Distance Attenuation Control of Spherical Loudspeaker Array

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***Abstract*—This paper describes control of distance attenuation using spherical loudspeaker array. Fisher *et al.* proposed radial filtering with spherical microphone to control the sensitivity to distance from a sound source by modeling the propagation of waves in spherical harmonic domain. Since transfer functions are not changed by swapping their inputs and outputs, we can use the same theory of radial filtering for microphone arrays to the filter design of distance attenuation control with loudspeaker arrays. Experimental results confirmed that the proposed method is effective in low frequencies.**

I. Introduction

Although loudspeaker playback does not constrain bodies of listeners unlike headphone playback, it suffers from leakage of sound. Thus limiting the area of the the sound of the loudspeaker playback is an important issue for acoustic insulation and speech privacy. The most popular approach to the area limitation is directivity control of loudspeakers. For example, various plane wave transducer devices are developed, e.g., [1]. Loudspeaker array is another solution for the directivity control [2]. Also, there has been a rapid progress in parametric speakers to form sharp directivity using ultrasonic waves, e.g., [3]. However, the magnitude behind the listeners cannot be maintained by directive loudspeakers, and the area limitation is also degraded by reflection on walls.

In this paper we propose sound area limitation by the control of distance attenuation using a spherical loudspeaker array so that we can limit the area of the sound with high magnitude is limited only around the loudspeaker array. For this purpose, we employ a theory to control sensitivity of spherical microphone arrays to distance from sound sources [4] proposed by Fisher *et al.* Utilizing a nature of transfer functions which are not changed by spwapping their inputs and outputs, the Fisher's method is used for design of filters to control disttance attenuation of a spherical loudspeaker array assuming a virtually inverted wave propagation model as if sound is emitted by a microphone and reaches at the loudspeaker array. As a result of a simulation in an anechoic environment, effectiveness of the proposed method in low frequencies is confirmed.

II. Proposed Method

A. Strategy

Here we review the outline of the Fisher's method to control the distance sensitivity of spherical microphone arrays, and

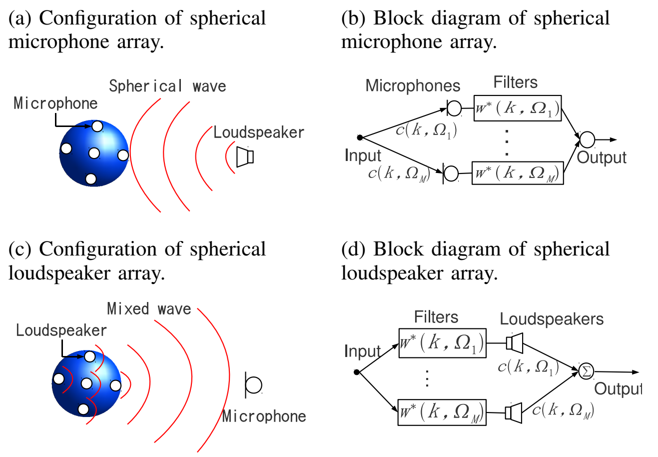

Fig. 1. Configuration and block diagram of spherical microphone array and spherical speaker array.

discuss how to apply it to distance attenuation control of spherical loudspeaker arrays.

The radial filtering proposed in [4] controls sensitivity of spherical microphone arrays to distance from the sources. As shown in Fig. 1 (a), assuming a spherical wave is radiated from a point source and reaches a microphone array with M microphone elements, the relation between the magnitude at each microphone element and the distance is modeled in the spherical harmonic domain. Using the modeled relation, filters to approximate arbitrary distance characteristics is designed. The observed signal at each element is filtered separately and summed to generate the output signal as shown in Fig. 1 (b). The transfer function $q_{\text{mic}}(k)$ between the source and the array output is given by

$$q_{\text{mic}}(k) = \sum_{j=1}^{M} w^*(k, \Omega_j)\, c(k, \Omega_j), \tag{1}$$

where $\{\cdot\}^*$ denotes complex conjugate, Ω_j $(j = 1, \ldots, M)$ is the angle of the j-th element in the three-dimensional polar coordinate with its origin at the center of the spherical microphone array, $w^*(k, \Omega_j)$ is the filter for the observed signal at the j-th element, $c(k, \Omega_j)$ is the transfer function between the source and the j-th element, and $k = f/v$ is the wave number with the frequency f and the velocity v.

