Nonlinear Signal Processing for Compensating Nonlinear Distortion of Louspeakers

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***Abstract*—In this paper, we propose a 3rd-order nonlinear IIR filter for compensating nonlinear distortions of loudspeaker systems. The 2nd-order nonlinear IIR filter based on the Mirror filter is used for reducing nonlinear distortions of loudspeaker systems. However, the 2nd-order nonlinear IIR filter cannot reduce nonlinear distortions at high frequencies because it does not include the nonlinearity of the self-inductance of loudspeaker systems. On the other hand, the proposed filter includes the effect of such self-inductance and thus can reduce nonlinear distortions at high frequencies. Experimental results demonstrate that the proposed filter can realize a reduction by 3.2 dB more than the conventional filter on intermodulation distortions at high frequencies.**

I. Introduction

The fundamental principle of loudspeaker systems has not changed since their invention. Loudspeaker systems employ a very complex structure to transform an electric signal into a mechanical vibration that generates acoustic waves. Nonlinear distortions are common in the vicinity of the lowest resonance frequency for electrodynamic loudspeaker systems that are widely used at present. This is because of the nonlinearity of the voice coil driving system and the mechanical nonlinearity of the edge and damper that support the diaphragm [1]. It is clear that these distortions lead to the degradation of sound quality. It seems impossible to compensate these distortions completely by only structural improvements. Therefore, some researchers have attempted to compensate nonlinear distortions by digital signal processing [2], [3], [4]. One interesting approach to compensating nonlinear distortions is to employ the 2nd-order nonlinear IIR filter [5] based on the Mirror filter [6], [7]. The 2nd-order nonlinear IIR filter is derived from a nonlinear differential equation of loudspeaker systems and includes the nonlinearities of the force factor and stiffness of such systems. However, it cannot compensate nonlinear distortions at high frequencies. This is because the 2nd-order nonlinear IIR filter does not include the nonlinearity of the self-inductance of loudspeaker systems. In this paper, we propose a 3rd-order nonlinear IIR filter to compensate nonlinear distortions at high frequencies. This filter includes the nonlinearity of the self-inductance of loudspeaker systems.

II. Third-order nonlinear IIR filter

The 3rd-order nonlinear IIR filter is based on Mirror filter [6]. Mirror filter employs nonlinear parameters that depend on the displacement of the diaphragm and cause the nonlinearity of loudspeaker systems. Mirror filter can compensate the nonlinearity of the force factor of the voice coil and magnetic circuit, the mechanical stiffness of the surround and spider, and the self-inductance of the voice coil. It is realized using the 2nd-order nonlinear IIR filter [5] derived from the nonlinear differential equation without the nonlinearity of self-inductance. Since the self-inductance governs a loudspeaker's behavior at high frequencies, the 2nd-order nonlinear IIR filter cannot reduce the nonlinear distortions at high frequencies. On the other hand, since the 3rd-order nonlinear IIR filter is derived from the nonlinear differential equation that includes the nonlinearity of self-inductance, it can reduce nonlinear distortions at high frequencies.

When the displacement of the diaphragm of a loudspeaker system is small, the vibration system of the loudspeaker system can be approximated as a single vibration system around the lowest resonance frequency. The motion equation is given by a 2nd-order linear differential equation with the linear parameters of the loudspeaker system as follows:

$$\begin{aligned} Bl_0 i(t) &= m_0\ddot{x} + K_0 x + R_m\dot{x}, &(1)\\ A_0 u(t) &= R_e i(t) + Bl_0\dot{x} + L_0\frac{di}{dt}, &(2)\end{aligned}$$

where $u(t)$ is the input voltage, $i(t)$ is the current, Bl_0 is the force factor, A_0 is the gain of the analogue part, R_e is the electrical resistance of the voice coil, m_0 is the mechanical mass, K_0 is the mechanical stiffness, R_m is the mechanical resistance, and L_0 is the self-inductance. In this case, the displacement of the diaphragm, $x(t)$, does not exhibit nonlinearity. From eqs. (1) and (2), the differential equation eq. (3) is derived as

$$G_0 u(t) = \ddot{x} + \omega_0^2 x + \frac{\omega_0}{Q_0}\dot{x} + \tau\frac{d}{dt}\left(\ddot{x} + \omega_0^2 x + \frac{\omega_0}{Q_m}\dot{x}\right),$$

