

No-reference Image Quality Assessment for Compressed Images based on DCT Coefficient Distribution and PSNR Estimation

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Abstract—We propose a new no-reference image quality assessment model nPSNR (No-reference PSNR) for JPEG compressed images. The model performs in DCT domain and the DCT coefficient distribution is used. This method estimates the MSQE (mean-squared quantization error) of a decoded image with the distributions of AC coefficients and DC coefficients of the encoded image, for PSNR estimation in DCT domain without reference. We test the proposed model on the selected images from the TID2008 database. Then, the computational scores (nPSNR) are compared with the ground truth values (MOS) based on three commonly-used performance criteria. Experimental results demonstrate that the proposed approach is more consistent with the subjective perception than the state-of-the-art full-reference image quality assessment schemes.

I. INTRODUCTION

Video coding is widely used in the transmission and recording of digital video systems. Since the original images are not available at the receiver, no-reference quality assessment is becoming increasingly important. The no-reference methods can be classified into two types: no-reference approaches on decoded signal (i.e., in spatial domain); no-reference approaches based on block-based discrete cosine transform (DCT) coefficient data (i.e., in compressed domain).

In [1], χ^2 test shows that GGFs (generalized Gaussian functions) model the distribution of DCT coefficients of images more accurately than Laplacian PDFs (probability density functions). Eude et al. found that AC coefficients of images are of neither a Cauchy distribution, nor a Laplacian distribution, but rather a mixture distribution of Laplace and Gaussian [2]. The DCT coefficients of the chrominance component were studied by Smoot and Rowe [3], and the results showed that the chrominance component from the MPEG-2 sequences exhibits the same distribution as for the luminance one. Lam et al. [4] offered a rigorous mathematical analysis using a doubly stochastic model of the images, which provided the appropriate theoretical explanations that allow us to investigate how certain changes in the image statistics could affect the DCT coefficient distributions.

The peak signal-to-noise ratio (PSNR) is used to calculate the error between original image and distorted one. Consider a distorted digital image $f(x, y)$ with size of $M \times N$ and its reference image $f_0(x, y)$, the PSNR (peak signal-to-noise ratio) of $f(x, y)$ is:

$$PSNR = 10 \lg \frac{f_{\max}^2}{MSE} \quad (1)$$

where $f_{\max}$ is the maximum gray value of image $f(x, y)$. For an 8 bit gray image, $f_{\max}$ value is 255; MSE is the mean square error, and it is defined as:

$$MSE = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - f_0(x, y)]^2}{MN} \quad (2)$$

PSNR is a most common quality evaluation index of degraded image. However, calculating the PSNR needs the original signal as a reference. In the absence of reference images, PSNR of compressed images can still be estimated based on DCT coefficient data. Turaga et al. [5] used the results of the statistical modeling of the DCT coefficients, combined with the PSNR model to calculate the quantization error, thus assesses the quality of the whole image. Ichigaya et al. [6] proposed a PSNR estimation for MPEG-2 encoded video sequences; The distributions of DCT coefficient were modeled according to Laplace distribution, and then PSNR values were estimated for I, B and P frames for compressed digital video. The estimated PSNR value from the experiment and the actual PSNR value may have high similarity (with correlation of 0.9). The authors of [6] later improved the compressed domain video quality assessment algorithm and presented an improved no-reference quality evaluation algorithm on bit stream [7]. The algorithm integrates the relation between distortion of DCT coefficients and the quantization error, and estimated the PSNR value. Experiment shows a strong correlation between the estimated value of PSNR and the true value of PSNR. Shim et al. [8] proposed an efficient method to estimate PSNR using the statistical distribution of the integer transform coefficients in an H.264

decoder, and assess the accuracy of the method by comparing the predicted PSNR and the real PSNR values. Ref. [9] evaluates the PSNR of the video through the detection of MPEG-2 video for the size and location, quantization step, and I-frame and other parameters, and reduces blocking and ringing effect according to the evaluation result. Experimental results show that the algorithm removes the ringing and blocking of the MPEG-2 video effectively, and PSNR is improved by 0.3dB.

In the proposed model, we first analyze the DCT coefficients distribution from the variance of the fixed block, then we apply the central limit theorem to show that the distribution of a distorted image is Gaussian distribution. Then, analysis is performed to show that the DCT coefficients follow Laplace distribution. And lastly the proposed nPSNR (No-reference PSNR) of compressed images is calculated with compressed images. The proposed model gives better results than that the relevant state-of-the-art full-reference image quality assessment metrics.

