matrix (M<<N), if the $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are incoherent.

$$\mathbf{p} = \boldsymbol{\Phi}\mathbf{s} = \boldsymbol{\Phi}\boldsymbol{\Psi}^{-1}\boldsymbol{\mu}, \qquad \|\boldsymbol{\mu}\|_0 = M << N \tag{3}$$

The image is reconstructed from the undersampled *k*-space data by solving the nonlinear optimization problem: minimize $\|\boldsymbol{\Psi}\mathbf{s}\|_1$ subject to $\|\boldsymbol{\Phi}\mathbf{s} - \mathbf{p}\|_2 < \varepsilon$, where ε is a small constant. Minimizing $\|\boldsymbol{\Psi}\mathbf{s}\|_1$, we use technique based on projection [8]. Algorithm of this class form by $\mathbf{s}$ successively projecting and thresholding;

$$\breve{\boldsymbol{\mu}}^{(i)} = \breve{\boldsymbol{\mu}}^{(i)} + \frac{1}{\beta}\boldsymbol{\psi}\boldsymbol{\Phi}^T\left(\mathbf{p} - \boldsymbol{\Phi}\boldsymbol{\psi}^{-1}\breve{\boldsymbol{\mu}}^{(i)}\right), \tag{4}$$

$$\breve{\boldsymbol{\mu}}^{(i+1)} = \begin{cases} \breve{\boldsymbol{\mu}}^{(i)}, & \left|\breve{\boldsymbol{\mu}}^{(i)}\right| \geq \tau^{(i)}, \\ 0 & else \end{cases} \tag{5}$$

where, $\boldsymbol{\Phi}^{\mathbf{T}}$ is orthonormal such that $\boldsymbol{\Phi}\boldsymbol{\Phi}^{\mathbf{T}}=\mathbf{I}$, β is scaling factor, $\tau^{(i)}$ is a threshold set appropriately at each iteration. The starting condition of Eq.(4) we used zero-filled reconstructed image for $\mathbf{s}^{(0)}$.

In our work, we adopt FREBAS transform[9] as a sparsifying transform matrix $\boldsymbol{\Psi}$. FREBAS transform consist of two different Fresnel transform algorithms, which offers multi-resolution image decomposition with highly directional representation. This higher degree of directional representation contributes to superior images in proposed CS reconstruction. Figure 3 shows the schematic of proposed CS with resolution enhancement in which FFT is used instead of calculating $\boldsymbol{\Phi}$ or $\boldsymbol{\Phi}^{\mathbf{T}}$.

The acquired PSFT signal (a) is filled with zero data for the trajectory where signals are not acquired and is extrapolated with zero data outside the signal to be 2N data points. Initial image (b) is obtained by applying inverse Fourier transformed to the zero-filled signal. Since the quadratic phase modulation is applied to the obtained image (b), demodulation of the quadratic phase is applied to cancel the quadratic phase modulation ((b) to (c)). The phase variation due to the inhomogeneous static magnetic field or third stage is recalculation of the PSFT signal using *a priori* information that the image is a real-value function. The real part of the obtained image (c) is FREBAS transformed and then applying the thresholding using the $\tau^{(i)}$. Inversely FREBAS transformed image (e) is phase-modulated by quadratic function $\exp(-j\gamma b\tau x^2)$ and then Fourier transformed to calculate the PSFT signal (g). In this step, signal will be restored in the zero-filled region outside the acquired signal. Signal on the sampling trajectory is replaced by the acquired signal (a) as true data. The updated signal (h) is inversely Fourier transformed to obtain updated image (b), and the procedure (b) to (h) is performed iteratively.

III. EXPERIMENTS

In the simulation experiments, PSFT signal is calculated using the numerical phantom according to the Eq.(1). The condition of experiments are as follows; $\gamma b\tau=\gamma b\tau_{\max}=1.23$ rad/cm, $\Delta x=\Delta y=0.1$ cm. The size of signal data is $256\times N_{\mathrm{PE}}$, where N_{PE} means the number of signal for phase encoding direction and the size of reconstructed image is 512×512.

