Ergodic Mutual Information of Amplify-and-Forward MIMO Relay Channels with LOS Components

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***Abstract*—In this paper, we address the ergodic mutual information of amplify-and-forward multiple-input multiple-output two-hop relay channels. In these channels, the source terminal, relay terminal, and destination terminal are equipped with a number of correlated antennas, and there presents a line-of-sight component on each link. The models have widel applications in the field of machine-type communication devices, such as meters and sensors. Given channel matrices with Gaussian entries, the mean of mutual information is derived under the large-system regimen, in which the number of antennas at the transmitter and the receiver go to infinity with a fixed ratio. Simulation results demonstrate that even for a moderate number of antennas at each end, the proposed analytical results provide undistinguishable results from those obtained by Monte-Carlo simulations. In addition, the well approximation property holds even if the entries of the channel matrices are non-Gaussian.**

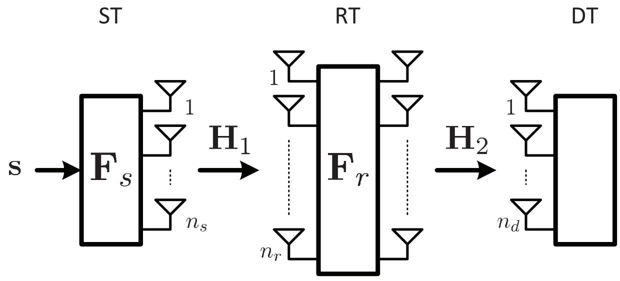


Fig. 1. A MIMO two-hop relay relay communication system.

I. Introduction

Combining wireless multiple-input multiple-output (MIMO) antenna systems with relay techniques has recently been recognized as a promising solution to extend communication coverage areas and to improve system throughput. This technique is particularly suitable for the field of machine-type communication devices, such as meters and monitors. Motivated by these possibilities, growing attention has been paid on understanding the achievable rates of relay channels under various protocols [1–8]. Among relaying protocols, the amplify-and-forward (AF) protocol receives more attention because of its simple implementation property. Following the same trend, our focus is on the mutual information analysis of the MIMO two-hop relay channels that employ the AF protocol as illustrated in Figure 1.

The considered two hop relay system consists of a source terminal (ST), a relay terminal (RT), and a destination terminal (DT). The transmission from the ST and the RT is done over two separate phases. The Kronecker correlated Rician fading model is considered for each communication link. We assume that the DT has perfect channel state information (CSI), while the ST and the RT do not have CSI. Although the setting above is common (or conventional), deriving analytical expressions for the average mutual information of finite-size MIMO relay channels is very difficult. For recent progress on a special channel setting please refer to [2]. To circumvent the mathematical difficulties, we consider the derivation under a large-system regimen, in which the numbers of antennas at the transmitter and the receiver go to infinity with a fixed ratio.

In the large system regimen, we can obtain an analytical approximation for the average mutual information of the MIMO relay channel. Simulation results indicate that the asymptotic regime is reached for a relatively small number of antennas. The derived analytical approximation provides a general formula that encompasses the known result of [4], wherein the line-of-sight (LOS) components are absent. In the Kronecker correlated Rician fading model, the random parts of the channel matrices are usually assumed to consist of independent and identically distributed (i.i.d.) standard complex Gaussian entries. We further show that the analytical approximation holds even when the considered random matrices are non-Gaussian. This extension makes the analytical expression more general, and thus our result can be used to cope with more general applications.

The remainder of this paper is structured as follows. Section II introduces the channel model of the two-hop relay system. Section III presents our main results along with a few discussions. Simulation results are provided in Section IV and conclusions are presented in Section V.

