Power Grid Vulnerability Measures to Cascading Overload Failures

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***Abstract*—Cascading failure in power grids has long been recognized as a sever security threat to national economy and society, which happens infrequent but can cause severe consequences. The causes of cascading phenomena can be extremely complicated due to the many different and interactive mechanisms such as transmission overloads, protection equipment failures, transient instability, voltage collapse, etc. In the literature a number of vulnerability measures to cascading failures have been proposed to identify the most critical components in the grid and evaluate the damages caused by the removal of such recognized components from the grid. In this paper we propose a novel power grid vulnerability measure called the *minimum safety time* after 1 line trip, defined based on the stochastic cascading failure model[1]. We compare its performance with several other vulnerability measures through a set of statistical analysis .**

I. Introduction

The future smart grid will incorporate more intermittent and volatile renewable generation and include more greater and sophisticated demand side participation. The system operator faces constant pressure to handle the generation/load uncertainties and maintain an economic and reliable operation. Under these circumstances, it would ideal to have some effective vulnerability measure to help recognize the most critical or stressed part in the grid so that the system operator can monitor and interpret what is going on in the system accurately, therefore enhance the system robustness by steering away from potential cascading threats. In the literature, many such measures have been proposed, such as *high traffic*, *high flow increase*, *high betweenness*,[2] [3]. Most of these measures were defined on the basis of deterministic grid operating conditions and network topology. In [1] we introduced a stochastic cascading failure models taking into account the statistical properties of generation/load, from which a novel vulnerability metric called the *expected safety time* is defined to evaluate by average how long a transmission line will stay connected given current operating conditions and network state. In this paper, we propose a novel power grid vulnerability measure called the *minimum safety time* after 1 line trip,and compare its performance with three other vulnerability measures through a statistical numerical analysis based on the stochastic cascading simulation model of [1].

The rest of the paper is organized as follows: Section II introduce the power grid model and the stochastic cascading analysis; Section III gives definitions of the proposed vulnerability measure and three other measures we are going to examine and compare; Section IV presents some numerical statistical results to compare the performance of all the examined measure; and Section V concludes the paper.

II. Power Grid Model and the Stochastic Analysis of Cascading Overload Vulnerability

A stochastic cascading failure models was introduced in [1] which incorporates the statistics of the generation and loads in the grid therefore to derive the statistics of the line flow process. For the tractability of the problem, we use the *DC power flow* approximation to characterize a power grid network, which is a standard approach widely used in optimizing flow dispatch and for assessing line overloads [4]. Consider a power grid transmission network with n nodes interconnected by m transmission lines, the network flow equation can be written as follows:

$$\begin{aligned} P(t) &= B'(t)\theta(t), \\ F(t) &= diag\left(y_l(t)\right)A\theta(t) \end{aligned} \tag{1}$$

where $P(t)$ represents the vector of injected real power, $\theta(t)$ the phase angles, and $F(t)$ the flows on the lines. The matrix $B'(t)$ is defined as

$$B'(t) = A^T diag\left(y_l(t)\right)A, \tag{2}$$

where $y_l(t) = s_l(t)/x_l$ with x_l the line reactance and $s_l(t)$ the line state; $s_l(t) = 0$ if line l is tripped, and $s_l(t) = 1$ otherwise; $diag\left(y_l(t)\right)$ represents a diagonal matrix with entries of $\{y_l(t),\ l = 1, 2, \cdots, m\}$. $A := (A_{l,k})_{m\times n}$ is the line-node incidence matrix, arbitrarily oriented, defined as: $A_{l,i} = 1$; $A_{l,j} = -1$, if the l^{th} link is from node i to node j and $A_{l,k} = 0, k \neq i, j$. The vector of line states, $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_m(t)]^T$ with $s_l(t) \in \{0,\ 1\}$, is defined as the network state.

The operating condition of the grid, represented by the real power injection $P(t) = [G(t)^T \quad -L(t)^T]^T$, where $G(t)$ is the generation and $L(t)$ is the load portion, assumes to be a conditionally Gaussian multivariate random process, given the network state of $\mathbf{s}(t)$. The probability density function of