The problem we discuss in this paper is the playback by the spherical loudspeaker array as shown in Fig. 1 (c). Suppose the M loudspeaker positions in Fig. 1 (c) is the same as the ones of microphones in Fig. 1 (a). Also, the source in Fig. 1 (a) is replaced by the microphone at the same position in Fig. 1 (c). Since the source is multiple loudspeakers on the surface of the sphere, the wave front is not a simple spherical wave but the more complicated superimposition of the waves from the multiple loudspeaker elements. However, the transfer functions between the spherical loudspeakers and the microphone in Fig. 1 (c) is the same as the ones between the source and the microphones, e.g., $c(k,\Omega_1),\dots,c(k,\Omega_M)$ because transfer functions are not changed by swapping their inputs and outputs. Thus, by outputting the signal filtered by the same filters $w^*(k,\Omega_j), j=1,\dots,M$, the transfer function $q_{\text{spk}}(k,\Omega_j)$ from the loudspeaker array input to the microphone is given by

$$\begin{aligned} q_{\text{spk}}(k) &= \sum_{j=1}^{M} c(k,\Omega_j)\, w^*(k,\Omega_j) \\ &= q_{\text{mic}}(k), \end{aligned} \tag{2}$$

results as the equivalent one as the transfer function $q_{\text{mic}}(k)$ of the spherical microphone array. Thus the distance characteristics of the loudspeaker output in Fig. 1 (c) is the same as that of the sensitivity of the microphone array in Fig. 1 (a). Therefore, we can design the distance characteristics of the spherical loudspeaker array by the Fisher's method using the inverted virtual microphone array model, where a single microphone emits the spherical wave and the elements of the loudspeaker array observes the wave emitted from the microphone.

B. Signal Expression in Spherical Harmonic Domain

Arbitrary sound pressure $g(k,\Omega)$ of the wave number k at a point of the angle Ω on a sphere is expressed by the following linear combination using the spherical harmonic $Y_n^m(\Omega)$ as the orthonormal bases.

$$g(k,\Omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} g_{nm}(k)\, Y_n^m(\Omega), \tag{3}$$

where $g_{nm}(k)$ is the coefficient corresponding to the basis $Y_n^m(\Omega)$, n is the order of harmonic, m is an integer to satisfy $-n \le m \le n$, and the detail of the spherical harmonic $Y_n^m(\omega)$ is described in Appendix A. Equation (3) is known as *inverse spherical Fourier transform*. The coefficient $g_{nm}(k)$ is obtained by the following integral on a 2-sphere S^2, referred to as *spherical Fourier transform*:

$$g_{nm}(k) = \int_{\Omega\in S^2} g(k,\Omega)\, Y_n^{m*}(\Omega)\, \mathrm{d}\Omega. \tag{4}$$

As can be seen in Eq. (3), infinite order of the spherical harmonic is required to express the pressure $g(k,\Omega)$ accurately. However, as discussed in [5], the order n to be approximated is bounded by the number M of the array elements as $M \ge (n+1)^2$. Thus we have to truncate the order by the maximum $N \le \sqrt{M}-1$, and the maximum wave number k_{max} to control has to satisfy

$$k_{\text{max}} \ll \frac{N}{a}, \tag{5}$$

where a is the radius of the array sphere. The theoretical estimate of the error caused by the truncation is discussed in [6], and in this paper we evaluate the error through the experiments.

C. Derivation of Radial Filter

The gain $y(k)$ of the array output of the microphone array to observe the sound field assuming near field is expressed as

$$y(k) = \sum_{n=0}^{N}\sum_{m=-n}^{n} w_{nm}^*(k)\, p_{nm}(k), \tag{6}$$

where $p_{nm}(k)$ is the spherical harmonic coefficients of the sound field $p(k,r,\Omega)$ with the near-field assumption, whose detail is described in Appendix B, is given by

$$\begin{aligned} p_{nm}(k,r) &= \int_{\Omega\in S^2} p(k,r,\Omega)\, Y_n^{m*}(\Omega)\, \mathrm{d}\Omega \\ &= b_n^{\text{s}}(k,r,r_{\text{s}})\, Y_n^{m*}(\Omega_{\text{s}}), \end{aligned} \tag{7}$$

where r_{s} and Ω_{s} are respectively the distance and the direction of the source, and $b_n^{\text{s}}(k,r,r_{\text{s}})$ is the near-field intensity given by

$$b_n^{\text{s}}(k,r,r_{\text{s}}) = i^{-n+1} k b_n(kr)\, h_n(kr_{\text{s}}), \tag{8}$$