where

$$G_0 = \frac{Bl_0 A_0}{R_e m_0} \quad \omega_0 = \sqrt{\frac{K_0}{m_0}} \quad Q_0 = \frac{\sqrt{m_0 K_0}}{R_m + Bl_0^2/R_e},$$

$$Q_m = \frac{\sqrt{m_0 K_0}}{R_m} \quad \tau = \frac{L_0}{R_e},$$

where τ is the time constant. From eq. (3), the linear displacement is obtained as

$$
\begin{aligned}
x(t) &= L^{-1}\{H_x(s)\} * x(t), && (3)\\
H_x(s) &= \left[G_0 \Big/ \left\{\left(s^2 + \frac{\omega_0}{Q_0}s + \omega_0^2\right)\right.\right.\\
&\qquad \left.\left. + \tau\left(s^3 + \frac{\omega_0}{Q_m}s^2 + \omega_0^2 s\right)\right\}\right], && (4)
\end{aligned}
$$

Then, from Eq. (4), the linear displacement as the discrete time is derived as

$$
\begin{aligned}
x(n) &= Z^{-1}\{H_x(z)\} * x(n), && (5)\\
H_x(z) &= G_0 \cdot \left[\frac{h_{x0} + h_{x1}z^{-1} + h_{x2}z^{-2} + h_{x3}z^{-3}}{1 + B_1 z^{-1} + B_2 z^{-2} + B_3 z^{-3}}\right], && (6)
\end{aligned}
$$

where

$$
\begin{aligned}
h_{x0} &= \frac{h_{x1}}{3} = \frac{h_{x2}}{3} = h_{x3} = \frac{1}{4f_s^2}\Big/\alpha,\\
\alpha &= \left\{1 + \frac{\omega_0}{2Q_0 f_s} + \frac{\omega_0^2}{4f_s^2}\right\}\\
&\qquad + \frac{2\tau}{T_s}\left\{1 + \frac{\omega_0}{2Q_m f_s} + \frac{\omega_0^2}{4f_s^2}\right\},\\
B_1 &= \left\{-1 + \frac{\omega_0}{2Q_0 f_s} + 3\frac{\omega_0^2}{4f_s^2}\right\}\Big/\alpha\\
&\qquad + \frac{2\tau}{T_s}\left\{-3 - \frac{\omega_0}{2Q_m f_s} + \frac{\omega_0^2}{4f_s^2}\right\}\Big/\alpha,\\
B_2 &= \left\{-1 - \frac{\omega_0}{2Q_0 f_s} + 3\frac{\omega_0^2}{4f_s^2}\right\}\Big/\alpha\\
&\qquad + \frac{2\tau}{T_s}\left\{3 - \frac{\omega_0}{2Q_m f_s} - \frac{\omega_0^2}{4f_s^2}\right\}\Big/\alpha,\\
B_3 &= \left\{1 - \frac{\omega_0}{2Q_0 f_s} + \frac{\omega_0^2}{4f_s^2}\right\}\Big/\alpha\\
&\qquad + \frac{2\tau}{T_s}\left\{-1 + \frac{\omega_0}{2Q_m f_s} - \frac{\omega_0^2}{4f_s^2}\right\}\Big/\alpha.
\end{aligned}
$$

$f_s = 1/T_s$ is the sampling frequency. In this case, the force factor, stiffness, and self-inductance of the voice coil become nonlinear parameters and cause nonlinear distortions in loudspeaker systems. The nonlinear parameters can be approximated using the following quadratic and cubic functions [1]:

$$
\begin{aligned}
Bl(x) &= Bl_0 b(x) = Bl_0(1 + b_1 x + b_2 x^2), && (7)\\
K(x) &= K_0 k(x) = K_0(1 + k_1 x + k_2 x^2), && (8)\\
L(x) &= L_0 l(x) = L_0(1 + l_1 x + l_2 x^2 + l_3 x^3), && (9)
\end{aligned}
$$

where $b(x)$, $k(x)$ and $l(x)$ represent the nonlinearities of the force factor, stiffness and self-inductance, respectively; these are all dimensionless. The differential equations eqs. (1) and (2) are rewritten as

$$
\begin{aligned}
Bl(x)i(t) &= m_0\ddot{x} + K(x)x + R_m\dot{x} - \frac{1}{2}i_L(t)^2\frac{dL(x)}{dx}, && (10)\\
A_0 u(t) &= R_e i(t) + Bl(x)\dot{x} + \frac{dL(x)i_L(t)}{dt}, && (11)\\
Bl(x)i_L(t) &= m_0\ddot{x} + K(x)x + R_m\dot{x}, && (12)
\end{aligned}
$$