II. DCT COEFFICIENT DISTRIBUTION

A. DCT coefficient distribution

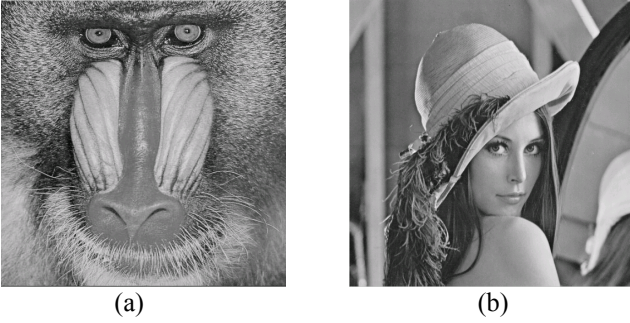


Fig. 1 Standard test images: (a) baboon (b) lena

For JPEG image coding, firstly, an image is divided into 8×8 non-overlapping blocks, and these blocks are used with DCT transform before quantization and entropy coding. Direct DCT transform without blocking step leads to less coefficient distribution features, and the coefficient distribution of DCT transform after blocking is not so smooth. To the later, the DCT coefficient distribution is obtained from the fixed-block variance; it has been found that the central limit theorem applies and this distribution is a Gaussian one. Then, through the statistical distribution of block difference, we analyze and find that the DCT coefficients accord with Laplace distribution.

Assume that the probability density function of the variance accord with the semi-Gaussian distribution, we can get:

$$p(I_{m,n}) = \frac{\sqrt{2/s}}{2} \exp\{-\sqrt{2/s} I_{m,n}\} \quad (3)$$

Here $p(x)$ stands for the probability density function of DCT coefficient distribution, and s for a DCT coefficient.

Therefore, the distribution meets the Laplace distribution, where $\mu = \sqrt{2/s}$. Fig. 2 shows a curve of $p(I_{m,n})$ when s takes two typical values. From Fig. 2, we can see the above approximation is reasonable. From this result, we can further infer if the distribution of the variance between the exponential distribution and semi-Gaussian distribution, the distribution of the results for $I_{m,n}$ and the Laplace distribution are very close.

Though block variance distribution is different from exponential distribution and Gaussian distribution significantly, we can still observe the higher-order statistics (see [12]) to obtain some conclusions about the $I_{m,n}$ distribution.

B. AC-coefficient MSQE (mean-squared quantization error) estimates

After DCT transform, the AC coefficients of image accord with the Laplace distribution. The distribution is shown in Figure 2. From the figure, for a fixed coefficient of X , the greater the value of the distribution's parameter λ , the greater

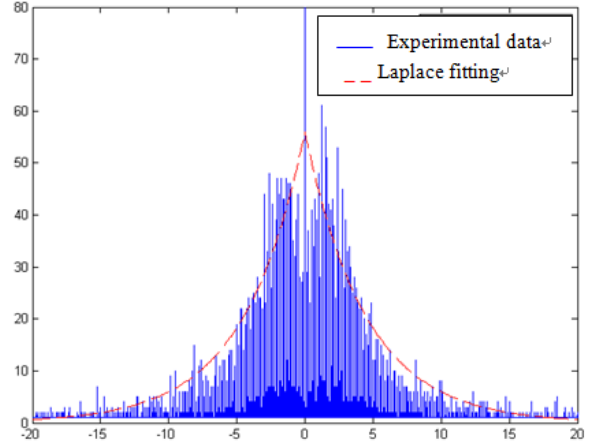


Fig. 2 "Lena" image AC coefficient maps, the red line is the Laplace function, $\lambda = 0.05$

the probability of $p(|x| \leq X)$, and the more zero values in distribution coefficients.

So it can be obtained:

$$\lambda = \sqrt{\frac{2}{D(x)}} \quad (4)$$

After getting distribution parameter λ of the probability density function of AC coefficients, we will calculate quantization error MSQE of a compressed image based on Laplace distribution.

Assume that the quantization step length is q , the value of an image is k_i after DCT coefficients quantization. For each AC frequency, a DCT coefficient value ranging between $[qk_i - q/2, qk_i + q/2]$ is quantified to qk_i . Therefore quantization error $\mathcal{E}_{k_i}^2$ in this range can be calculated by integral, shown in Eq. (5):

$$\varepsilon_{k_i}^2 = \frac{\lambda / 2 \int_{qk_i - q/2}^{qk_i + q/2} (x - qk_i)^2 e^{-\lambda x} dx}{P(qk_i - q/2 < x < qk_i + q/2)} \quad (5)$$

And the last AC quantization error can be calculated as:

$$\varepsilon_{AC}^2 = \varepsilon_1^2 + \varepsilon_2^2 = \frac{2}{\lambda^2} - \frac{2q \cdot e^{-\lambda q/2}}{\lambda \cdot (1 - e^{-\lambda q})} \quad (6)$$

where

$$\varepsilon_1^2 = \frac{e^{-\lambda q}}{\lambda^2 (1 - e^{-\lambda q})} \left\{ e^{\lambda q/2} \left[\left(\frac{\lambda \cdot q}{2} \right)^2 - \lambda \cdot q + 2 \right] - e^{-\lambda q/2} \left[\left(\frac{\lambda \cdot q}{2} \right)^2 + \lambda \cdot q + 2 \right] \right\} \quad (7)$$

$$\varepsilon_2^2 = \frac{1}{\lambda^2} \left\{ 2 - e^{-\lambda q/2} \left[\left(\frac{\lambda \cdot q}{2} \right)^2 + \lambda \cdot q + 2 \right] \right\} \quad (8)$$