Notations—Throughout this paper, for any matrix $\mathbf{A}$, $[\mathbf{A}]_{i,j}$ indicates the (i,j)th entry of $\mathbf{A}$, $\mathbf{A}^\dagger$ represents the conjugate transpose of $\mathbf{A}$, $\mathbf{A}^{\frac{1}{2}}$ represents the principal square root of $\mathbf{A}$, and $\mathtt{Tr}(\mathbf{A})$ represents the trace of $\mathbf{A}$. Also, $\mathbf{I}$ denotes the identity matrix, $\mathbf{0}$ denotes the zero matrix, $\|\cdot\|$ denotes the

Euclidean norm, $\mathbb{E}[\cdot]$ denotes the expectation operator, $\log(\cdot)$ is the natural logarithm, and $\otimes$ denotes the Kronecker product.

II. System Model

We consider the two-hop MIMO AF relay system consisting of a ST, a RT, and a DT (see Figure 1). The transmission from the SD and the RT is done over two separate phases. We assume that there is no direct communication link between ST and DT. First, the ST transmits a symbol vector $\mathbf{s} \in \mathbb{C}^{n_s}$ through a precoding matrix $\mathbf{F}_s \in \mathbb{C}^{n_s \times n_s}$, and the received signal at the RT is[1]

$$\mathbf{y}_r = \mathbf{H}_1 \mathbf{F}_s \mathbf{s} + \mathbf{n}_r, \tag{1}$$

where $\mathbf{H}_1 \in \mathbb{C}^{n_r \times n_s}$ denotes the channel between the ST and the RT (referred to as the first-hop channel), and $\mathbf{n}_r \in \mathbb{C}^{n_r}$ are the i.i.d. standard (zero-mean unit-variance) complex Gaussian noise vectors at the RT.

In the second phase, the RT is required to transmit the received signal in the first phase $\mathbf{y}_1$ to the DT. Before being forwarded to the DT, the received signal $\mathbf{y}_1$ should be weighted by a forwarding matrix $\mathbf{F}_r$ in order to adjust the forwarding direction and to satisfy the power constraint of the RT. Hence, the signal received by the DT in the second phase can be written as

$$\mathbf{y} = \mathbf{H}_2 \mathbf{F}_r \left(\mathbf{H}_1 \mathbf{F}_s \mathbf{s} + \mathbf{n}_r \right) + \mathbf{n}_d, \tag{2}$$

where $\mathbf{H}_2 \in \mathbb{C}^{n_d \times n_r}$ denotes the channel matrix of the RD link and $\mathbf{n}_d \in \mathbb{C}^{n_d}$ is the standard complex Gaussian noise vector at the DT.

For the channel model, we consider the Kronecker correlated Rician fading channel as:

$$\begin{aligned} \mathbf{H}_1 &= \mathbf{R}_r^{\frac{1}{2}} \mathbf{H}_1^{(w)} \mathbf{T}_s^{\frac{1}{2}} + \bar{\mathbf{H}}_1, \\ \mathbf{H}_2 &= \mathbf{R}_d^{\frac{1}{2}} \mathbf{H}_2^{(w)} \mathbf{T}_r^{\frac{1}{2}} + \bar{\mathbf{H}}_2, \end{aligned}$$

where $\mathbf{R}_r \in \mathbb{C}^{n_r \times n_r}$, $\mathbf{T}_s \in \mathbb{C}^{n_s \times n_s}$, $\mathbf{R}_d \in \mathbb{C}^{n_d \times n_d}$, and $\mathbf{T}_r \in \mathbb{C}^{n_r \times n_r}$ are the spatial correlation matrices at the respective terminals; $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ are the LOS component of the channel matrix for the respective communication links; $\mathbf{H}_1^{(w)} \in \mathbb{C}^{n_r \times n_s}$ and $\mathbf{H}_2^{(w)} \in \mathbb{C}^{n_d \times n_r}$ denote the channel components that are i.i.d. standard complex Gaussian entries.