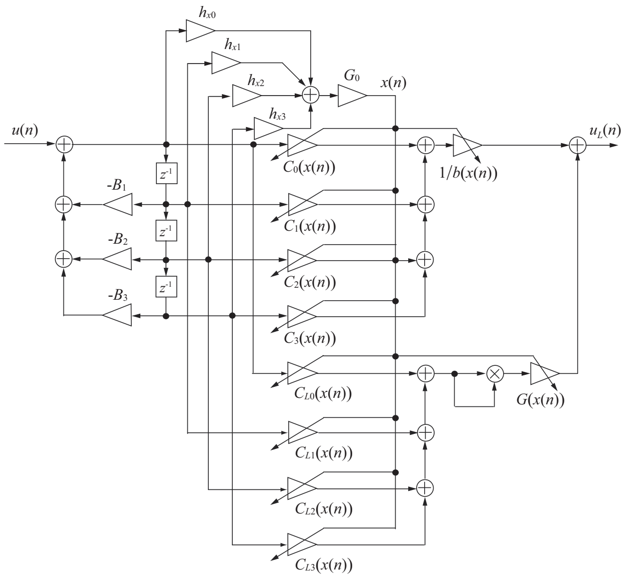


Fig. 1. Block diagram of the 3rd-order nonlinear IIR filter.

where $i_L(t)$ is the compensation current for self-inductance. From eqs. (10) and (11), the following equation is derived.

$$
\begin{aligned}
G_0 b(x) u_L(t) &= \ddot{x} + \omega_0^2 k(x)x\\
&+ \left\{1 + \left(1 - \frac{Q_0}{Q_m}\right)\left(b(x)^2 - 1\right)\right\}\frac{\omega_0}{Q_0}\dot{x}\\
&+ \tau\frac{dl(x)}{dt}\left\{\ddot{x} + \frac{\omega_0}{Q_m}\dot{x} + \omega_0^2 k(x)x\right\}\\
&- \tau\frac{l(x)}{b(x)}\frac{db(x)}{dt}\left\{\ddot{x} + \frac{\omega_0}{Q_m}\dot{x} + \omega_0^2 k(x)x\right\}\\
&+ \tau l(x)\left\{\dddot{x} + \frac{\omega_0}{Q_m}\ddot{x}\right.\\
&\qquad \left. + \omega_0^2 k(x)\dot{x} + \omega_0^2\frac{dk(x)}{dt}x\right\}\\
&- \frac{1}{2m_0}\left\{\frac{A_0}{R_e G_0 b(x)}\right\}^2\\
&\qquad \left(\ddot{x} + \frac{\omega_0}{Q_m}\dot{x} + \omega_0^2 k(x)x\right)^2\frac{dL(x)}{dx}.
\end{aligned}
\qquad (13)
$$

The nonlinear motion of loudspeaker systems is represented by eqs. (10) $\sim$ (12). In these equations, the displacement x shows a nonlinear behavior. On the other hand, the displacement x shows a linear behavior in eqs. (1) and (2). If the displacement x of eqs. (10) $\sim$ (12) shows a linear behavior, these equations can be treated as equations that show a linear behavior. Therefore, the 3rd-order nonlinear IIR filter can be derived by substituting the linear displacement eq. (3) into the nonlinear differential equation eq. (13). Figure 1 shows the block diagram of the 3rd-order nonlinear IIR filter derived according to the above procedure. The coefficients in Fig. 1

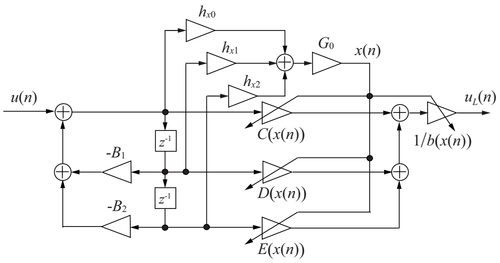


Fig. 2. Block diagram of the 2nd-order nonlinear IIR filter.

