

Area Spectral Efficiency of Cooperative Network With DF and AF Relaying

Lei Zhang

Dept. of Elec.& Comp. Engr.
University of Victoria
Victoria, BC, V8W 3P6, Canada
Email: leizhang@ece.uvic.ca

Hong-Chuan Yang

Dept. of Elec.& Comp. Engr.
University of Victoria
Victoria, BC, V8W 3P6, Canada
Email: hy@uvic.ca

Mazen O. Hasna

Dept. of Elec. Engr.
Qatar University
Doha, Qatar
Email: hasna@qu.edu.qa

Abstract—Most performance metrics for cooperative networks focus on the qualification of either spectrum efficiency or link reliability, without considering the spatial effect of radio transmission. Area spectral efficiency (ASE) was first introduced to qualify the spatial spectral utilization efficiency of cellular systems. In this paper, we generalize the definition of ASE and investigate this performance metric in a three-node cooperative network with decode-and-forward (DF) and amplify-and-forward (AF) relaying. We derive the mathematical expression of ASE with the consideration of path-loss and fading effects. We show through selected numerical examples that ASE provides a new perspective on the spectrum utilization efficiency and transmission power design.

Index Terms—cooperative network, area spectral efficiency, relay.

I. INTRODUCTION

Cooperative relay transmission is one of the most effective approaches to improve the capacity and coverage of wireless networks. Typically, two types of relaying protocols can apply [1], [2]. Non-regenerative relay stations (RSs) amplify and forward (AF) the received signals, while regenerative RSs decode and forward (DF) the received signals to the destination. In general, RSs are usually assumed to operate in half-duplex mode where the relay transmissions are carried out in two steps [3]: source to relay step and then relay to destination step. Both steps require appropriate time/spectral resource allocation, which leads to a certain penalty to the overall link spectral efficiency. A considerable amount of research has been carried out on cooperative networks (see for example [4]–[10]). Shannon capacity of opportunistic cooperative system with AF relaying over Rayleigh fading channels was derived in [4]. The outage performance of cooperative network with DF relaying over Nakagami fading channels was studied in [5]. In [6], the authors derived the average channel capacity and outage probability for four types of adaptive transmission techniques with AF relaying.

Most performance metrics for cooperative networks mainly focus on the quantification of either bandwidth utilization efficiency or link reliability. In particular, ergodic capacity [7] and average spectral efficiency [8] serve as the examples for the former whereas outage probability [9] and average error rate [10] for the latter. However, these performance metrics fail to capture the benefit of the usually smaller “footprint” of relay

transmission. In this paper, we evaluate the spectral efficiency of cooperative networks while taking the spatial property of relay transmission into account.

Area spectral efficiency (ASE) performance metric was introduced in [11] to quantify the spectrum utilization efficiency of cellular wireless systems. In cellular systems, the radio spectrum is systematically reused at different geometrical areas, i.e. co-channel cells [12]. Specifically, the co-channel cells will be separated by a minimum reuse distance of D such that their transmissions will not seriously interfere with one another. As such, the same spectrum will be used only once over an area of the size of $\pi D^2/4$. The ASE of cellular systems was therefore defined as the maximum data rate per unit bandwidth of a user randomly located in cell coverage area, over the area of $\pi D^2/4$, with unit being $b/s/(Hz \cdot m^2)$. Recently, the ASE metric was applied to characterize the performance of two-tier cellular networks [13]. Note that the regular frequency reuse based on typical hexagon cell structure greatly facilitates the ASE analysis [14]. Meanwhile, to the best knowledge of the authors, the ASE of cooperative systems has not been investigated in the literature.

In this work, we generalize the ASE definition and apply it to performance analysis of relay networks. In particular, ASE is defined as the ratio of overall effective ergodic capacity of the transmission over its affected area. The *affected area* refers to the area where a significant amount of transmission power is observed and parallel transmission over the same frequency is prohibited due to severe interference from/to target transmission. Note that the size of the affected area depends on various of factors, including transmission power, propagation environment, as well as receiver sensitivities. The average radius of the affected area will be proportional to the transmission power. Therefore, ASE can also quantify the transmission power utilization efficiency in achieving unit capacity. In cooperative networks, due to different power allocation scheme on the source and relay node, the direct and relaying hops may have different affected area. Using ASE metric to analyze cooperative networks will better capture the benefit of smaller footprint due to relaying.

