An Improved LLR Approximation Algorithm for Low-Complexity MIMO Detection Towards Green Communications

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***Abstract*—As the number of transmit/receive antennas gets large in wireless communication systems, the drastically-increasing complexity in MIMO detection imposes significant challenges in implementing green communications while achieving high spectral efficiency. The winner-path-extension (WPE) K-best algorithm is an efficient detection algorithm in uncoded MIMO systems, known for its stable throughput and excellent symbol-error-rate (SER) and bit-error-rate (BER) performances under relatively low complexity. However, when applying the WPE K-best algorithm into coded MIMO systems, where soft-output information such as log-likelihood ratio (LLR) is required, missing counter-hypotheses issue in LLR calculation often degrades the error performance. To solve this problem, in this paper we propose an improved LLR approximation algorithm, such that WPE K-best algorithm can be well suited to coded MIMO systems. Specifically, when a counter-hypothesis misses, we set a metric threshold for the missing counter-hypothesis by calculating the metric of the bit flipping vector, and then randomly choose a value below the threshold as the approximation. We conduct simulation evaluations for our proposed algorithm in an 8×8 MIMO multiplexing system employing 16QAM modulation and Turbo coding. Simulation results show that compared with other existing LLR approximation schemes, our proposed approach can effectively improve the block-error-rate (BLER) performance as well as reducing the complexity in the tree search of WPE K-best algorithm. Moreover, we use a look-up table method to determine the Schnorr-Euchner (SE) enumeration order, which can further decrease the computational complexity of WPE K-best algorithms.**

***Index Terms*—WPE K-best algorithm; coded MIMO systems; LLR approximation; SE enumeration**

I. Introduction

Recently, green wireless communications have attracted wide attention towards the developing future wireless networks. MIMO technology plays a critically important role in green wireless communications in light of its high spectral efficiency. However, the high complexity required for MIMO detector also imposes great demands on energy consumption and causes the generation of more battery wastes. Although there have been several low-complexity MIMO detection algorithms, e.g., zero forcing (ZF) algorithm, minimum mean square error (MMSE) algorithm, and successive interference cancellation (SIC) algorithm, their error performances cannot meet the application requirement. Consequently, the quasi-maximum likelihood (ML) detectors have been gradually used in realistic systems. Quasi-maximum likelihood detectors based on tree-search can be categorized into breadth-first, depth-first and metric-first search algorithms. Sphere detection (SD) algorithm [1]-[2] is a depth-first tree-search algorithm. SD algorithm is not propitious for hardware implementation due to unstable throughput. On the other hand, metric-first stack algorithm [3] suffers from frequent back-search process and the high store complexity. As one of breadth-first search algorithms, WPE K-best algorithms [4]-[6] have heavy computation burden in coded MIMO systems, especially when K is large. These tree-search algorithms all need a ML hypothesis and a counter-hypothesis to calculate the LLR value for every output bit. However, the counter-hypotheses of certain bits may be missing due to pruning. As a consequence, the LLR values for certain bits cannot be calculated resulting in further performance degradation.

Some schemes have been proposed to solve the problem of missing counter-hypotheses. Sizhong Chen et al. [7] proposed to use the difference between the largest metric and the smallest metric to approximate the LLR or assign a predefined LLR, such as +6 and -6. In [8], a bit flipping method was proposed which flips the desired bit of the ML hypothesis. The advantage of the above three methods is that they do not have to expand the searching range. However, the BLER performance of these methods is not satisfactory. The smart candidate adding (SCA) algorithms were proposed in [9]-[12], which use an unconstrained search for ML hypothesis and multiple constrained searches for counter-hypotheses. In [13], the authors proposed to make use of the partial metric to compute the approximated LLR at every layer instead of at the last layer only. A compressing based transformation method was developed in [14], which can achieve significant performance improvement by compressing the dubious observation of the metrics. The contributions of our work include two folds. First, unlike the aforementioned algorithms, we propose an improved LLR approximation algorithm by applying the metric threshold of the missing counter-hypothesis. The proposed scheme can improve the accuracy of LLR values without

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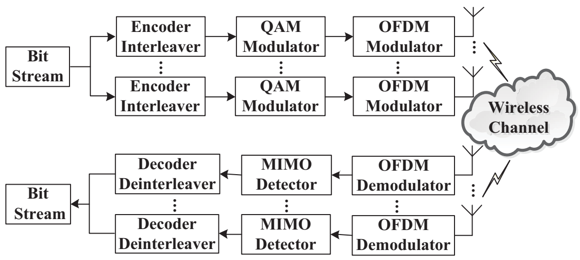

Fig. 1. Block diagram of the coded MIMO system

expanding the searching range. Second, a SE enumeration method is developed. These two approaches can effectively reduce the complexity of MIMO detectors as compared to the existing yet widely used WPE K-best algorithms, thus easing the implementation of MIMO detectors towards green communications in wireless networks.