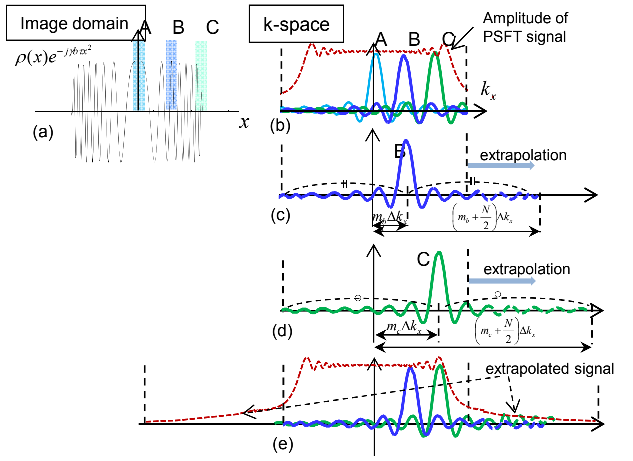

Figure 1. Schematic of PSFT signal extrapolation. Consider small width segmented images A, B C in the image domain in which quadratic phase modulation is applied. Spectrums of those segmented images are shifted in *k*-space in accordance with the spatio-temporal frequency given on the segmented images. The shifted spectrum can be restored so as to be symmetry with the position of peak of the shifted spectrum when the localized images B, C are supposed to be real-value functions. Resultant PSFT signal has the bandwidth beyond the sampling length.

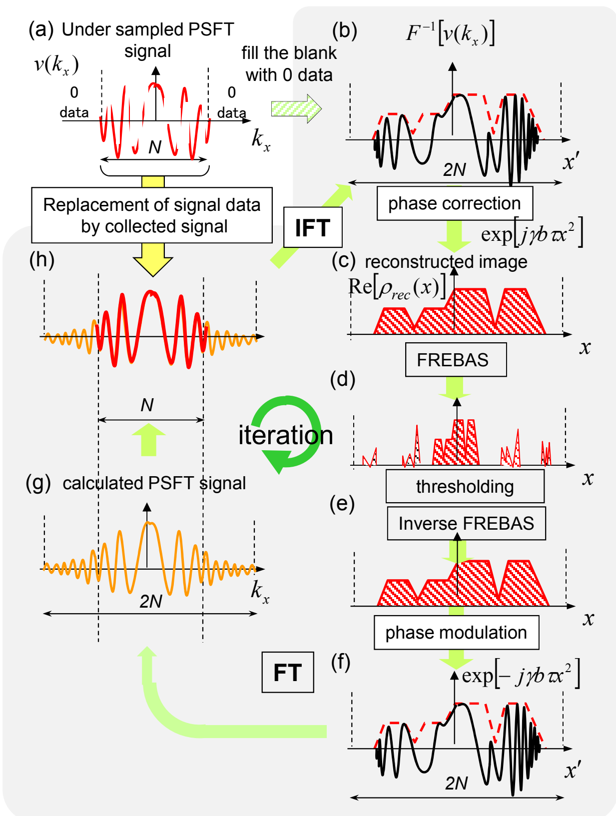

Figure 2. Schematic of compressed sensing with signal extrapolation in PSFT imaging.

The FREBAS scaling parameter D used in CS calculation is 5 at which favorable images were reconstructed. We used Cartesian sampling because it is by far the most widely used in practice. k-space signal for the phase encoding direction is randomly picked to simulate a given reduction factor. Figure 3 show the one-dimensional profiles of PSFT signal after resolution enhancement. It was shown that signals are restored in the zero-filled region. Fig 4 shows the results of simulation experiment for no phase variation model. Figs.(b)-(d) show the close-up of original model, fully scanned image with Fourier Transform imaging (FT), CS image using 35% FT signal, respectively. Figs.(e)-(h) show the images using 100%, 35%, 30%, 25% signal in proposed method. Comparison of pointed areas by arrows A and B reveal that the proposed CS images using 25%-35% signal have higher resolution and much more details than that of fully scanned image in standard FT imaging (c). Figure 5 shows the distribution of resolution improvement for phase encoding direction as a function of signal reduction factor. The resolution improvement is normalized by the resolution of fully-scanned image by standard FT imaging. It was shown from these results that the resolution improvement is possible for the undersampled signal and the resultant resolution can be superior to fully scanned FT images for the data containing more than 35% of k-space signal.