where $h_n(\cdot)$ describes the spherical Hankel function whose details are described in Appendix C, and $b_n(\cdot)$ is the far-field intensity described in Appendix D. Here we consider the following filter coefficient $w_{nm}^*(k)$ to cancel the terms unrelated to the distance r_{s}:

$$w_{nm}^* = \frac{d_n(k)}{i^{-n+1} k b_n(ka)} Y_n^m(\Omega_{\text{l}}), \tag{9}$$

where $i=\sqrt{-1}$ and Ω_{l} is the look direction of the source and $d_n(k)$ is the coefficient to design. Its array output gain $y(k,r_{\text{s}},\Omega_{\text{l}})$ is given by

$$\begin{aligned} y(k,r_{\text{s}},\Omega_{\text{l}}) &= \sum_{n=0}^{N}\sum_{m=-n}^{n} w_{nm}^*(k)\, p_{nm}(k,a)\, Y_n^m(\Omega_{\text{l}}) \\ &= \sum_{n=0}^{N} \frac{2n+1}{4\pi} d_n(k)\, h_n(kr_{\text{s}})\, P_n(\cos\Theta), \end{aligned} \tag{10}$$

using the spherical harmonics addition theorem [7]:

$$\sum_{m=-n}^{n} Y_n^{m*}(\Omega_{\text{s}})\, Y_n^m(\Omega_{\text{l}}) = \frac{2n+1}{4\pi} P_n(\cos\Theta), \tag{11}$$

where Θ is the angle between Ω_{l} and Ω_{s}, and $P_n(\cdot)$ is the Legendre function. It can be seen in Eq. (10) that the directivity of the control is affected only by the Legendre function $P_n(\cos\theta)$, whose plots with low orders are shown in Fig. 2. With any order $P_n(\cos\theta)$ is one if $\Theta = 0, \pi$ and the directivity is not so strong for other directions with the low orders. Thus the radial filtering by Eq. (10) does not have

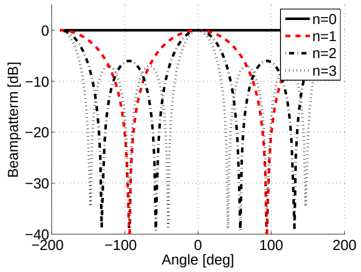

Fig. 2. Legendre function $P_n(\cos\theta)$ with orders $n = 0, 1, 2, 3$.

strong directivity.

D. Filter Design by Least Squares Estimate

Suppose the listener is at the distance of r_{lis} from the center of the sphere, referred to as *listening distance*, and we control the gain $y(k, r_{\text{s}}, \Omega_{\text{l}})$ to be close to the desired gain $y_{\text{d}}(k, r_{\text{s}}, \Omega_{\text{l}})$, which equals to 1 at $r_{\text{s}} = r_{\text{lis}}$ and equals to zero at the distance farther than r_{sup} as

$$y(k, r_{\text{s}}, \Omega_{\text{l}}) \approx y_{\text{d}}(k, r_{\text{s}}, \Omega_{\text{l}}) = \begin{cases} 1 & (r_{\text{s}} = r_{\text{lis}}), \\ 0 & (r_{\text{s}} \geq r_{\text{sup}}). \end{cases} \tag{12}$$

To design the filter coefficients $d_n(k)$ to satisfy Eq. (12), formulate the least squares problem with the sampled distances.

We sample the $(L-1)$ distances from r_{sup} to r_{supE} $(> r_{\text{sup}})$, and we define the L sample distances r_l, $l = 1, \ldots, L$ together with r_{lis} as

$$r_1 = r_{\text{lis}}, \ \ r_2 = r_{\text{sup}}, \ \ r_L = r_{\text{supE}}, \ \ r_{l-1} < r_l < r_{l+1}. \tag{13}$$

Note that $L > N$. Next, we define the $L \times (N+1)$ matrix $\mathbf{H}(k)$ composed of the right term of Eq. (10) except the coefficient $d_n(k)$ and Legendre functions as

$$\mathbf{H}(k) = \left[\frac{2n+1}{4\pi} h_{m-1}(kr_l)\right]_{lm}, \tag{14}$$

where $[x]_{lm}$ denotes the matrix which has the entry x in the l-th row and m-th column. Also with the vector $\mathbf{d}(k)$ composed of the coefficients $d_n(k)$, given by

$$\mathbf{d}(k) = [d_0(k), \ldots, d_N(k)]^{\mathsf{T}}, \tag{15}$$

where $\{\cdot\}^{\mathsf{T}}$ denotes the matrix transposition, the L-dimensional vector form $\mathbf{y}(k)$ of the series of the array output gain $y(k, r_l, \Omega_{\text{l}})$ is expressed as

$$\begin{aligned} \mathbf{y}(k) &= [y(k, r_1, \Omega_{\text{l}}), \ldots, y(k, r_L, \Omega_{\text{l}})]^{\mathsf{T}} \\ &= \mathbf{H}(k)\,\mathbf{d}(k). \end{aligned} \tag{16}$$