In this paper, we investigate the ASE in a three-node cooperative network with DF and AF relaying. First, we present a transmission mode selection criterion to determine whether

direct transmission or relay transmission should be used. We then obtain the average ASE of this three-node network through statistical analysis with the consideration of these two transmission modes. Finally, the optimal transmission power and location of the relay node is investigated with respect to the performance metric of ASE using selected numerical examples.

The remainder of this paper is organized as follows. Section II presents the system and channel model. Based on the model, mathematical analysis of ASE is given in Section III. Numerical examples are presented in Section IV with related discussions. Finally, section V concludes the paper.

II. SYSTEM MODEL

A. Three-node cooperative network under Rayleigh fading

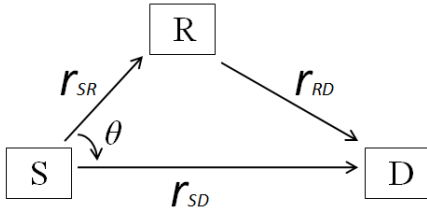


Fig. 1. Network Architecture.

As shown in Fig. 1, we consider a cooperative network that consists of three nodes: the source, the destination and the relay. The source node can communicate with the destination node via the direct link, whose distance is r_{SD} , or with the help of the relay node. The distances from source to relay and relay to destination are denoted by r_{SR} and r_{RD} , respectively. The transmission power for the source node is denoted by $\mathcal{P}_s$ while that for the relay node is $\mathcal{P}_r$. We assume that the transmission experiences slow Rayleigh fading. The PDF of received signal-to-noise ratio (SNR) Γ_{ij} at distance r_{ij} over Rayleigh fading channel is given by

$$f_{\Gamma_{ij}}(\gamma) = \frac{1}{\bar{\gamma}_{ij}} e^{-\gamma/\bar{\gamma}_{ij}}, \quad (1)$$

where $\bar{\gamma}_{ij}$ is the average received SNR at distance r_{ij} decided by path loss, r_{ij} ($i \in \{S, R\}, j \in \{R, D\}$ and $i \neq j$) is the distance from the transmitter i to receiver j . With the simplified path-loss model, $\bar{\gamma}_{ij}$ can be expressed as

$$\bar{\gamma}_{ij} = \frac{\mathcal{P}_i}{N \cdot K \cdot r_{ij}^\alpha}, \quad (2)$$

where K is a constant parameter and α is the path loss exponent, ranging from 2 to 6, N is the received noise power. We also assume that accurate estimation of the received SNR can be obtained and sent back to the source and relay via an error-free feedback path. The time delay in this feedback path is negligible. Thus, the source and relay can adapt their transmission rates to the actual channel state. For the sake of simplicity, we ignore the effect of shadowing in the analysis.

We consider a time division multiplexing (TDM) system where the transmission occurs in time slots of equal duration T . In addition, the relay node employs a half-duplex relaying operation, as it cannot transmit and receive at the same time. For that purpose, the time slots for relay transmission are further divided into two subslots of equal duration. Specifically, in the first subslot, relay node receives transmission from source node; in the second subslot, the DF relay decodes the transmission while the AF relay amplifies the received signal, and then forwards it to the destination node.

B. Transmission mode selection

We assume that the source node will select either direct or relay link to communicate with the destination node. In order to maximize the overall network capacity, the source node chooses the transmission mode with larger instantaneous capacity. In particular, the instantaneous capacity of direct transmission mode is given by

$$C_d = \log_2(1 + \Gamma_{SD}), \quad (3)$$

where Γ_{SD} is the received SNR at the destination node over the direct link. Under Rayleigh fading environment, Γ_{SD} of the cooperative network under consideration is exponential distributed with $\mathbb{E}\{\Gamma_{SD}\} = \bar{\gamma}_{SD}$. In relay transmission mode, the data to a destination node needs to be transmitted over the same bandwidth in two subslots. Therefore, the instantaneous capacity of relay transmission mode is given by

$$C_r = \frac{1}{2} \log_2(1 + \Gamma_r), \quad (4)$$

where Γ_r is the equivalent received SNRs at the destination node over the relay link, and the factor $\frac{1}{2}$ is due to the half duplexing constraint. After performing transmission mode selection, the instantaneous capacity of the three-node cooperative network is the maximal value of C_d and C_r , i.e.