are given by

$$\begin{aligned}
C_i(x(n)) &= h_{ai} + \omega_0^2 k(x(n)) h_{xi} \\
&\quad + \left\{1 + \left(1 - \frac{Q_0}{Q_m}\right)\left(b(x(n))^2 - 1\right)\right\} \frac{\omega_0}{Q_0} h_{vi} \\
&\quad + \tau \left\{ \Delta\left\{l(x(n))\right\} - \frac{l(x(n))}{b(x(n))} \Delta\left\{b(x(n))\right\} \right\} \\
&\quad \times \left\{ h_{ai} + \frac{\omega_0}{Q_m} h_{vi} + \omega_0^2 k(x(n)) h_{xi} \right\} \\
&\quad + \tau l(x(n)) \left\{ h_{ji} + \frac{\omega_0}{Q_m} h_{ai} + \omega_0^2 k(x(n)) h_{vi} \right. \\
&\quad \left. + \omega_0^2 \Delta\left\{k(x(n))\right\} h_{xi} \right\} \quad (i = 0,\ 1,\ 2,\ 3),
\end{aligned}$$

$$\begin{aligned}
h_{v0} &= h_{v1} = -h_{v2} = -h_{v3} = \frac{1}{2f_s} \Big/ \alpha, \\
h_{a0} &= -h_{a1} = -h_{a2} = h_{a3} = 1 \Big/ \alpha, \\
h_{j0} &= -\frac{h_{j1}}{3} = \frac{h_{j2}}{3} = -h_{j3} = 2f_s \Big/ \alpha, \\
C_{Li}(x(n)) &= h_{ai} + \frac{\omega_0}{Q_m} h_{vi} + \omega_0^2 k(x(n)) h_{xi}, \\
&\qquad\qquad (i = 0,\ 1,\ 2,\ 3), \\
G(x(n)) &= \frac{A_0 \tau}{2Bl_0} \frac{1}{b(x(n))^3} \left\{ l_1 + 2l_2 x(n) + 3l_3 x(n)^2 \right\},
\end{aligned}$$

where "$\Delta\{\ \}$" is the difference value. This filter generates a compensation signal in two steps. First, the linear displacement $x(n)$ is calculated. Then, the coefficients depending on the displacement $x(n)$ are calculated. These coefficients include the effects of the linear displacement, velocity, acceleration and derivation of acceleration. If the self-inductance of the loudspeaker system is ignored, the block diagram shown in Fig. 1 is reduced to that shown in Fig. 2, which represents the 2nd-order nonlinear IIR filter, that is, the proposed nonlinear IIR filter includes the conventional nonlinear IIR filter.

III. Experimental results

We conducted experiments on compensating the nonlinear distortion of a loudspeaker system. The specifications of the loudspeaker system are shown in Table I. The 2nd- and 3rd-order nonlinear IIR filters need the linear and nonlinear parameters of the loudspeaker system. These parameters were estimated by the parameter estimation method for a closed-box loudspeaker system using Volterra kernels [8]. This method is based on the calculation of the compensation amount of nonlinear distortions of the nonlinear IIR filter. The initial linear parameters were determined from impedance characteristics, as shown in Table II. The nonlinear parameters were estimated as

$$\begin{aligned}
Bl(x) &= Bl_0(1 + 21x - 50800x^2), && (14) \\
K(x) &= K_0(1 + 61x + 49900x^2), && (15) \\
L(x) &= L_0(1 - 231x - 6200x^2 + 55500x^3). && (16)
\end{aligned}$$

TABLE I
Specifications of a loudspeaker system.

Diameter	6.6 cm
Rated power	5 W
Electrical resistance	7.78 Ω
Enclosure volume	0.7 l
Enclosure type	Closed-box

TABLE II
Initial linear parameters determined from impedance characteristics.

ω_0	1892 rad/s
Q_0	2.31
Q_m	4.37
R_e	7.78 Ω
R_m	0.32 Ns/m
m_0	0.74 $\times 10^{-3}$ kg
K_0	2663 N/m
Bl_0	1.50 Wb/m
L_0	0.18 mH