The signal-to-noise-ratios (SNRs) of the communication links are defined as

$$\mathsf{SNR}_s = \frac{\mathsf{tr}\big(\mathsf{E}\{\mathbf{H}_1 \mathbf{H}_1^H\}\big)}{\mathsf{E}\{\mathbf{n}_r^H \mathbf{n}_r\}}, \tag{3a}$$

$$\mathsf{SNR}_d = \frac{\mathsf{tr}\big(\mathsf{E}\{\mathbf{H}_2 \mathbf{H}_2^H\}\big)}{\mathsf{E}\{\mathbf{n}_d^H \mathbf{n}_d\}}. \tag{3b}$$

Note that the above definitions cannot reflect the true SNR of the communication links because the true ones should depend on the input signals at the respective terminals. Therefore, the definitions are merely for convenience. With a simple algebraic calculation, we have

$$\mathsf{tr}\big(\mathsf{E}\{\mathbf{H}_1 \mathbf{H}_1^H\}\big) = \mathsf{tr}(\mathbf{R}_r)\mathsf{tr}(\mathbf{T}_s) + \mathsf{tr}(\bar{\mathbf{H}}_1 \bar{\mathbf{H}}_1^H) \tag{4}$$

Therefore, to satisfy (3), we assume without loss of generality that $\mathbf{R}_r$, $\mathbf{T}_s$, and $\bar{\mathbf{H}}_1$ are normalized in such a way that

$$\begin{cases} \mathsf{tr}(\mathbf{T}_s) = 1 \\ \mathsf{tr}(\mathbf{R}_r) = n_r \dfrac{1}{\mathcal{K}_1 + 1} \mathsf{SNR}_s \\ \mathsf{tr}(\bar{\mathbf{H}}_1 \bar{\mathbf{H}}_1^H) = n_r \dfrac{\mathcal{K}_1}{\mathcal{K}_1 + 1} \mathsf{SNR}_s, \end{cases} \tag{5}$$

where $\mathcal{K}_1$ (Rician factor) determines the relative power of the channel's LOS and non-LOS components. The similar normalization is used for $\mathbf{H}_2$.

III. Main Results

We assume that the DT knows the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ perfectly, and $\mathbf{s}$ is Gaussian with covariance $\mathbf{I}$. The mutual information between $\mathbf{x}$ and $\mathbf{y}$ is thus given by

$$I = \log\det\left(\mathbf{R}_n + \mathbf{R}_s\right) - \log\det\left(\mathbf{R}_n\right) \tag{6}$$

with

$$\begin{aligned} \mathbf{R}_n &= \mathbf{I}_{n_d} + \mathbf{H}_2 \mathbf{F}_r \mathbf{F}_r^H \mathbf{H}_2^H \\ \mathbf{R}_s &= \mathbf{H}_2 \mathbf{F}_r \mathbf{H}_1 \mathbf{F}_s \mathbf{F}_s^H \mathbf{H}_1^H \mathbf{F}_r^H \mathbf{H}_2^H. \end{aligned}$$

For ease of expression, $(\mathbf{F}_s, \mathbf{F}_r)$ are incorporated into the channel correlation matrices resulting in

$$\begin{cases} \tilde{\mathbf{T}}_s^{\frac{1}{2}} := \mathbf{T}_s^{\frac{1}{2}} \mathbf{F}_s \\ \tilde{\mathbf{T}}_r^{\frac{1}{2}} := \mathbf{T}_r^{\frac{1}{2}} \mathbf{F}_r \\ \bar{\mathbf{H}}_1 := \bar{\mathbf{H}}_1 \mathbf{F}_s \\ \bar{\mathbf{H}}_2 := \bar{\mathbf{H}}_2 \mathbf{F}_r. \end{cases} \tag{7}$$

For convenience, we still use $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ to denote those matrices that include the effect of $(\mathbf{F}_s, \mathbf{F}_r)$.