Fig. 6 shows the results using MR image model with phase variations and noises. MR volunteer image data were collected on Toshiba 1.5T MRI scanner with Fourier transform based 3D gradient echo sequence. PSFT signals were numerically calculated according to the Eq.(1) using the acquired MR image data with noises and phase variations. The imaging parameters were same as Fig.4. The CS reconstruction algorithm for phase varied signal consists of two-steps. First step estimates the phase variation on the image by producing low resolution images, and the second step executes the iterative reconstruction according to the algorithm of Fig.3 using the estimated phase variation. In the estimation of phase variation, we use Wiener filter as a linear filter instead of nonlinear hard thresholding to preserve the phase on the image. Figs 6 (a) and (b) show the fully scanned image and its phase variation (real-part of image). Figs (c) and (d) show the acquiring signal trajectory and CS images using 30% signal in proposed method. Fig. (f) is the 1-dimentional profile of signal (d) at the center of signal space. It was shown that signal is extrapolated beyond the acquired signal length. Figs.(f)-(i) show the close-up of obtained images; (f) is the full scanned gold standard image and (g) is the Fourier transform-based CS image using 30% signal for comparison, (h) and (i) are proposed images using 30% and 40% signal, respectively. Details of the subject are much more preserved in the proposed method (h).(i) as pointed in arrows. These results indicate that propose method can be applicable to MR signal with phase variation and noises.

MR images have sometimes large non-uniformities due to susceptibility artifact. The estimation of phase variation is very important and is the key to improve the spatial resolution in proposed technique.

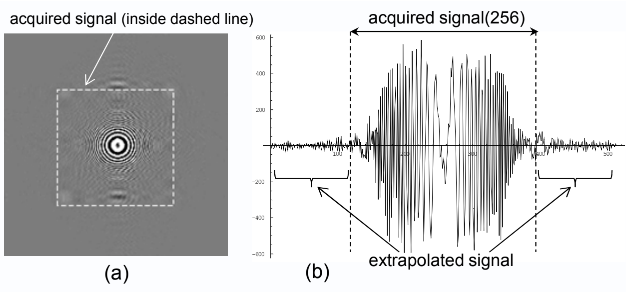

Figure 3. PSFT signal after CS reconstruction; (a) real part of the extrapolated PSFT signal, (b) 1-D profile of extrapolated signal. Signals are restored in the zero-filled region outside the acquire signal.