The rest of this paper is organized as follows. In Section II, the MIMO system model is described. An improved LLR approximation algorithm and a look-up table method to determine the SE enumeration order are described in Section III. We analyze the complexity of our scheme and evaluate the error performance through simulations in Sections IV and Section V, respectively. The paper concludes with Section VI.

II. System Model

We consider a coded MIMO system as depicted in Fig. 1. The input bit stream is encoded and interleaved to become the coded bits, then the coded bits are mapping to the QAM constellation. The complex signals are orthogonal frequency division multiplexing (OFDM) modulated before transmitting. The length of cyclic prefix is longer than the largest multipath delay. The signal transmissions can be modeled by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is an $N_r \times 1$ signal vector received by the receive antennas, $\mathbf{H}$ is an $N_r \times N_\mathrm{t}$ channel matrix following the spatial channel model (SCM), and $\mathbf{n}$ is a noise vector whose elements are independent circularly symmetric complex Gaussian random variables with mean zero and variance σ^2. If we let

$$\begin{cases} \mathbf{y}_R &= \left[\mathrm{Re}^T(\mathbf{y}) \quad \mathrm{Im}^T(\mathbf{y})\right]^T; \\ \mathbf{H}_R &= \begin{bmatrix} \mathrm{Re}(\mathbf{H}) & -\mathrm{Im}(\mathbf{H}) \\ \mathrm{Im}(\mathbf{H}) & \mathrm{Re}(\mathbf{H}) \end{bmatrix}; \\ \mathbf{n}_R &= \left[\mathrm{Re}^T(\mathbf{n}) \quad \mathrm{Im}^T(\mathbf{n})\right]^T, \end{cases} \tag{2}$$

the complex-valued system model given in (1) can be written in a real-valued form as $\mathbf{y}_R = \mathbf{H}_R\mathbf{x} + \mathbf{n}_R$, where $\mathbf{x}$ is a real-valued transmission vector, $\mathbf{x} \in \mathbf{X}^{2N_t}$, and $\mathbf{X}$ is the set of symbols in the real-valued constellation, e.g. $\mathbf{X} = \{-3, -1, 1, 3\}$ in the case of 16QAM. In this paper, we let $2N_t = 2N_r = N$ for convenience. The objective of MIMO detection is to find a vector $\mathbf{x}_{\mathrm{ML}}$, such that its transformed vector $\mathbf{H}_R\mathbf{x}$ has the minimum Euclidean distance to the received vector $\mathbf{y}_R$, i.e.,

$$\mathbf{x}_{\mathrm{ML}} = \arg\min_{\mathbf{x}\in\mathbf{X}^N} \|\mathbf{y}_R - \mathbf{H}_R\mathbf{x}\|_2^2 \tag{3}$$

Let $\mathbf{p} = \mathbf{H}_R^{-1}\mathbf{y}_R$, $\mathbf{e} = \mathbf{p} - \mathbf{x}$, and $\mathbf{H}_R^T\mathbf{H}_R = \mathbf{R}^T\mathbf{R}$, where $\mathbf{R}$ is an upper triangular matrix, the ith element of $\mathbf{e}$ is e_i. Also, we set $\mathbf{Q}$ as an upper triangular matrix, where $\mathbf{Q}_{i,i} = \mathbf{R}_{i,i}^2$, $\mathbf{Q}_{i,j} = \mathbf{R}_{i,j}/\mathbf{R}_{i,i}$, an $i = 1, 2, ..., N$, $j = i+1, i+2, ..., N$. Eq. (3) can be rewritten as

$$\begin{aligned} \mathbf{x}_{\mathrm{ML}} &= \arg\min_{\mathbf{x}\in\mathbf{X}^N} \|\mathbf{H}_R(\mathbf{p}-\mathbf{x})\|_2^2 \\ &= \arg\min_{\mathbf{x}\in\mathbf{X}^N} \sum_{i=1}^{N} Q_{i,i}\left(e_i + \sum_{j=i+1}^{N} Q_{i,j}e_j\right)^2 \\ &= \arg\min_{\mathbf{x}\in\mathbf{X}^N} \sum_{i=1}^{N} D_i \end{aligned} \tag{4}$$

where $D_i = Q_{i,i}\left(e_i + \sum_{j=i+1}^{N} Q_{i,j}e_j\right)^2$ is the metric increment of the ith dimension.