The mutual information of the Kronecker Rician fading MIMO relay channel, I, is also a random variable because $\mathbf{H}_1$ and $\mathbf{H}_2$ are random. Deriving an analytical distribution of I for finite-size MIMO relay channels is difficult and unsolvable. This difficulty is circumvented by considering the large-system regime where $\{n_s, n_r, n_d\}$ approach to infinity at fixed ratios: $\frac{n_r}{n_s}$ and $\frac{n_d}{n_r}$.

Proposition 1: The mean of I can be asymptotically approximated by

$$\begin{aligned} \mathsf{E}\{I\} \sim\ & \log\det\left(\mathbf{I}_{n_s} + \mathbf{\Phi}_1\right) + \log\det\left(\mathbf{I}_{n_r} + \mathbf{\Phi}_2 \mathbf{\Theta}_1\right) \\ & + \log\det\left(\mathbf{\Theta}_2\right) - \log\det\left(\mathbf{\Theta}_3\right) - \log\det\left(\mathbf{I}_{n_r} + \mathbf{\Phi}_3\right) \\ & - \left(s_1 t_1 + \tilde{s}_2 t_2 - s_3 t_3\right) \end{aligned} \tag{8}$$

[1] In this paper, we aim to complement the results of [4] by extending the asymptotic analysis to the case of separately correlated Rician fading MIMO channels. For the readers convenience, similar notations as those of [4] are used.

with

$$t_1 = \mathrm{tr}\Big\{(\mathbf{I}_{n_s} + \mathbf{\Phi}_1)^{-1}\tilde{\mathbf{T}}_s\Big\} \tag{9a}$$

$$t_2 = \mathrm{tr}\Bigg\{\Big(\mathbf{\Theta}_2 + \bar{\mathbf{H}}_2\mathbf{\Omega}(\mathbf{I}_{n_r} + t_2\tilde{\mathbf{T}}_r\mathbf{\Omega})^{-1}\bar{\mathbf{H}}_2^H\Big)^{-1}\mathbf{R}_d\Bigg\} \tag{9b}$$

$$t_3 = \mathrm{tr}\Big\{(\mathbf{I}_{n_r} + \mathbf{\Phi}_3)^{-1}\tilde{\mathbf{T}}_r\Big\} \tag{9c}$$

$$s_1 = \mathrm{tr}\big\{(\mathbf{I}_{n_r} + \mathbf{\Phi}_2\mathbf{\Omega})^{-1}\mathbf{\Phi}_2\mathbf{R}_r\big\} \tag{9d}$$

$$\tilde{s}_2 = \mathrm{tr}\Big\{(\mathbf{I}_{n_r} + \mathbf{\Omega}\mathbf{\Phi}_2)^{-1}\mathbf{\Omega}\tilde{\mathbf{T}}_r\Big\} \tag{9e}$$

$$s_3 = \mathrm{tr}\big\{(\mathbf{I}_{n_d} + \mathbf{\Psi}_3)^{-1}\mathbf{R}_d\big\} \tag{9f}$$

and

$$\begin{cases} \mathbf{\Theta}_1 \triangleq \mathbf{I}_{n_r} + t_1\mathbf{R}_r \\ \mathbf{\Theta}_2 \triangleq \mathbf{I}_{n_d} + \tilde{s}_2\mathbf{R}_d \\ \mathbf{\Theta}_3 \triangleq \mathbf{I}_{n_d} + t_3\mathbf{R}_d \\ \mathbf{\Xi}_1 \triangleq \mathbf{I}_{n_s} + s_1\tilde{\mathbf{T}}_s \\ \mathbf{\Xi}_3 \triangleq \mathbf{I}_{n_r} + s_3\tilde{\mathbf{T}}_r \\ \mathbf{\Phi}_1 \triangleq s_1\tilde{\mathbf{T}}_s + \bar{\mathbf{H}}_1^H\mathbf{\Phi}_2(\mathbf{I}_{n_r} + \mathbf{\Theta}_1\mathbf{\Phi}_2)^{-1}\bar{\mathbf{H}}_1 \\ \mathbf{\Phi}_2 \triangleq t_2\tilde{\mathbf{T}}_r + \bar{\mathbf{H}}_2^H\mathbf{\Theta}_2^{-1}\bar{\mathbf{H}}_2 \\ \mathbf{\Phi}_3 \triangleq s_3\tilde{\mathbf{T}}_r + \bar{\mathbf{H}}_2^H\mathbf{\Theta}_3^{-1}\bar{\mathbf{H}}_2 \\ \mathbf{\Psi}_1 \triangleq t_1\mathbf{R}_r + \bar{\mathbf{H}}_1\mathbf{\Xi}_1^{-1}\bar{\mathbf{H}}_1^H \\ \mathbf{\Psi}_3 \triangleq t_3\mathbf{R}_d + \bar{\mathbf{H}}_2\mathbf{\Xi}_3^{-1}\bar{\mathbf{H}}_2^H \\ \mathbf{\Omega} \triangleq \mathbf{\Theta}_1 + \bar{\mathbf{H}}_1\mathbf{\Xi}_1^{-1}\bar{\mathbf{H}}_1^H. \end{cases}$$