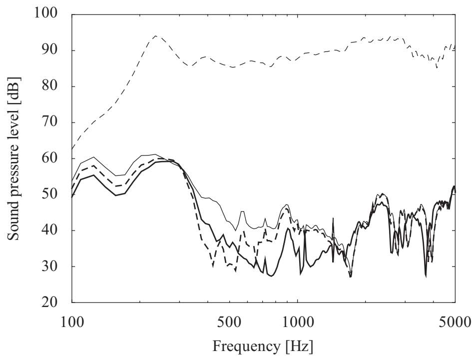
The 2nd- and 3rd-order nonlinear IIR filters are realized using the above parameters, and the effectiveness of compensating the nonlinear distortion of the loudspeaker system is compared between these filters. The measurement conditions are shown in Table III. The sound pressure characteristics of nonlinear distortions are shown in Fig. 3, and the average nonlinear distortion compensation amounts are shown in Table IV. As observed in Fig. 3 and Table IV, the 3rd-order nonlinear IIR filter can reduce the intermodulation distortions by about 3.2 dB at high frequencies and is superior to the 2nd-order nonlinear IIR filter. However, the harmonic distortion is not reduced at high frequencies. This is because the harmonic distortion is smaller than the intermodulation distortions. On the other hand, the 3rd-order nonlinear IIR filter can also reduce nonlinear distortions at low frequencies and is superior to the 2nd-order nonlinear IIR filter. Hence, the 3rd-order nonlinear IIR filter is effective for compensating nonlinear distortions of the loudspeaker system.

IV. Conclusions

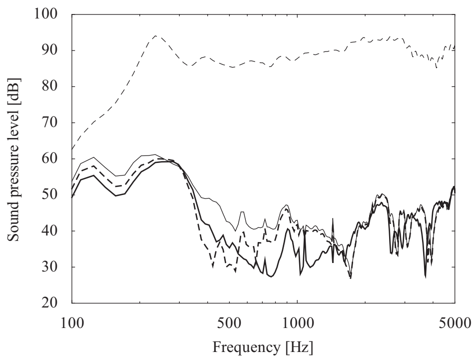
In this paper, we proposed a 3rd-order nonlinear IIR filter, and compared its compensation ability for nonlinear distortions of a loudspeaker system with that of the 2nd-order nonlinear IIR filter. Experimental results indicated that the

TABLE III
MEASUREMENT CONDITIONS FOR COMPENSATING NONLINEAR DISTORTIONS.

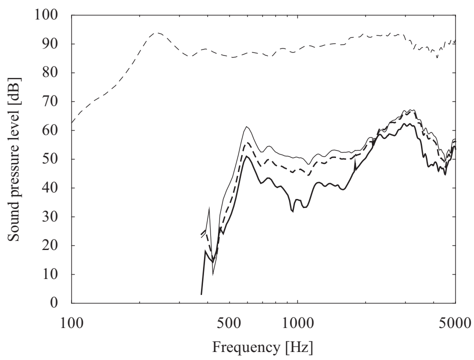
Input signal	Swept sinusoidal wave
Sampling frequency f_s	32000 Hz
Fixed frequency m_1	350 Hz
Swept frequency m_2	100 - 5000 Hz
Average	15
Input voltage	3.5 V



(a) $2m_2$ characteristic.



(b) $m_1 + m_2$ characteristic.



(c) $m_2 - m_1$ characteristic.

- - - Linear characteristic
—— Before compensation
- - - After compensation (2nd-order)
—— After compensation (3rd-order)

Fig. 3. Comparison of the compensation abilities of nonlinear distortions between the 2nd- and 3rd-order nonlinear IIR filters.

TABLE IV
COMPARISON OF AVERAGE NONLINEAR DISTORTION COMPENSATION AMOUNTS BETWEEN THE 2ND- AND 3RD-ORDER NONLINEAR IIR FILTERS.

	2nd-order	3rd-order
$2m_2$ characteristic		
100Hz $\sim$ 700Hz	6.0 dB	6.8 dB
700Hz $\sim$ 5kHz	0.8 dB	2.0 dB
$m_1 + m_2$ characteristic		
100Hz $\sim$ 700Hz	10.8 dB	6.3 dB
700Hz $\sim$ 5kHz	3.2 dB	4.2 dB
$m_2 - m_1$ characteristic		
100Hz $\sim$ 700Hz	4.7 dB	9.7 dB
700Hz $\sim$ 5kHz	1.8 dB	7.1 dB

3rd-order nonlinear IIR filter can reduce the intermodulation distortion more effectively than the 2nd-order nonlinear IIR filter. Hence, we conclude that the 3rd-order nonlinear IIR filter is effective for compensating nonlinear distortions of loudspeaker systems. In the future, we should improve the parameter estimation method to better compensate such nonlinear distortions.

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