$$C_{\text{inst}} = \max \{C_d, C_r\} \quad (5)$$

Substituting (3) and (4) into (5), the instantaneous capacity specializes to

$$C_{\text{inst}} = \frac{1}{2} \log_2 \left\{ 1 + \max \{ \Gamma_{SD}^2 + 2\Gamma_{SD}, \Gamma_r \} \right\}. \quad (6)$$

Here $\Gamma \triangleq \max \{ \Gamma_{SD}^2 + 2\Gamma_{SD}, \Gamma_r \}$ can be viewed as the equivalent received SNR. The average ergodic capacity can be derived by averaging the instantaneous capacity over the distribution of Γ , i.e.

$$\bar{C} = \int_0^\infty \frac{1}{2} \log_2(1 + \gamma) \cdot f_\Gamma(\gamma) d\gamma, \quad (7)$$

where $f_\Gamma(\gamma)$ is the PDF of Γ . Meanwhile, the probability that the system performs direct transmission is equal to the probability that $C_d > C_r$, i.e.

$$P_{\text{direct}} = \Pr \left\{ \Gamma_{SD}^2 + 2\Gamma_{SD} > \Gamma_r \right\}. \quad (8)$$

Since Γ_{SD} is exponential distributed with mean $\bar{\gamma}_{SD}$, then $\mathbf{P}_{\text{direct}}$ can be written as

$$\mathbf{P}_{\text{direct}} = \frac{1}{\bar{\gamma}_{SD}} \int_0^\infty F_{\Gamma_r}(x^2 + 2x) \exp(-x/\bar{\gamma}_{SD}) dx, \quad (9)$$

where $F_{\Gamma_r}(\gamma)$ is CDF of Γ_r , which is determined by the relaying protocol applied. Accordingly, the probability that the system performs relay transmission is given by $\mathbf{P}_{\text{relay}} = 1 - \mathbf{P}_{\text{direct}}$.

III. ASE ANALYSIS

In this section, we derive the analytical expression of ASE in a three-node cooperative network. In conventional cellular network, ASE is defined to be the average data rate per unit bandwidth per unit area supported by a base station [11]. In cooperative network, we generalize the definition of ASE, denoted by η , to be the ratio of average ergodic capacity $\bar{C}$, over the affected area of the transmission $\mathbf{A}_{\text{aff}}$, i.e.

$$\eta = \frac{\bar{C}}{\mathbf{A}_{\text{aff}}}. \quad (10)$$

In particular, the affected area is defined as the area where a significant amount of transmission power is observed and parallel transmission over the same frequency is prohibited due to severe interference from/to target transmission. With fading, the probability that an area of distance r from the transmitter is affected equals to the probability that the received signal power $\mathcal{P}_{\text{rec}}$ is greater than $\mathcal{P}_{\text{min}}$. It follows that the affected area can be determined as

$$\mathbf{A}_{\text{aff}} = \int_0^\infty \Pr[\mathcal{P}_{\text{rec}} \geq \mathcal{P}_{\text{min}}] r dr. \quad (11)$$

For the Rayleigh fading environment, the affected area specializes to

$$\mathbf{A}_{\text{aff}}^i = \frac{1}{\alpha} \Gamma\left(\frac{2}{\alpha}\right) \left(\frac{\mathcal{P}_i}{NK\mathcal{P}_{\text{min}}}\right)^{2/\alpha}, \quad i \in \{S, R\}, \quad (12)$$

where $\Gamma(\cdot)$ denotes the Gamma function. Note that if the transmission powers of the source node and relay node are not equal, the affected area in direct and relay transmission mode should be not the same. Therefore, the ASE of three-node cooperative network can be calculated as the weighted summation of those of direct transmission mode and relay transmission mode. While noting that the affected area of each relay hop might be different, it can be written as

$$\eta = \mathbf{P}_{\text{direct}} \cdot \frac{\bar{C}_d}{\mathbf{A}_{\text{aff}}^S} + \mathbf{P}_{\text{relay}} \cdot \frac{1}{2} \left(\frac{\bar{C}_r}{\mathbf{A}_{\text{aff}}^S} + \frac{\bar{C}_r}{\mathbf{A}_{\text{aff}}^R} \right), \quad (13)$$

where $\bar{C}_d$ and $\bar{C}_r$ are the average ergodic capacity under direct and relay transmission mode. In what follows, we will calculate $\bar{C}_d$ and $\bar{C}_r$ for DF and AF relaying protocols respectively.