Soft-output WPE K-best algorithm is one of breadth-first search algorithms. The detector keeps all the K best survivors as a candidate list, and the LLR of the bit c_i is calculated according to Eq.(5) if there is no priori information:

$$\mathrm{LLR}(c_i|\mathbf{y}) = \log\frac{P\left[c_i = 1|\mathbf{y}\right]}{P\left[c_i = 0|\mathbf{y}\right]} \tag{5}$$

With Max-log approximation [2], Eq.(5) can be written as

$$\begin{aligned} &\mathrm{LLR}(c_i|\mathbf{y}) \\ &\approx \min_{\mathbf{x}\in L_i^0}\left\{\frac{\|\mathbf{y}_R - \mathbf{H}_R\mathbf{x}\|_2^2}{\sigma^2}\right\} - \min_{\mathbf{x}\in L_i^1}\left\{\frac{\|\mathbf{y}_R - \mathbf{H}_R\mathbf{x}\|_2^2}{\sigma^2}\right\} \end{aligned} \tag{6}$$

where L_i^0 and L_i^1 represent all vectors with the bit c_i being 0 and 1, respectively. If we call the vector which has the minimum metric as ML hypothesis and the vectors which have at least one different bit from the ML hypothesis as counter-hypotheses, then the LLR value is the difference between the metric of the ML hypothesis and that of the counter-hypothesis. However, in the soft-output WPE K-best algorithm, if K is not large enough, we may find sometimes all the candidates have the same bit at a certain position, which is called missing counter-hypotheses problem. Now we can't calculate the LLR values using Eq. (6). An improved LLR approximation algorithm will be proposed to solve this issue.

III. Low-complexity soft-output WPE K-best Algorithm

WPE K-best algorithm expands the search tree using WPE method. When cooperating with accurate SE enumeration order, WPE method can decrease the complexity of K-best algorithm (see [4] for more details). In order to further reduce the complexity of WPE K-best algorithm, we propose a look-up table method to determine the SE enumeration order and an improved LLR approximation method, which are elaborated in Section III-A and Section III-B, respectively.

A. Determination of Accurate SE Enumeration Order

We can express the metric increment at the ith layer as

$$D_i = Q_{i,i}\left(p_i + \sum_{j=i+1}^{N} Q_{i,j}e_j - x\right)^2 = Q_{i,i}(S_i - x)^2 \tag{7}$$

TABLE I
SE ENUMERATION ORDER TABLE FOR 16QAM

-3	-1	1	3
-1	-3	1	3
-1	1	-3	3
1	-1	3	-3
1	3	-1	-3
3	1	-1	-3

where $S_i = p_i + \sum_{j=i+1}^{N} Q_{i,j} e_j$ is called extension center similar to [15], it is easy to find that the metric increment D_i is determined by the distance between S_i and x. In order to extend the node which has the minimum metric increment firstly, we need to find the constellation point x which is nearest to S_i. So, the accurate SE enumeration order is to sort the constellation points by their distance to S_i ascendingly, and a table similar to [16] is used to store the possible SE enumeration order. Let the real-valued constellation set of M-QAM be expressed as

$$\Omega = \{-(\sqrt{M}-1), -(\sqrt{M}-3), \ldots, -1, \\ 1, 3, \ldots, \sqrt{M}-3, \sqrt{M}-1\}, \tag{8}$$

the row number for lookup table can be calculated using Eq. (9), where $\lceil . \rceil$ denotes the ceiling function. Taking 16QAM as the typical example, Eq.(9) can be simplified as Eq.(10) as follows, and the accurate SE enumeration order is given in Table I, where

$$rn_i = \begin{cases} 1, & if\ S_i \le -\sqrt{M}+2; \\ 2(\sqrt{M}-1), & if\ S_i \ge \sqrt{M}-2; \\ \lceil S_i \rceil + (\sqrt{M}-1), & \text{otherwise.} \end{cases} \tag{9}$$