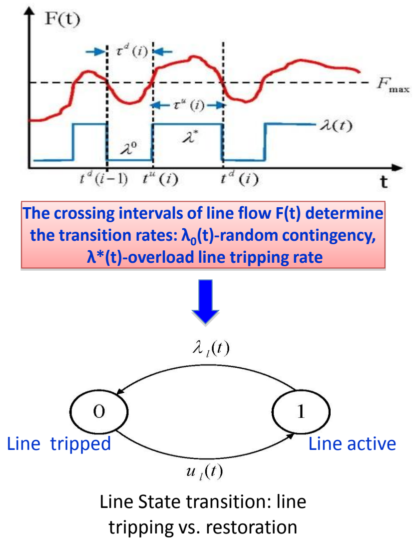


Fig. 1. The flow process and the line state transition

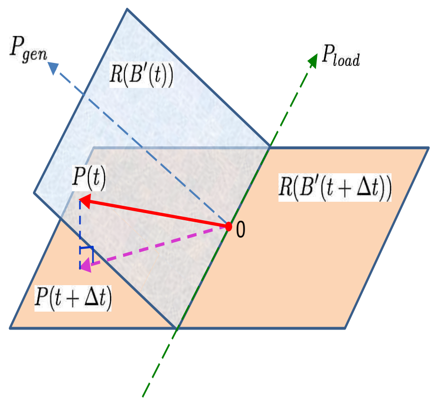


Fig. 2. The power injection adjustments to keep the system balanced

$\{P(t)|\mathbf{s}(t)\}$ is, therefore, fully specified giving its conditional mean and covariance, denoted as follows:

$$\mu_P(t) = \left[\begin{array}{c} \mu_g(t) \\ -\mu_l(t) \end{array} \right], C_P(t) = \left[\begin{array}{cc} \Sigma_{gg}(t) & \Sigma_{gl}(t) \\ \Sigma_{lg}(t) & \Sigma_{ll}(t) \end{array} \right]. \quad (3)$$

The time dependence is due to the fact that the random process is intrinsically non-stationary, e.g., the generation settings in the power grid are adjusted periodically to balance the loads during the day. The covariance reflects the uncertainty in the load/generation settings coming from multiple sources such as the forecasting deviation, the measurement errors, and the volatility caused by demand response and renewable generation.

We can then compute the statistics of line flow process as

$$\begin{array}{l} \mu_F(t) = \sqrt{y_t}(\widetilde{A}_t^T)^\dagger \mu_P(t) \\ C_F(t) = \sqrt{y_t}(\widetilde{A}_t^T)^\dagger C_P(t)(\widetilde{A}_t)^\dagger \sqrt{y_t}, \end{array} \quad (4)$$

with $\widetilde{A}(t) = \sqrt{y(t)}A = \mathbb{USV}^T$ and $\sqrt{y_t} = diag\{\sqrt{y_l(t)}\}$. Note that during a cascading process, in some cases the grid network may become disconnected and some necessary power adjustments have to be made to keep the system balanced. In this model we assume the primary generation and load controls in the grid could take a least-squared adjustment within their capacity limit. Here we use Fig. 2 to illustrate the process: as indicated by the system's power flow equation (1), the power injection vector $P(t)$ needs to stay in $R(B'(t))$ the range space of the grid's $B'(t)$ in order to satisfy the power balance constraints. Therefore when some line outage(s) in the grid steers away the range space to $R(B'(t+\Delta t))$, we need to project the power injection vector to $P(t+\Delta t)$, back into the new range space along the shortest path which corresponds to the least-squared power adjustment.

A line is considered as *overloaded* if the power flow through it exceeds the line limit determined by its thermal capacity or static/dynamic stability conditions, i.e., which is called the line's *overload threshold*, where $F_l^{\max}$. Therefore the normalized overload distance for a line flow can be written as $a_l = \left(F_l^{\max} - \mu_{F_l(t)}\right)/\sigma_{F_l(t)}$ with $\sigma_{F_l} = \sqrt{C_F(l,l)}$. And its overload probability can be approximated as $\rho_l(t) \approx \mathcal{Q}(a_l)$.

The persistent overload condition may cause a line to trip shortly, consequently, a transition in the state $\mathbf{s}(t)$. Fig.1 illustrates a line flow process for which two kinds of sojourn intervals can be defined: the *overload intervals* (when the flow magnitude stays above its threshold: $|F_l(t)| \geq F_l^{\max}$), and the *normal-load intervals* (when $|F_l(t)| \leq F_l^{\max}$), which are associated with an overload and normal-load line-tripping rate respectively, denoted as λ_l^* and λ_l^0. Obviously $\lambda_l^* \ll \lambda_l^0$. Assuming that $F_l(t)$ is Gaussian and differentiable, one can compute the average level crossing rate (i.e. the expected number of crossings of the threshold $F_l^{\max}$ in either direction) [5] as $\gamma_l = \frac{W}{\pi} e^{-a_l^2/2}$, where $W = \sqrt{-R''_{F_l}(0)/R_{F_l}(0)}$ is the equivalent bandwidth.