A. Decode-and-forward relaying protocol

For DF relaying protocol, the relay node first decodes the transmitted signal from the source node and then forwards to the destination node. The instantaneous capacity of relay transmission is given by

$$\mathbf{C}_r^{\text{DF}} = \frac{1}{2} \min \left\{ \log_2(1 + \gamma_{\text{SR}}), \log_2(1 + \gamma_{\text{RD}}) \right\}, \quad (14)$$

where γ_{SR} and γ_{RD} are the instantaneous received SNR at the relay node and the destination node, respectively. The equivalent received SNR over the relay link is then given by

$$\Gamma_r^{\text{DF}} = \min\{\Gamma_{\text{SR}}, \Gamma_{\text{RD}}\}. \quad (15)$$

Under Rayleigh fading channel, Γ_{SR} and Γ_{RD} are exponential distributed with mean $\bar{\gamma}_{\text{SR}}$ and $\bar{\gamma}_{\text{RD}}$, respectively. Then CDF of Γ_r^{DF} can be obtained

$$F_{\Gamma_r^{\text{DF}}}(\gamma) = 1 - e^{-\alpha_1 \gamma}, \quad (16)$$

where $\alpha_1 = \frac{1}{\bar{\gamma}_{\text{SR}}} + \frac{1}{\bar{\gamma}_{\text{RD}}}$. Substituting (16) into (9), we can obtain the probability that the system performs direct transmission with DF relaying protocol, as

$$\mathbf{P}_{\text{direct}}^{\text{DF}} = 1 - \frac{1}{\bar{\gamma}_{\text{SD}}} \mathfrak{D}(\infty; \alpha_1, \alpha_2), \quad (17)$$

where $\alpha_2 = \frac{2}{\bar{\gamma}_{\text{SR}}} + \frac{2}{\bar{\gamma}_{\text{RD}}} + \frac{1}{\bar{\gamma}_{\text{SD}}}$, and $\mathfrak{D}(x; \alpha_1, \alpha_2)$ is defined to be

$$\begin{aligned} \mathfrak{D}(x; \alpha_1, \alpha_2) &\triangleq \int_0^x e^{-\alpha_1 t^2 - \alpha_2 t} dt \\ &= \frac{1}{2} \sqrt{\frac{\pi}{\alpha_1}} e^{\frac{\alpha_2^2}{4\alpha_1}} \left[\text{erf}(\sqrt{\alpha_1} \cdot x + \frac{\alpha_2}{2\sqrt{\alpha_1}}) - \text{erf}\left(\frac{\alpha_2}{2\sqrt{\alpha_1}}\right) \right], \end{aligned}$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Taking derivative of (16) with respect to γ , we arrive at the PDF of Γ_r^{DF}

$$f_{\Gamma_r^{\text{DF}}}(\gamma) = \alpha_1 e^{-\alpha_1 \gamma}. \quad (18)$$

The equivalent SNR of the cooperative network with DF protocol is given by

$$\Gamma^{\text{DF}} = \max \left\{ \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}}, \Gamma_r^{\text{DF}} \right\}. \quad (19)$$

It can be shown that the CDF under the condition $\Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_r^{\text{DF}}$ is given by

$$F_{\Gamma^{\text{DF}}}(\gamma \mid \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_r^{\text{DF}}) = \frac{\bar{\gamma}_{\text{SD}} \cdot F_{\Gamma_{\text{SD}}}(\xi) - \mathfrak{D}(\xi; \alpha_1, \alpha_2)}{\bar{\gamma}_{\text{SD}} - \mathfrak{D}(\infty; \alpha_1, \alpha_2)}, \quad (20)$$

where $\xi = \sqrt{\gamma + 1} - 1$. Taking derivative of (20) with respect to γ , we can obtain the conditional PDF of Γ^{DF} as

$$f_{\Gamma^{\text{DF}}}(\gamma \mid \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_r^{\text{DF}}) = \frac{\bar{\gamma}_{\text{SD}} \cdot f_{\Gamma_{\text{SD}}}(\xi) \cdot F_{\Gamma_r^{\text{DF}}}(\gamma)}{2(\xi + 1) \cdot (\bar{\gamma}_{\text{SD}} - \mathfrak{D}(\infty; \alpha_1, \alpha_2))}. \quad (21)$$