Consider a Gaussian vector channel with the following input-output relationship

$$\check{\mathbf{y}} = \mathbf{\Phi}_1^{1/2}\check{\mathbf{x}} + \check{\mathbf{n}} \tag{10}$$

where $\check{\mathbf{y}} \in \mathbb{C}^{n_s}$, $\mathbf{\Phi}_1^{1/2} \in \mathbb{C}^{n_s \times n_s}$ is the deterministic channel matrix, and $\mathbf{n} \in \mathbb{C}^{n_s}$ is a standard complex Gaussian random vector. Clearly, the first term $\log\det(\mathbf{I}_{n_s} + \mathbf{\Phi}_1)$ in (8) corresponds to the mutual information between $\check{\mathbf{x}}$ and $\check{\mathbf{y}}$ over the Gaussian vector channel (10). Note that $\mathbf{\Phi}_1$ consists of the two parts $s_1\tilde{\mathbf{T}}_s$ and $\bar{\mathbf{H}}_1^H\mathbf{\Phi}_2(\mathbf{I}_{n_r} + \mathbf{\Theta}_1\mathbf{\Phi}_2)^{-1}\bar{\mathbf{H}}_1$, wherein one is related to the random part of the channel and the other is related to the deterministic part of the channel (i.e, LOS component). Finally, we note that Propositions 1 provides a general formula that encompasses the known result of [4], wherein the LOS components are absent.

Proposition 2: Proposition 1 is true even if the entries of $\mathbf{H}_1^{(w)}$ and $\mathbf{H}_2^{(w)}$ are non-Gaussian.

The proofs of Propositions 1&2 are omitted due to the space limitation.

IV. Numerical Results

All of our analyses are based on the assumption that the dimensions of the channel matrices are large. In this subsection, we provide simulation results that show the asymptotic regime is reached for a relatively small number of antennas. For the simulations, we assumed that the spatial correlation is generated by

$$[\mathbf{T}_s]_{ij} = \left(\frac{1}{2}\right)^{|i-j|}, \ [\mathbf{R}_r]_{ij} = \left(\frac{1}{3}\right)^{|i-j|},$$
$$[\mathbf{T}_r]_{ij} = \left(\frac{1}{4}\right)^{|i-j|}, \ [\mathbf{R}_d]_{ij} = \left(\frac{1}{5}\right)^{|i-j|}.$$

In addition, the LOS components $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ are generated randomly.