The probabilistic distribution of crossing intervals and line states can then be derived by using the Rice's results on Gaussian random process [6] and compute the expected safety time for each line to stay connected given current network topology and operating state. More details of the derivation can be found in [1]. Given the overload probability ρ_l of the l-th line, the level crossing density γ_l at $F_l^{\max}$, and the line tripping rates λ_l^* for an overload line and λ_l^0 caused by random contingencies, the expected number of crossings after which the l^{th} line finally gets tripped is $\bar{\kappa}_l = [(1+\beta_l) - (\beta_l - \alpha_l)\rho_l]/(1-\alpha_l\beta_l)$, and the expected life-time of the line is:

$$\mathcal{T}_l = (\bar{\kappa}_l - 1)/\gamma_l + E\{\Delta t_l\}, \quad (5)$$

where $E\{\Delta t_l\}$ is the mean duration of the *last* interval $E\{\Delta t_l\} = \left[\Delta T_l^* + \Delta T_l^0\right]/(1-\alpha_l\beta_l)$, with $\alpha_l = E\left\{p\left\{s_l\left(t_l^d(i)\right) = 1\right\}\right\}$, $\beta_l = E\left\{p\left\{s_l\left(t_l^u(i)\right) = 1\right\}\right\}$, $\Delta T_l^* = (1-\alpha_l)\left[\beta_l + (1-\beta_l)\rho_l\right]/\lambda_l^*$ and $\Delta T_l^0 = (1-\beta_l)\left[1-(1-\alpha_l)\rho_l\right]/\lambda_l^0$.

III. Definition of the Vulnerability Measures

In this paper we wish to evaluate and compare the damaging impacts when some *critical line* in the grid is lost. First step is to define an effective measure to recognize such critical lines.

Based on the Markovain-transition cascading failure model proposed in [1], we define an effective novel vulnerability measured of the *minimum safety time*.

A. The Minimum Safety Time

Given a specific operating condition, $\mathcal{T}_l$ is a safety metric for the line, and $\min_l \mathcal{T}_l(t_0)$ can be used as a global metric for the safety of the grid under current network condition, while the line at greatest risk can be identified as $l^* = \arg\ \min_l \mathcal{T}_l$. If the network transfers from one state to another, however, the $\mathcal{T}_l$'s all change, because the flows will be redistributed, and most likely this minimum shrinks further. We wish to examine how a line-outage contingency affects the system's minimum line safety time. That is, with only 1 line tripped, e.g., the k-th line in the network, to evaluate the minimum safety time of the remaining system as follows:

$$\min_l \mathcal{T}_l|_k = \min_l \{\mathcal{T}_l |\ s_k(0){=}0, s_{i\neq k}(0){=}1\}. \qquad (6)$$

And the corresponding vulnerability measure is to identify the line whose outage will cause the smallest $\min_l \mathcal{T}_l|_k$.

B. Other Vulnerability Measures

In the literature there are other measures such as *high traffic*, *high flow increase*, and *high betweenness*, whose definitions are listed as follows.

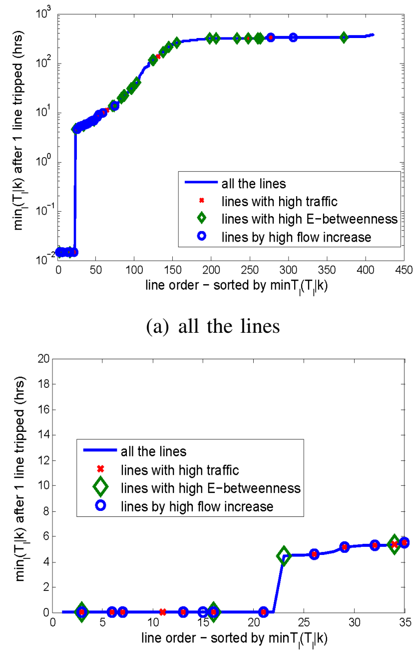
1) High Traffic: The criterion of *high traffic* is to select the lines which carries the largest flows in the original network.

2) High Flow Increase: The criterion of *high flow increase* after tripping is to compute the flow increase after 1 line tripping by using the line outage distribution factor factor (LODF) [7] and select the lines tripping which will cause the largest flow increase in the remaining network.

3) High E-Betweenness: High betweenness is to select the lines which connects most shortest-paths among all the possible pairs of nodes in a network. This centrality measure used to a purely graphic one, i.e., fully dependent on the network topology. [2] improved this graphic measure by defining "electrical" distance so as to incorporate the transmission line impedance into the definition. Henceforth the improved criterion we name as *high E-betweenness*.