This new method can be used in all M-QAM modulations, and the row number is easy to determine with two comparison operations at most. Therefore, the proposed method has a lower complexity than the algorithm in [15]. The SE enumeration order determined in [16] is inaccurate due to the inaccuracy of ZF solution. However, the SE enumeration order determined by the proposed method is accurate and can guarantee that the metric increment changes from small to large.

$$rn_i = \begin{cases} 1, & if\ S_i \le -2; \\ 6, & if\ S_i \ge 2; \\ \lceil S_i \rceil + 3, & \text{otherwise.} \end{cases} \tag{10}$$

B. Proposed LLR Approximation Algorithm

If K is not large enough, soft-output WPE K-best algorithm may suffer from missing counter-hypotheses problem when calculating the LLR values after the tree search. We don't know the accurate metric of the missing counter-hypothesis, but we can get the metric of the bit flipping vector by flipping the desired bit of ML hypothesis. Because there is correlation between signal layers, there will most likely exist a better candidate satisfying the bit requirement [9]. So we define the metric of the bit flipping vector as the upper bound of the metric of the missing counter-hypothesis. Different from other works, this paper firstly considers the metric threshold of the missing counter-hypothesis, and then approximates the LLR by making use of the determined threshold.

TABLE II
AN IMPROVED LLR APPROXIMATION ALGORITHM

Step 1: Getting the bit flipping vector by flipping the desired bit of the ML hypothesis, and calculating the metric of the bit flipping vector, expressed as γ.

Step 2: If $\gamma > \gamma_K$, we can select a metric γ_s between γ_K and γ randomly, let $\mathrm{LLR}(c_i|y) = \begin{cases} (\gamma_1 - \gamma_s)/\sigma^2, & c_i^{\mathrm{ML}} = 0 \\ (\gamma_s - \gamma_1)/\sigma^2, & c_i^{\mathrm{ML}} = 1 \end{cases}$.

Step 3: If $\gamma \le \gamma_K$, though this condition happens with low probability, let $\mathrm{LLR}(c_i|y) = \begin{cases} (\gamma_1 - \gamma)/\sigma^2, & c_i^{\mathrm{ML}} = 0 \\ (\gamma - \gamma_1)/\sigma^2, & c_i^{\mathrm{ML}} = 1 \end{cases}$.

Step 4: For all LLR, if $\mathrm{LLR} > 6$, then let $\mathrm{LLR} = 6$; If $\mathrm{LLR} < -6$, then let $\mathrm{LLR} = -6$.

We can use the steps in Table II to approximate the LLR values. Suppose the metrics of K best candidate vectors which can get from tree search are $\gamma_1, \gamma_2, \ldots, \gamma_K$ from small to large, and the metric of the bit flipping vector is γ. If $\gamma > \gamma_K$, we consider the metric of the missing counter-hypothesis is between γ and γ_K, so we select a metric γ_s between γ and γ_K randomly as the metric of the missing counter-hypothesis. If $\gamma \le \gamma_K$, though this condition happens with low probability, we use γ as the metric of the missing counter-hypothesis. As we may overestimate the metrics of all the counter-hypotheses, we clip all the LLR values at a magnitude of ± 6.

Although the proposed LLR approximation method can not determine the accurate metrics of the missing counter-hypotheses, this method considers the metric thresholds of the missing counter-hypotheses. Compared to other methods, this approximation method can achieve better performance.

C. Calculation of the Metric for the Bit Flipping Vector

As the bit flipping vector only has one symbol different from the vector of ML hypothesis, we can use the following low-complexity scheme to calculate the metric of the bit flipping vector. The metric of a vector $\mathbf{x}$ can be expressed as

$$\|\mathbf{y}_R - \mathbf{H}_R\mathbf{x}\|_2^2 = \sum_{i=1}^{N} \left(y_{R_i} - \sum_{j=1}^{N} H_{Ri,j} x_j \right)^2. \tag{11}$$