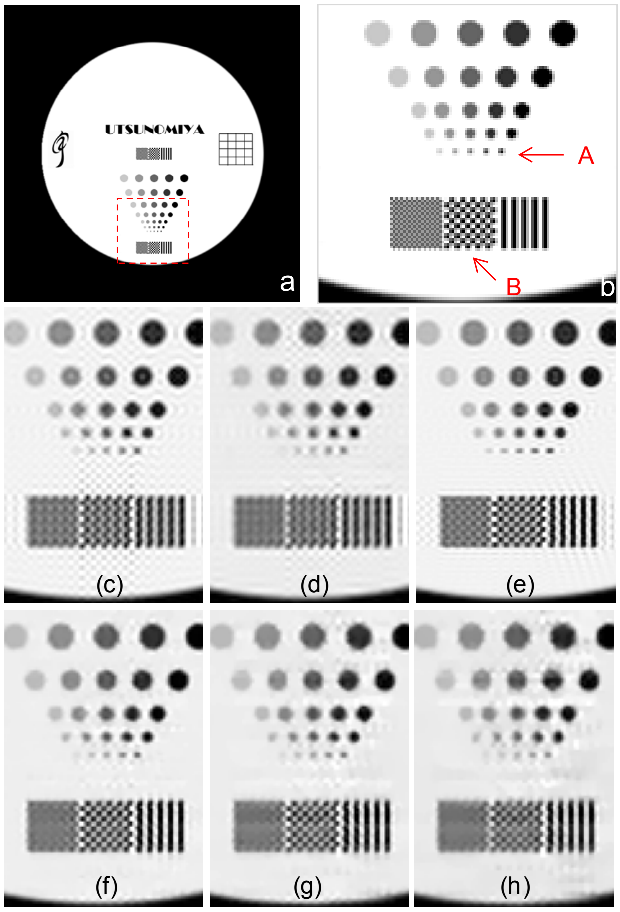

Figure 4. Comparison of CS images;(a) original image(512x512), (b) close up of image (a), (c) fully scanned(256x256) image by standard FT, (d) FT-CS image using 35% signal, (e), (f),(g),(h) PSFT-CS images with band extrapolation using 100%, 35%, 30%, 25% signal, respectively.

The calculation cost of proposed method is higher than standard CS because the size of image is enlarged to double size. Standard CS calculation takes 12s for reconstruction of single image, whereas proposed method takes 28s. Our future work is the reduction of calculation time.

IV. Conclusion

A new magnetic resonance compressed sensing reconstruction in which spatial resolution is enhanced by the

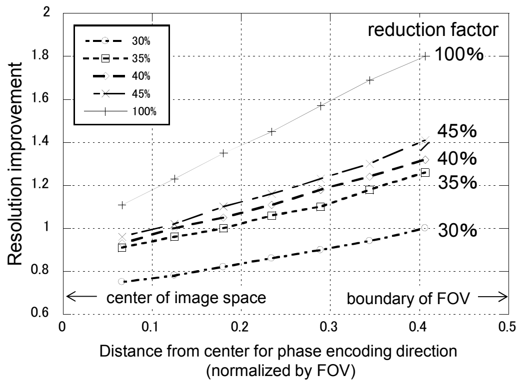

Figure 5. Resolution improvement for phase encoding direction after signal extrapolation in PSFT-CS.

extrapolation of k-space signal is proposed and demonstrated. The proposed method uses quadratic phase modulation for the subject to shift the spectrum of imaging subject. It was shown that spatial resolution was improved in accordance with the distance from the center of quadratic phase function and the resolution depends on the reduction factor of signal. Simulation studies revealed that the resultant resolution can be superior to that of fully-scanned image by Fourier transform imaging and the proposed method can be applied to the signal with phase variations.

ACKNOWLEDGMENT

This work was supported partly by Grant-in-Aid for Scientific Research (C) (24560510), Tateishi science and technology foundation and Telecommunications advancement foundation.

REFERENCES

[1] M. Lustig M, et al., "Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging," Magnetic Resonance in Medicine, vol.58, no.6, pp.1182-1195,2007

[2] D.L. Donoho, "Compressed sensing," IEEE Trans. Inform. Theory, vol. 52, no. 4, pp. 1289–1306, 2006.

[3] A.A. Maudsley, "Dynamic range improvement in NMR imaging using phase scrambling," J Magn Reson, vol. 76, pp.287-305, 1988.

[4] V.J. Wedeen,Y.S. Chao, J.L. Ackerman, "Dynamic Range Compression in MRI by Means of a Nonlinear Gradient Pulse," Magn. Reson. Med., vol.6, pp.287-295, 1988.

[5] S. Ito, Y. Yamada, "Improvement of Spatial resolution in Magnetic Resonance Imaging Using Quadratic Phase Modulation," IEEE ICIP 2009, Cairo, Egypt, pp.2497-2500 2009.

[6] S. Ito, K. Osada, Y. Yamada, "Compressed Sensing using FREBAS transform in Phase-scrambled Fourier Imaging Technique," The 37th Japanese Society for Magnetic Resonance in Medicine, P-1-26, p.317(in Japanese), Japan, 2009.