The mean of the mutual information results is plotted versus SNR_d for $n_s = n_d = 2, 4, 8$ and $n_r = 2n_s$ in Figure 2, when $\mathsf{SNR}_s = 20$ dB and $\mathcal{K}_1 = \mathcal{K}_2 = 0$. In this figure, we show 1) the average mutual information $\mathsf{E}\{I\}$, which is obtained by 10,000 Monte Carlo simulations, and 2) its asymptotic

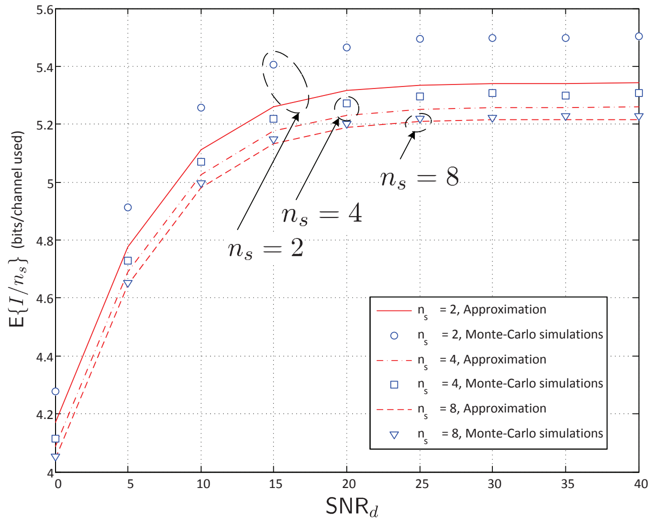


Fig. 2. Ergodic mutual information versus SNR_d for the MIMO relay channel with $\mathcal{K}_1 = \mathcal{K}_2 = 0$ and $n_r = 2n_s = n_d = 4, 8, 16$. The lines plot the analytical results, while the markers plot the exact results.

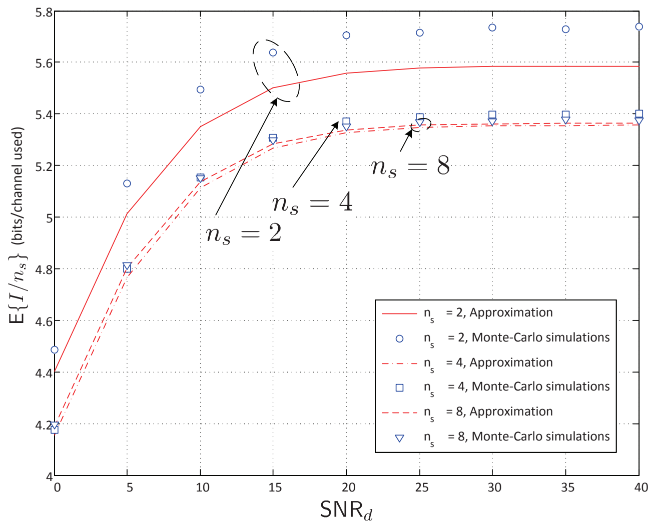
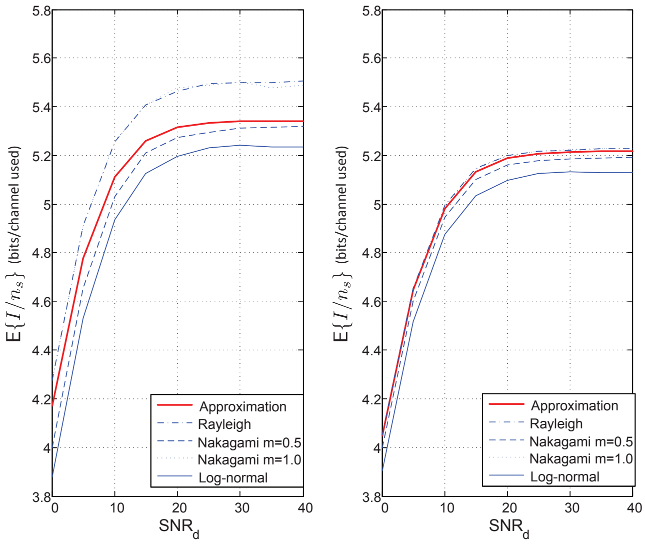
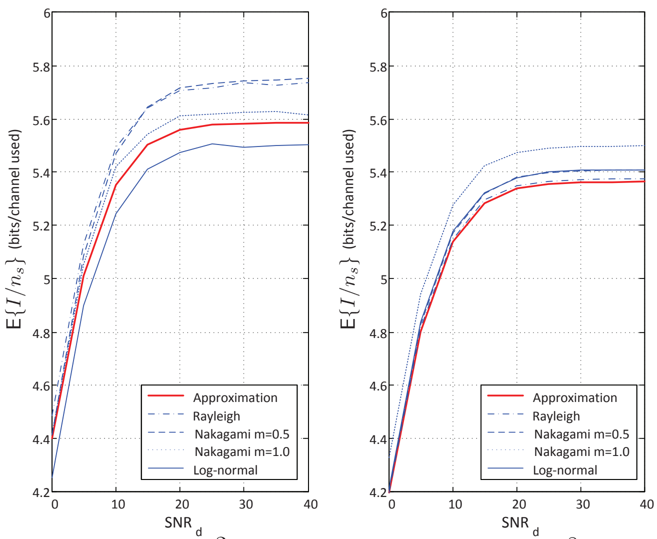