Also the L-dimensional vector form $\mathbf{y}_{\text{d}}(k)$ of the desired gain $y_{\text{d}}(k, r_l, \Omega_{\text{l}})$ is given by

$$\mathbf{y}_{\text{d}}(k) = [y_{\text{d}}(k, r_1, \Omega_{\text{l}}), \ldots, y_{\text{d}}(k, r_L, \Omega_{\text{l}})]^{\mathsf{T}}. \tag{17}$$

Thus the least squares solution of the coefficient $d_n(k)$ to minimize $\|\mathbf{y}_{\text{d}}(k) - \mathbf{y}(k)\|^2$ is obtained as

$$\mathbf{d}(k) = \mathbf{H}(k)^{\dagger}\,\mathbf{y}_{\text{d}}(k), \tag{18}$$

TABLE I
PARAMETERS OF SIMULATION

Number L of sampled distances	100
Maximum order N	2
Number M of loudspeaker elements	12
Radius a of spherical loudspeaker array	0.05 m
Maximum controlled distance r_{supE}	5 m
Listening distance r_{lis}	0.1 m, 0.2 m

where $\{\cdot\}^{\dagger}$ denotes the pseudo inverse matrix. However, the designed distance characteristics $y(k, r_{\text{s}}, \Omega_{\text{l}})$ is just a theoretical curve only within the truncated order $n \leq N$ is considered. the performance degraded by the truncation is evaluated in the simulation of the following section.

By substituting the designed coefficient $d_n(k)$ in Eq. (9) and applying the inverse spherical Fourier transform to $w_{\text{mn}}(k)$, the frequency characteristics of the array filter $w^*(k, \Omega_j)$ for $j = 1, \ldots, M$ is obtained as

$$w^*(k, \Omega_j) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \frac{d_n(k)}{i^{-n+1} k b_n(ka)} Y_n^m(\Omega_{\text{l}})\, Y_n^m(\Omega_j). \tag{19}$$

With the inverse discrete Fourier transform of $w^*(k, \Omega_j)$, we can obtain the array filter for the l-th element in the form of finite impulse response.

III. SIMULATION

A. Experimental Conditions

We simulate the distance attenuation control in anechoic environment comparing the performance in different configurations. We compared the simulated responses of the spherical loudspeaker array of the designed filter given by Eq. (19) with theoretical values given by Eq. (6) to evaluete the error caused by the truncation of the order, and with the simulated responses of a single loudspeaker output to evaluate the effectiveness of the proposed method. The used spherical loudspeaker array has 12 elements positioned at the vertexes of the icosahedron inscribed within the open sphere. The simulation parameters are listed on Table I.

B. Results

The results of the simulation are shown in Fig. 3. Comparing (a)–(c) or (d)–(f), the control in the low frequency is successful, but in the high frequencies the curve of the processed gain is far worse than the theoretical curve, the gain in the spherical harmonic domain, and approaches to the curve of the unprocessed signal. For example, in 148 Hz shown in (a) the suppression of about 40 dB is obtained at 1 m compared with the unprocessed output, but it degrades to worse than 10 dB in 1000 Hz as in (c). Thus the control of the high frequency is hard because of the condition in Eq. (5) is hardly satisfied.

Comparing (a) and (d), (b) and (e) or (c) and (f) reveals that the contol is easy when the listening distance is close to the array surface.