Substituting (21) into (7) and making some manipulations, we can obtain the average ergodic capacity of direct transmission

mode as

$$\bar{C}_d^{\text{DF}} = \frac{1}{\ln 2} \cdot \frac{1}{\bar{\gamma}_{\text{SD}} - \mathfrak{D}(\infty; \alpha_1, \alpha_2)} \left\{ \bar{\gamma}_{\text{SD}} \cdot e^{\frac{1}{\bar{\gamma}_{\text{SD}}}} E_1\left(\frac{1}{\bar{\gamma}_{\text{SD}}}\right) - \int_0^\infty \ln(1+t) e^{-\alpha_1(t^2+2t) - \frac{1}{\bar{\gamma}_{\text{SD}}}t} dt \right\}. \quad (22)$$

Following the same procedure, we can arrive at the PDF of Γ^{DF} under the condition $\Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} < \Gamma_r^{\text{DF}}$ as

$$f_{\Gamma^{\text{DF}}}(\gamma | \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} < \Gamma_r^{\text{DF}}) = \frac{\bar{\gamma}_{\text{SD}} \cdot f_{\Gamma_r^{\text{DF}}}(\gamma) \cdot F_{\Gamma_{\text{SD}}}(\xi)}{\mathfrak{D}(\infty; \alpha_1, \alpha_2)}. \quad (23)$$

It follows that the average ergodic capacity of relay transmission mode is given by

$$\bar{C}_r^{\text{DF}} = \frac{1}{\ln 2 \cdot \mathfrak{D}(\infty; \alpha_1, \alpha_2)} \left\{ \bar{\gamma}_{\text{SD}} \cdot e^{\alpha_1} E_1(\alpha_1) - \int_0^\infty \ln(1+t) \cdot e^{-\alpha_1 t - \frac{1}{\bar{\gamma}_{\text{SD}}}(\sqrt{1+t}-1)} dt \right\}. \quad (24)$$

B. Amplify-and-forward relaying protocol

With AF relaying protocol, the transmitted signal from source node is amplified by a factor G at the relay node and then forwards to the destination node. The relay gain G is usually selected to be [15]

$$G^2 = \frac{\mathcal{P}_r}{\mathcal{P}_d \cdot |h|^2 + N}, \quad (25)$$

where h is the channel gain between the source node and relay node, which leads to the equivalent received SNR of the relay link with AF protocol is given by

$$\Gamma_r^{\text{AF}} = \frac{\Gamma_{\text{SR}} \cdot \Gamma_{\text{RD}}}{\Gamma_{\text{SR}} + \Gamma_{\text{RD}} + 1}. \quad (26)$$

For mathematical tractability, we use the following tight upper bound [16]

$$\Gamma_r^{\text{AF}} = \frac{\Gamma_{\text{SR}} \cdot \Gamma_{\text{RD}}}{\Gamma_{\text{SR}} + \Gamma_{\text{RD}}}, \quad (27)$$

whose CDF was given in [16] as

$$F_{\Gamma_r^{\text{AF}}}(\gamma) = 1 - 2\beta_1 \gamma e^{-\beta_2 \gamma} K_1(2\beta_1 \gamma), \quad (28)$$

where $\beta_1 = \frac{1}{\sqrt{\bar{\gamma}_{\text{SR}} \cdot \bar{\gamma}_{\text{RD}}}}$, $\beta_2 = \frac{1}{\bar{\gamma}_{\text{SR}}} + \frac{1}{\bar{\gamma}_{\text{RD}}}$ and $K_1(\cdot)$ is the first order modified Bessel function of the second kind. Substituting (28) into (9), we can obtain the probability that the relay performs direct transmission with AF protocol, denoted by $\mathbf{P}_{\text{direct}}^{\text{AF}}$, i.e.