[7] Y. Wiaux, G. Puy, R. Gruetter et al., Spread spectrum for compressed sensing techniques in magnetic resonance imaging. Proc. IEEE ISBI, pp. 756–759, 2010.

[8] J. Haupt and R. Nowak, Signal reconstruction from noisy random projections, IEEE Trans. Infor. Theory, vol.52, no.9, pp.4036-4048, 2006.

[9] S. Ito, Y. Yamada, "Multiresolution Image analysis using dual Fresnel transform Pairs and Application to Medical Image Denoising," IEEE ICIP 2003, Barcelona, Spain, MA-P8.7, 2003.

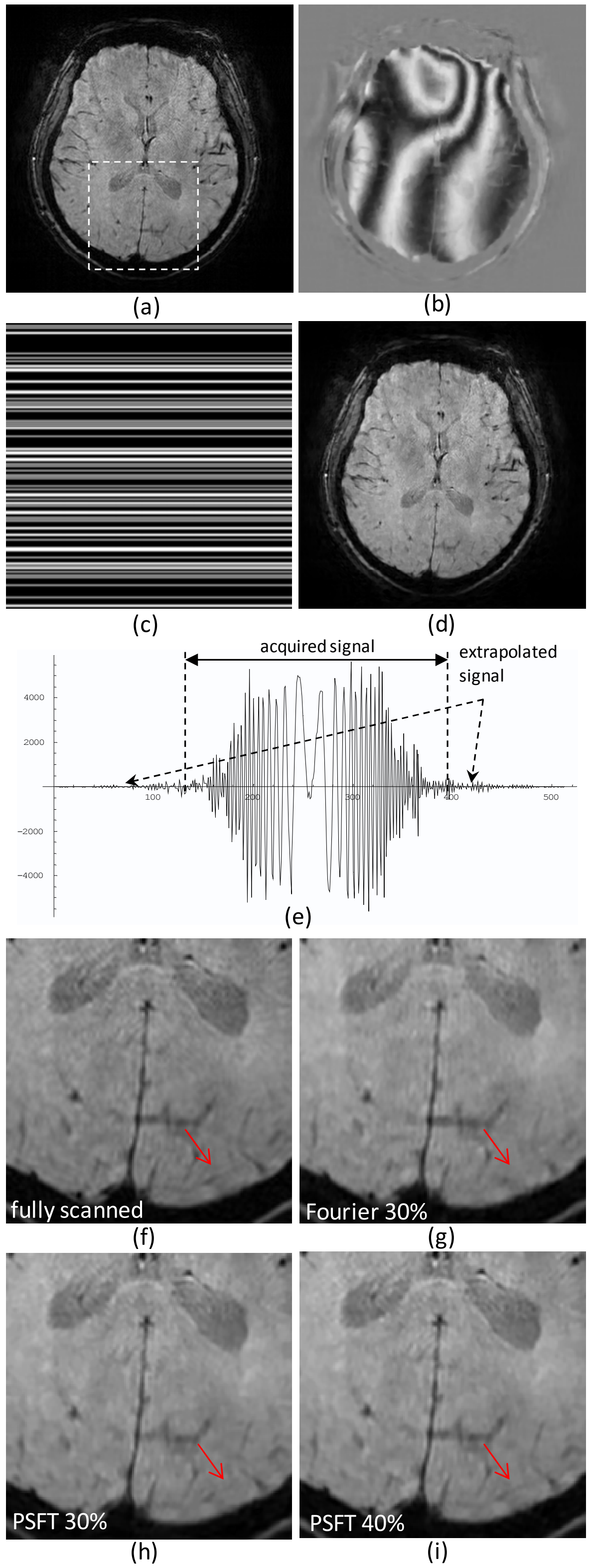

Figure 6. Application to MR model with phase variation; (a) fully scanned image (b) phase variation, (c) signal trajectory (vertical is the phase-encoding direction), (d) proposed method using 30% signal, (e) real part of PSFT signal after proposed method, (f)-(i) close-up of rectangular region in image (a),for different imaging method and amount of signal.