To reduce the complexity of calculating the bit flipping vector, we first calculate matrix $\mathbf{H}'$ and vector $\mathbf{A}$ using ML hypothesis vector $\mathbf{x}_{\mathrm{ML}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}^T$. $\mathbf{H}'$ is an $N \times N$ matrix and $\mathbf{A}$ is an $N \times 1$ vector

$$H'_{i,j} = H_{Ri,j} x_j, \tag{12}$$

$$A_i = \sum\nolimits_{j=1}^{N} H_{Ri,j} x_j, \tag{13}$$

where $i = 1, 2, \cdots, N, j = 1, 2, \cdots, N$. If the bit flipping vector is $\mathbf{x'}_{\mathrm{ML}} = \begin{bmatrix} x_1 & x'_2 & \cdots & x_N \end{bmatrix}^T$. We can calculate the metric of the bit flipping vector using Eq. (14), as

$$\gamma' = \sum_{i=1}^{N} \left(y_{Ri} - A_i + H'_{i,2} - H_{Ri,2} {x'}_2 \right)^2. \tag{14}$$

By making use of the relation between ML vector and the bit flipping vector, the complexity of calculating the metric of the bit flipping vector is little.

IV. Complexity Analysis

The computational complexity of MIMO detectors includes metric computation, LLR approximation and sorting operations. The computational complexity is measured using the number of real-valued operations, such as addition and multiplication. Sorting complexity is measured by the number of compare operations, and bubble sort algorithm is adopted for its fixed complexity. To select m smallest numbers out of n candidates, if $m < n$, bubble sort algorithm needs $(2n - m - 1)m/2$ compare operations; if $m = n$, the number of compare operations is $(n-1)n/2$. The cost of one compare operation is similar to one addition operation. So we add the number of compare operations to the number of addition operations.

In this paper, we use BLER to measure the performance of coded MIMO systems. We compare the performance and complexity of three LLR approximation algorithms. The first algorithm uses the difference between the largest metric and the smallest metric to approximate the LLR values, we call it DLS method [7]. The second algorithm assigns a predefined clipping value, such as +6 and -6, as the approximated LLR. The third method is the proposed LLR approximation algorithm that will be referred to as random selection (RS) method, which randomly selects a metric below the determined metric threshold as the approximated metric of missing counter-hypothesis.

The complexity of clipping method is equal to the complexity of DLS method, as when calculating the LLR values, both these two methods don't need extra complexity. But the proposed RS method need to calculate the metrics of bit flipping vectors. However, the extra complexity of RS method is low by making use of the relation between ML vector and the bit flipping vectors. And different from calculating the metrics during the tree-search process, calculating the metrics of the bit flipping vectors doesn't need sorting operations. By applying the metric threshold of the missing counter-hypotheses, RS LLR approximation method can achieve better approximated performance. Then WPE K-best algorithm with RS method need a smaller K value than other methods when achieving the same BLER performance. From Section V, we can see the proposed RS LLR approximation algorithm has a lower complexity than other LLR approximation methods without BLER performance degradation.

V. Simulation Evaluations

In this paper, a coded MIMO system is considered and the simulation parameters are given in Table III. The detector runs only once, i.e. we do not employ iterative detection-decoding.

TABLE III
SIMULATION PARAMETERS

Number of transmit/receive antennas	8/8
Modulation type	16QAM
Channel model	SCM (six path)
Propagation scenario	Urban micro
Base/mobile station arrangement	10λ/ 0.5λ spacing
Channel estimation	Ideal
Channel coding/decoding	Turbo (1/3 rate)/Max-log-Map

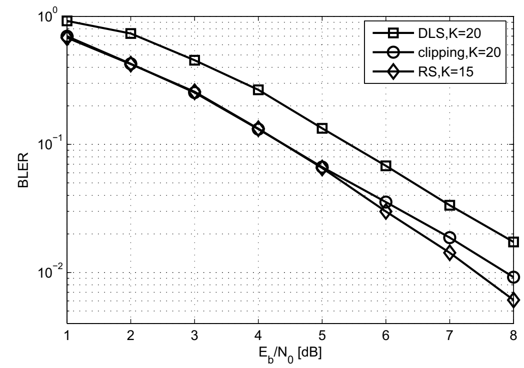

Fig. 2. BLER of clipping, DLS and RS methods when K=15 and 20

All the algorithms compared in this paper employ a sorted QR decomposition for preprocessing.

Fig. 2 shows the BLER performance comparison of WPE K-best algorithm with clipping, DLS and RS LLR approximation methods when K=15 and 20. As the phenomenon of missing counter-hypothesis happens randomly, we compare the complexity of different LLR approximation algorithms in a statistic way. The number of additions and multiplications at different E_b/N_0 are compared in Fig. 3 and Fig. 4, respectively. From Fig. 2, it is seen that clipping approximation method has better BLER performance than DLS method for the same K=20. The proposed RS approximation algorithm with $K = 15$ achieves better BLER performance than other two approximation algorithms with $K = 20$. The complexity comparison at this performance is recorded in Table IV (E_b/N_0=5dB). From Table IV, we can see RS method with K=15 leads to a reduction of 22.56% addition operations and 1.04% multiplication operations compared to clipping or DLS approximation methods with K=20.