$$\mathbf{P}_{\text{direct}}^{\text{AF}} = 1 - \frac{1}{\bar{\gamma}_{\text{SD}}} \mathfrak{A}(\infty; \beta_1, \beta_3), \quad (29)$$

where $\mathfrak{A}(x; \beta_1, \beta_3)$ is defined to be

$$\mathfrak{A}(x; \beta_1, \beta_3) = \int_0^x 2\beta_1(t^2+2t) e^{-\beta_3(t^2+2t)} K_1(2\beta_1(t^2+2t)) dt, \quad (30)$$

where $\beta_3 = \frac{1}{\bar{\gamma}_{\text{SD}}} + \frac{1}{\bar{\gamma}_{\text{SR}}} + \frac{1}{\bar{\gamma}_{\text{RD}}}$. Taking derivative of (28) with respect to γ , we can obtain PDF of Γ_r^{AF} given by

$$f_{\Gamma_r^{\text{AF}}}(\gamma) = 2\beta_1 \gamma e^{-\beta_2 \gamma} \left\{ \beta_2 K_1(2\beta_1 \gamma) + 2\beta_1 K_0(2\beta_1 \gamma) \right\}, \quad (31)$$

where $K_0(\cdot)$ is the zero-order modified Bessel function of the second kind.

The equivalent SNR of the cooperative network with AF protocol is given by

$$\Gamma^{\text{AF}} = \max \left\{ \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}}, \Gamma_r^{\text{AF}} \right\}. \quad (32)$$

The PDF of Γ^{AF} under the condition $\Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_r^{\text{AF}}$ can be obtained as

$$f_{\Gamma^{\text{AF}}}(\gamma | \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_r^{\text{AF}}) = \frac{\bar{\gamma}_{\text{SD}} \cdot f_{\Gamma_{\text{SD}}}(\xi) \cdot F_{\Gamma_r^{\text{AF}}}(\gamma)}{2(\xi+1) \cdot (\bar{\gamma}_{\text{SD}} - \mathfrak{A}(\infty; \beta_1, \beta_3))}. \quad (33)$$

Correspondingly, we can calculate the average ergodic capacity of direct transmission mode $\bar{C}_d^{\text{AF}}$ by averaging the instantaneous capacity over the PDF of Γ^{AF} under the condition $\Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_r^{\text{AF}}$. Similarly, the PDF of Γ^{AF} under the condition $\Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} < \Gamma_r^{\text{AF}}$ is given by

$$f_{\Gamma^{\text{AF}}}(\gamma | \Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} < \Gamma_r^{\text{AF}}) = \frac{\bar{\gamma}_{\text{SD}} \cdot f_{\Gamma_r^{\text{AF}}}(\gamma) \cdot F_{\Gamma_{\text{SD}}}(\xi)}{\mathfrak{A}(\infty; \beta_1, \beta_3)}, \quad (34)$$

which can be applied to the calculation of $\bar{C}_r^{\text{AF}}$.

IV. NUMERICAL EXAMPLE

In this section, we present several numerical examples to study the ASE performance of the three-node cooperative network. We focus on the effect of relay position as well the transmission power of source and relay nodes.

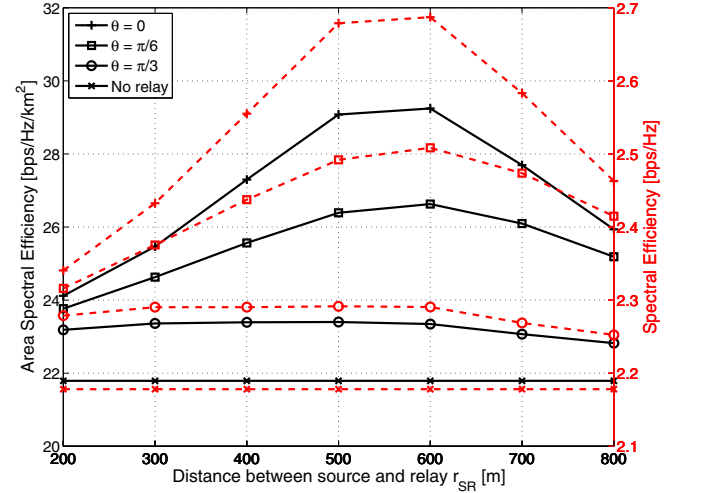


Fig. 2. The effect of the distance between source and relay node r_{SR} on ASE and spectral efficiency with DF relaying protocol for different angle θ . ($\mathcal{P}_s = 22$ dBm, $\mathcal{P}_r = 19$ dBm, $\mathcal{P}_{\text{min}} = -80$ dBm, $N = -100$ dBm, $K = 0.31$, $\alpha = 4$, $r_{\text{SD}} = 1000$ m.)