C. The minimum safety time after one critical line lost

In this paper we used the IEEE 300 bus system to examine and compare the performance of different vulnerability measures. The IEEE 300 bus system is synthesized from the New England power system and has a topology with 300 nodes and 411 links. The initial operating equilibrium and conditions are taken and derived from the power flow solution of the system data from [8]. Taking the mean of $P(0)$ as $\mu_P(0) = [G(0)^T, -L(0)^T]^T$, we set the standard deviation of the loads as $\sigma_L = 0.07|L(0)|$, but ignore the variance in G. For simplicity, we assume that the loads and generation are statistically independent of one other. The line overload thresholds are set as $F^{\max} = 1.20|F(0)|$. Here we take $F(0)$ as the rational flow distribution under normal operating conditions and assume that the line capacity allows a 20%



(a) all the lines

(b) the most critical lines

Fig. 3. The minimum safety time after 1 line tripped: the IEEE-300 system

load increase [9]. The overload and normal-load line tripping rates are set as $\lambda^* = 1.92 \cdot 10^{-2}$Hz and $\lambda^0 = 7.70 \cdot 10^{-11}$Hz respectively. Analysis on the load record from realistic power grids [10] has shown that the load process can be approximated as a low-pass Gaussian process with an equivalent bandwidth of $W \approx 10^{-5}$Hz. Since the flow process in a grid can be seen as a linear projection from the load process, we can apply the equivalent bandwidth W to the flow processes in the grid.

Fig. 3 demonstrates all the $\min_l \mathcal{T}_l|_k$'s after 1-line trip sorted according to the magnitude. It can be seen that for about 50% of the lines in the network, if an $(n-1)$ contingency happens, the system's minimum safety time will not be noticeably changed. However, if the 1-line tripping occurs upon the other portion in the network, the system safety will be worsened more or less. Most interestingly, there is a set of 22 lines, denoted as $\mathcal{L}$, in the network that can be classified as extremely "critical", under current specific condition. If one line trips, a second line will trip on average within minutes, which may indicate an imminent cascading process. Fig. 3 also indicates the $\min_l \mathcal{T}_l|_k$ corresponding to the first most vulnerable 22 lines located by the three other vulnerability measures as mentioned above. It shows that by utilizing the criterion of high flow increase after 1 line tripping, one is able to locate 7 out of the 22 "critical" lines in $\mathcal{L}$; by utilizing the criterion of high traffic, one is also able to locate 7 "critical" lines from $\mathcal{L}$; while utilizing the criterion of high electrical betweenness, is most ineffective, only able to capture 2 critical lines from $\mathcal{L}$.

The analysis in the paper has indicated that the overall vulnerability of a power grid network in terms of cascading overload failures depends on the following four factors: (1)

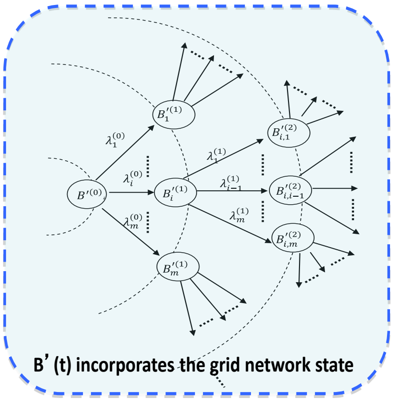
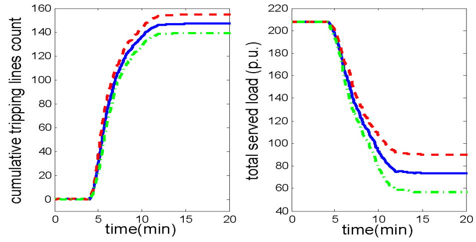


Fig. 4. The transition of the grid network state in $B'(t)$

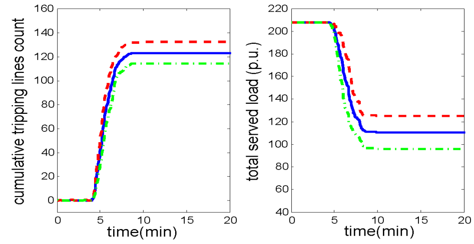
the network condition which includes the connecting topology, and the line impedances, (2) the line capacities; (3) the operating condition which includes the generation dispatch and load settings; and (4) the statistics of the line flows which can be derived from the statistics of injected power. The criterion based on the minimum safety time has taken into account of all four factors therefore it reflects the system vulnerability with most accuracy. Both the criterion of *high flow increase* after 1 trip and of *high traffic* consider only (1) and (3); however, the former takes one step further to compute the post-contingency flow redistribution therefore is able to locate a few more most critical lines than the latter. In contrast, the criterion of *high electrical betweenness* only considers (1) regardless of specific operating conditions, line capacities, or flow statistics, hence it is ineffective in locating the critical lines in terms of vulnerability. At the same time we can say the other criterions such as high nodal degrees, defined based on only topological information of power grid networks, will have very limited ability to locate the really critical components.