Fig. 5 shows the BLER of clipping, DLS and RS LLR approximation methods when K=15, 20 and 30. Compared to clipping method with K=30, RS approximation algorithm with $K = 15$ has a performance loss less than 0.2dB, however, RS method leads to a reduction of 58.95% addition operations and 31.54% multiplication operations as showed in Table IV. From Fig. 5, it is seen that the proposed RS approximation method with K=20 has the best BLER performance. Compared to clipping or DLS approximation methods with K=30, RS method with K=20 has a reduction of 41.17% addition operations and 16.13% multiplication operations. Finally, we conclude that the proposed RS LLR approximation algorithm can both reduce the complexity of WPE K-best algorithm and

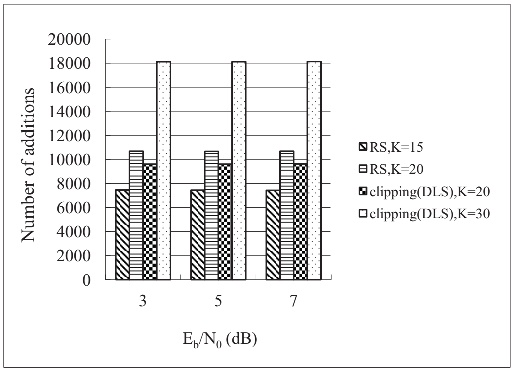

Fig. 3. The number of additions in WPE K-best algorithm with different LLR approximation algorithms

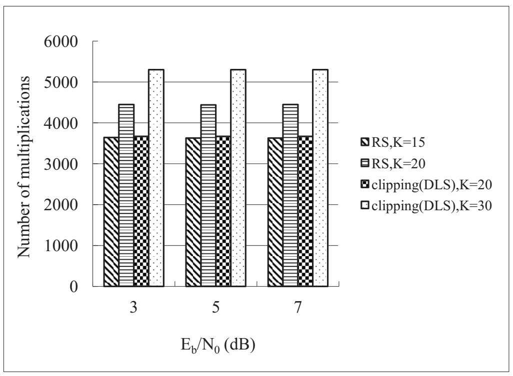

Fig. 4. The number of multiplications in WPE K-best algorithm with different LLR approximation algorithms

improve the BLER performance.

VI. Conclusions

In this paper, an improved LLR approximation algorithm was proposed to solve the problem of missing counter-hypotheses. Moreover, we also used a look-up table method to determine the SE enumeration order, which can further reduce the computational complexity of WPE K-best algorithm for MIMO detection. From simulation results, we can see that the proposed LLR approximation algorithm can improve the BLER performance of soft-output WPE K-best algorithm and decrease the computational complexity at the same time, thus easing the implementation of MIMO detectors towards green communications. Furthermore, this improved LLR approximation algorithm can be applied to other MIMO detectors which use a candidate list to calculate the LLR values, such as M algorithm and semi-definite relaxation algorithm.

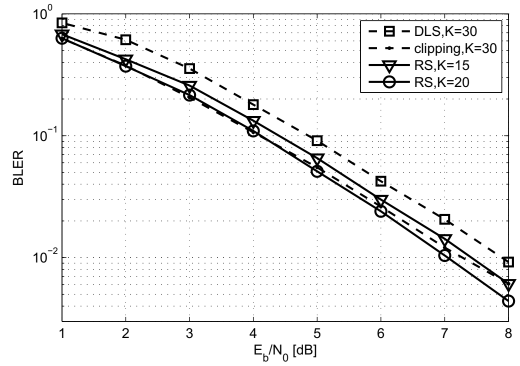

Fig. 5. BLER of clipping, DLS and RS methods when K=15, 20 and 30

TABLE IV
Complexity reduction of scheme B when compared to Scheme A

Scheme A	clipping/DLS, K=20	clipping/DLS, K=30	
Scheme B	RS, K=15	RS, K=15	RS, K=20
Addition	22.56%	58.95%	41.17 %
Multiplication	1.04%	31.54%	16.3%

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