In Fig. 2, we plot ASE as function of the distance between the source and the relay node r_{SR} for DF relaying protocol. For comparison, we plot the spectral efficiency with the same configuration. It clearly shows that the relay position has similar effect on ASE and spectral efficiency. Another

interesting observation is that when the angle between source and relay node θ is small, ASE varies dramatically as the position of relay changes. The peak of ASE is obtained when the relay is near the midpoint of the line between source and destination. When θ is large, the distance r_{SR} has little effect on ASE. Meanwhile, large θ has worse ASE performance than small θ . Similar observation can be observed for AF-based relay networks.

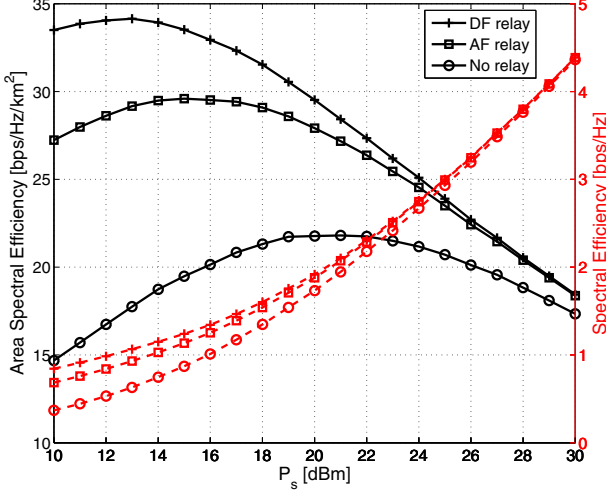


Fig. 3. The effect of the source node transmission power P_s on ASE with DF and AF relaying protocol. ($P_r = 10$ dBm, $r_{SD} = 1000$ m, $r_{SR} = r_{RD} = 500$ m, $\theta = 0$, $P_{\min} = -80$ dBm, $N = -100$ dBm, $K = 0.31$, $\alpha = 4$.)

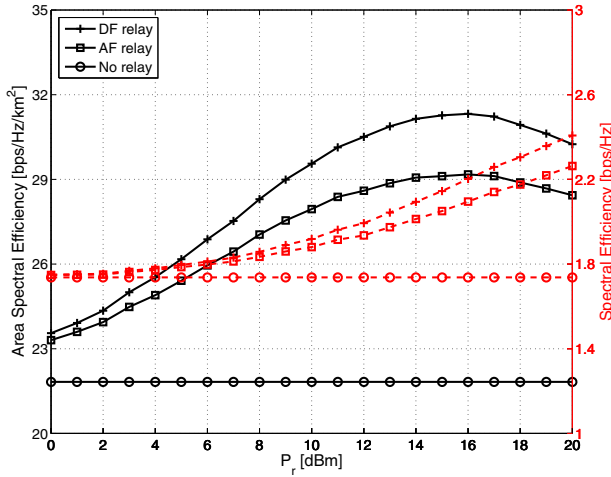


Fig. 4. The effect of the relay node transmission power P_r on ASE with DF and AF relaying protocol. ($P_s = 20$ dBm, $r_{SD} = 1000$ m, $r_{SR} = r_{RD} = 500$ m, $\theta = 0$, $P_{\min} = -80$ dBm, $N = -100$ dBm, $K = 0.31$, $\alpha = 4$.)

In Fig. 3 we plot the ASE and spectral efficiency as function of the source node transmission power P_s for DF and AF relaying protocol. For comparison, we include the ASE curve

of conventional system without relays. It shows that the relay-enhanced systems always enjoy better ASE performance than the conventional system. Meanwhile, the DF relaying protocol has slightly better overall ASE performance than AF relaying protocol. However, this performance gain shrinks as the transmission power P_s increases. Unlike spectral efficiency, whose performance curves are monotonically increasing function with respect to P_s , the ASE curves show a peak as the transmission powers increase. When P_s is small, ASE is an increasing function of transmission power, which indicates the capacity increases faster than the affected area. When P_s is large, ASE decreases as the transmission power increases, due to the faster increase of the affected area than capacity. This observation tells us that increasing the transmission power can lead to a higher spectral efficiency but can not necessarily increase ASE. Therefore, ASE provides a new perspective on transmission power selection. Another interesting observation is that to achieve optimal ASE, conventional system needs much higher transmission power than relay-enhanced systems, which implies the relay-enhanced system is more power efficient with respect to ASE.