Fig. 3. Ergodic mutual information versus SNR_d for the MIMO relay channel with $\mathcal{K}_1 = \mathcal{K}_2 = 1$ and $n_r = 2n_s = n_d = 4, 8, 16$. The lines plot the analytical results, while the markers plot the exact results.



(a) $n_s = 2$ (b) $n_s = 8$

Fig. 4. Ergodic mutual information versus SNR_d for the MIMO relay channel under different types of fading distributions with $\mathcal{K}_1 = \mathcal{K}_2 = 0$.



(a) $n_s = 2$ (b) $n_s = 8$

Fig. 5. Ergodic mutual information versus SNR_d for the MIMO relay channel under different types of fading distributions with $\mathcal{K}_1 = \mathcal{K}_2 = 1$.

approximation (8). We see the trend that the approximation results come close to the simulation results asymptotically when the numbers of antennas increase. Under the same setting as the above simulations but with $\mathcal{K}_1 = \mathcal{K}_2 = 1$, Figure 3 illustrates the similar observation.

Recall that Proposition 1 is true even if the entries of $\mathbf{H}_1^{(w)}$ and $\mathbf{H}_2^{(w)}$ are non-Gaussian in the large-system regimen. Next, we intend to clarify if the mean of the mutual information approximation is also insensitive to different types of fading distributions when the number of antennas is not *so large*. For different types of fading distributions, $[\mathbf{H}_1^{(w)}]_{ij}$ (or $[\mathbf{H}_2^{(w)}]_{ij}$) is assumed to be of the form $W_{ij}\exp(j\theta_{ij})$, where θ_{ij} is the phase modeled as a uniform distribution over $[0,\, 2\pi]$, and W_{ij} is the amplitude fading drawn from a distribution with $\mathsf{E}\{W_{ij}^2\} = 1$. In Figures 4 and 5, we evaluate $\mathsf{E}\{I\}$ when W_{ij} is drawn from common probability density functions including the Rayleigh, Nakagami, or log-normal distributions [9, 10]. When the number of antennas grows large (e.g., $n_s = n_d = 8$, $n_r = 2n_s$) all of the curves tend to overlap regardless of the distributions. Thus, this invariance phenomenon of the ergodic mutual information in the large-system limit agrees with Proposition 2.

V. Conclusions

This paper investigated the approximated mutual information expression for the MIMO relay channel under the large-system assumption. Our result addressed the general MIMO relay model where the correlation matrices are generally nonnegative definite, an LOS component is presented on each link, and the channel entries are non-Gaussian distributed. The approximation result seems to provide a realizable estimate under all types of fading distribution. From the practical perspectives, the mean of the mutual information approximation is important to the design of capacity achieving input covariance matrix and the relay amplifying matrix. This is a promising future work.

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