IV. Simulations Experiments

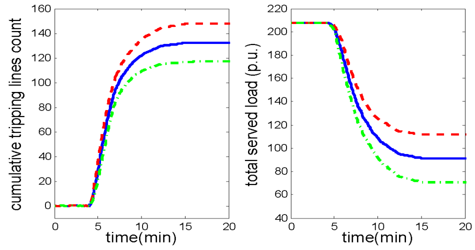
We also perform Monte Carlo experiments on the IEEE 300 bus system to evaluate the statistical impacts by using the Markovian-transition model developed in [1], as shown in Fig.4. First five most important lines have been identified by using each of the four vulnerability measures discussed in section III. Then a Cascading simulations can be started by tripping one of such lines in the system. Each experiment is repeated 10 times and the statistical results are aggregated for each measure to compare the damaging impacts. Fig. 5 shows the mean values of the cumulative line loss and the resulting load served during the cascading evolution process where $1\ p.u. = 100MW$. The red and green dashed lines in the figure show the half standard deviation range of the corresponding statistical results. It is found that removal of these critical lines recognized by the four different measures will all trigger cascading overload failures in the system, but with different extents of damages in the system. terms of the . Table I summarize the average cumulative line outages and load unserved after the cascading process calms down. The comparison shows that removal of the lines picked by the *minimum safety time* $\min_l \mathcal{T}_l|_k$ causes the most severe loss in the system, about 147 (35.8% of the total) lines outages and 134.3 p.u. load (65% of the total) loss by average; removal of the lines picked up *high E-betweeness* causes the least severe damages among the group, about 103 (25%) line outages and 65.6 p.u. load (31.6% of the total) unserved. The damage extents cause by the lines specified by *high flow increase* and *high traffic* lie in the middle with the former a little bit heavier than the latter. These simulation results are consistent with the analysis regarding the effectiveness of each vulnerability measure in the last section.



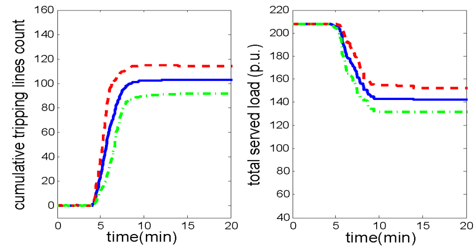
(a) $\min_l \mathcal{T}_l|_k$



(b) high traffic



(c) high flow increase



(d) high E-betweenness

Fig. 5. The cascading process in the IEEE-300 system triggered by tripping one of the most critical lines identified the four vulnerability measures as indicated below each sub-figure: the mean value of line outages and load served (blue); the range of half standard deviations (red and green).

TABLE I

THE AVERAGE LINE OUTAGES AND LOAD UNSERVED IN THE IEEE 300 BUS SYSTEM AFTER CASCADING PROCESS TRIGGERED BY REMOVAL OF A CRITICAL LINE IDENTIFIED BY DIFFERENT VULNERABILITY MEASURES

measures	total line outages (with percentage)	total load unserved (with percentage)
$\min_l \mathcal{T}_l\|_k$	147 (35.8%)	134.3(65%)
high traffic	123 (29.9%)	97.4(46.9%)
high flow increase	132 (32.2%)	116.4 (56.0%)
high E-betweenness	103 (25%)	65.6 (31.6%)

V. CONCLUSIONS

In this work we propose a power grid vulnerability measure called the *minimum safety time* after 1 line trip, $\min_l \mathcal{T}_l|_k$, which is defined based on the stochastic cascading-failure model [1] and compare its performance with three other measures, i.e., *high traffic*, *high flow increase*, *high E-betweenness*. It is found that the $\min_l \mathcal{T}_l|_k$ is the most effective measure to recognize the critical lines in a system whose loss may trigger an imminent cascading process. The reason for its effectiveness is because the proposed measure has integrated all the four factors that relates with the grid's overall vulnerability to cascading overload failures, namely the network condition which includes the connecting topology, and the line impedances, the line capacities; the operating condition which includes the generation dispatch and load settings; and the statistics of the line flows. While the other three only consider one or two of the listed factors. Simulation experiments have been performed on the IEEE 300 bus system and the statistical results also verified the effectiveness comparison of the vulnerability measures we discussed.

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