In Fig. 4 we plot ASE and spectral efficiency as function of the relay node transmission power P_r for DF and AF relaying protocol. For small P_r , both DF and AF relaying schemes show slightly better ASE performance than the conventional system. This performance benefit increases as we enlarge P_r . Meanwhile, the relay-enhanced systems show an optimal P_r value to achieve the maximal value of ASE.

V. CONCLUSION

In this paper, we generalized the concept of ASE to study the performance of cooperative network with both DF and AF relaying. We first presented a transmission mode selection scheme to determine whether direct transmission or relay transmission is performed. Based on this scheme, we derived the conditional statistics of received SNR under each mode, which is then applied to calculate the ASE. Through selected numerical examples, we showed ASE provides a new perspective to the performance of cooperative network. Moreover, we showed that there is an optimal transmission power level for source and relay nodes in terms of maximizing ASE. Our future research would concentrate on the utilization of the ASE performance metric to analyze arbitrary wireless network with the consideration of interference.

ACKNOWLEDGEMENT

This work was supported by a grant from Qatar Telecom (Qtel).

REFERENCES

- [1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [2] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE Journal on Selected Areas in Commun.*, vol. 22, no. 6, pp. 1099-1109, Aug. 2004.
- [3] W. Nam, W. Chang, S. Y. Chung, and Y. H. Lee, "Transmit optimization for relay-based cellular OFDMA systems," in *Proc. IEEE Int. Conf. on Commun.*, pp. 5714-5719, 2007.

- [4] M. Torabi, and D. Haccoun, "Capacity analysis of opportunistic relaying in cooperative systems with outdated channel information," *IEEE Commun. Lett.*, vol. 14, no. 12, pp. 1137-1139, Dec. 2010.
- [5] F. Xu, F. C. M. Lau, Q. F. Zhou, and D. -W. Yue, "Outage performance of cooperative communication systems using opportunistic relaying and selection combining receiver," *IEEE Signal Process. Lett.*, vol. 16, no. 4, pp. 237-240, Apr. 2009.
- [6] M. Torabi, D. Haccoun, and J. -F. Frigon, "On the performance of AF opportunistic relaying with adaptive transmission over Rayleigh fading channels," *2011 IEEE Pacific Rim Conf. on Commun., Computers and Signal Process. (PacRim)*, pp. 173-178, 23-26 Aug. 2011.
- [7] Y. Ding, J. -K. Zhang, and K. M. Wong, "Ergodic channel capacities for the amplify-and-forward half-duplex cooperative systems," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 713-730, Feb. 2009.
- [8] J. -W. Kwon, Y. -C. Ko, and H. -C. Yang, "Maximum spectral efficiency of amplify-and-forward cooperative transmission with multiple relays," *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, pp. 49-54, Jan. 2011.
- [9] H. Yu, I. -H. Lee, and G. L. Stuber, "Outage probability of decode-and-forward cooperative relaying systems with co-channel interference," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 266-274, Jan. 2012.
- [10] S. S. Ikki, and M. H. Ahmed, "Performance analysis of incremental-relaying cooperative-diversity networks over rayleigh fading channels," *IET Commun.*, vol. 5, no. 3, pp. 337-349, Feb. 2011.
- [11] M. S. Alouini, and A. J. Goldsmith, "Area spectral efficiency of cellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1047-1066, 1999.
- [12] A. Goldsmith, *Wireless Communacations*, Cambridge University Press, 2005.
- [13] Y. Kim, T. Kwon, and D. Hong, "Area spectral efficiency of shared spectrum hierarchical cell structure networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 8, pp. 4145-4151, Oct. 2010.
- [14] D. R. Basgeet, and Y. C. Chow, "Uplink performance analysis for a relay based cellular system," in *Proc. IEEE Veh. Technol. Conf.*, pp. 132-136, 2006.
- [15] J. N. Laneman, and G. W. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," *IEEE Wireless Commun. and Networking Conf.*, vol. 1, pp. 7-12, 2000.
- [16] M. O. Hasna, and M. -S. Alouini, "Performance analysis of two-hop relayed transmissions over Rayleigh fading channels," *Proc. IEEE Veh. Technol. Conf.*, vol. 4, pp. 1992- 1996, Sep. 2002.
- [17] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 6th ed. New York: Academic Press, 2000.