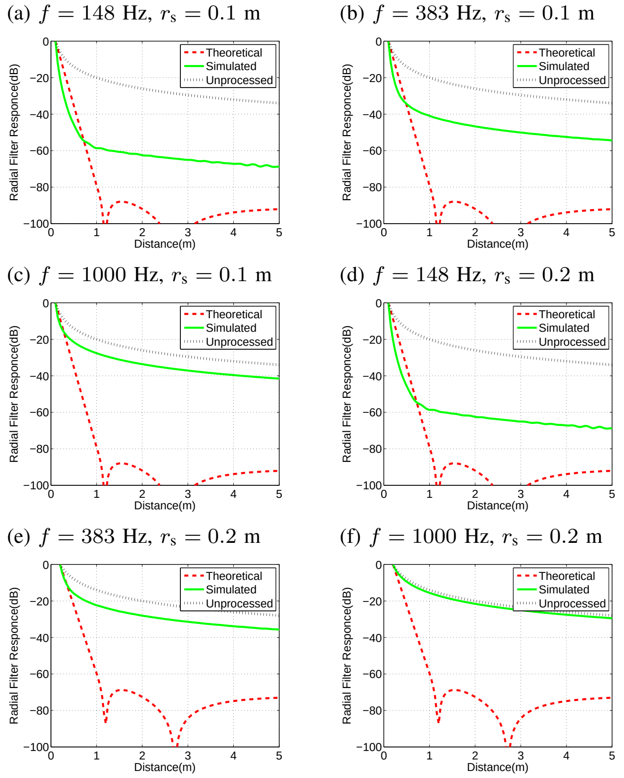

Fig. 3. Experimental results.

IV. CONCLUSIONS

This paper described the control of the area where loudspeaker output is audible using loudspeaker array. The filter coefficients are designed in the spherical harmonic domain according to the inverted model of the wave propagation which is treated as if a microphone emits the spherical wave and the loudspeaker array observes it. Effectiveness of the proposed method in the low frequency is ascertained in the simulation.

ACKNOWLEDGMENT

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APPENDIX

A. Spherical Harmonic

The spherical harmonic is the solution of three-dimensional Laplace equation in spherical coordinate, given by

$$Y_n^m(\Omega) = \sqrt{\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta)\, e^{im\phi}, \tag{20}$$

where $P_n^m(\cdot)$ is the associated Legendre function given by

$$P_n^m(x) = (-1)^m \left(1-x^2\right)^{m/2} \frac{\mathrm{d}^m}{\mathrm{d}x^m} P_n(x), \tag{21}$$

with the Legendre function given by

$$P_n(x) = \frac{1}{2^n n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x^2-1\right)^n\right]. \tag{22}$$

B. Sound Field in Spherical Harmonic Domain

The sound filed with the near-field assumption is given as follows in the frequency domain.

$$p(k,r) = \frac{e^{-2\pi ik}\,|\mathbf{r}-\mathbf{r}_\mathrm{s}|}{|\mathbf{r}-\mathbf{r}_\mathrm{s}|}, \tag{23}$$

where $\mathbf{r} = (r,\Omega)$ and $\mathbf{r}_\mathrm{s} = (r_\mathrm{s},\Omega_\mathrm{s})$ and $|\mathbf{r}-\mathbf{r}_\mathrm{s}|$ denotes the Euclidean distance between the vectors $\mathbf{r}$ and $\mathbf{r}_\mathrm{s}$. Applying the spherical Fourier transform, the sound field $p(k,r,\Omega)$ in the spherical harmonic domain is given as

$$p(k,r,\Omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} b_n^\mathrm{s}(k,r,r_\mathrm{s})\, Y_n^{m*}(\Omega_\mathrm{s})\, Y_n^m(\Omega_\mathrm{s}), \tag{24}$$

whose double sum is canceled by the orthonormal property of $Y_n^m(\Omega)$ and reduces to Eq. (7).

C. Spherical Hankel and Bessel functions

The spherical Hankel and Bessel functions h_n and j_n and the derivatives of them h_n' and j_n' are given by

$$h_n(kr) = \sqrt{\frac{\pi}{2kr}} H_{n+\frac{1}{2}}^{(1)}(kr) \tag{25}$$

$$j_n(kr) = \sqrt{\frac{\pi}{2kr}} J_{n+\frac{1}{2}}(kr) \tag{26}$$

$$h'(kr) = \frac{n}{kr} h_n(kr) - h_{n+1}(kr) \tag{27}$$

$$j_n'(kr) = \frac{n}{kr} j_n(kr) - j_{n+1}(kr), \tag{28}$$

where $H_{n+\frac{1}{2}}^{(1)}$, $J_{n+\frac{1}{2}}$ and $Y_{n+\frac{1}{2}}$ are the Hankel function of the first kind, the Bessel function of the first kind and the Bessel function of the second kind, respectively.

D. Intensity Function

The intensity $b_n(kr)$ of the far-field assumption is given for open and rigid spheres as

$$b_n(kr) = 4\pi i^n \begin{cases} j_n(kr) & \text{(open sphere)}, \\ j_n(kr) - \frac{j_n'(ka)}{h_n'(kr)} h_n(kr) & \text{(rigid sphere)}. \end{cases} \tag{29}$$

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