If AC coefficients of DCT are all zero, then $\lambda = +\infty$. By Eq. (6), the error is equal to zero. In fact, the true value of the error is not zero, but very small and close to zero. We select λ to be a big integer, for example 10,000, when $\lambda = +\infty$. The purpose of this treatment is to make the final error not equal to zero.

C. DC-coefficient MSQE estimates

The approach for calculating the DC coefficient quantization error is the same as that of AC. A DC coefficient ranging in $[qk_i - q/2, qk_i + q/2]$ is quantified to qk_i . When computing the difference of DC coefficients of adjacent blocks, each DC coefficient subtracts that of the previous adjacent block and of that of the next one, respectively; this is different from the calculation of AC coefficients. The DC coefficient of the j^{th} 8×8 DCT block is recorded as DC_j , and the difference between this j^{th} 8×8 DCT block and the $(j+1)^{\text{th}}$ block is denoted by $\overline{DC_j}$, where $j=1,2,\dots,J$; and the difference between the adjacent DC coefficients is calculated as follows:

$$\overline{DC_1} = DC_2 - DC_1, \overline{DC_2} = DC_3 - DC_2, \dots, \overline{DC_{J-1}} = DC_J - DC_{J-1}$$

After the DC coefficient quantization error ε_{DC}^2 between adjacent blocks is calculated, it is easy to get the quantization error ε_{DC}^2 of the DC coefficient itself as follows:

$$\varepsilon_{DC}^2 = \frac{2}{\lambda^2} - \frac{4q \cdot e^{-\lambda q}}{\lambda \cdot (1 - e^{-2\lambda q})} \quad (9)$$

$$\varepsilon_{DC}^2 = (\frac{\varepsilon_{\overline{DC}}}{2})^2 = \frac{\varepsilon_{\overline{DC}}^2}{4} \quad (10)$$

When $\lambda = \infty$, the λ can be replaced with a large number, such as 10,000, as for the AC case.

III. THE PROPOSED NO-REFERENCE MODEL nPSNR

The degradation of image quality due to JPEG compression is closely related with DCT coefficient quantization. As mentioned above, the traditional full-reference image quality assessment method PSNR is calculated using Eq. (1) to Eq. (2). According to the Parseval's Theorem, the mean square error in the pixel domain is equivalent to the DCT-domain mean square quantization error. Because the DCT transform

is a standard orthogonal transformation, the PSNR in DCT domain is estimated as:

$$PSNR = 20 \log_{10} \left(\frac{S_{p-p}}{\sqrt{MSQE}} \right) \quad (11)$$

$$MSQE = \frac{1}{J} \sum_{i=1}^J (X_i - X'_i)^2$$

Where X_i and X'_i are the DCT coefficients of source signal and decoded signal respectively, and S_{p-p} is the peak of the signal amplitude (for an 8-bit images, the value is 255).

Since we focus on no-reference video quality assessment, and the DCT coefficients of the source signal are not available. A solution is to calculate the quantization error in the DCT domain using statistical method and considering DCT coefficients distribution of the encoded image. The calculation of AC and DC MSQE has been shown in Section II, and the overall MSQE is calculated as:

$$MSQE = \left(\frac{\alpha \cdot \varepsilon_{DC}^2 + \beta \cdot \sum_{i=1}^{63} \varepsilon_{AC_i}^2}{\alpha + \beta} \right) / 64 \quad (12)$$

where α and β are weighted parameters. In this paper, $\alpha = 0.3$ and $\beta = 0.7$.

After quantization error of the whole DCT coefficients is calculated according to AC coefficients and DC coefficients, the overall no-reference PSNR (nPSNR for short) as the quality score is calculated as:

$$nPSNR = 10 \log_{10} \left(\frac{S_{p-p}^2}{MSQE} \right) \quad (13)$$

A brief overview of the steps of the nPSNR values being calculated in the DCT domain are as follows.

- (1) To obtain the quantization matrix and the DCT coefficients for the coded image;
- (2) To determine the distribution of DCT coefficients, AC coefficients distribution and the distribution of the adjacent block DC coefficient differences;
- (3) Using the distribution of DCT coefficients, to calculate quantization errors for the DC coefficient and 63 AC coefficients and;
- (4) To calculate the overall quality score (nPSNR) of the entire image.

IV. THE EXPERIMENTAL RESULTS

To validate the proposed model, tests have been performed on selected 310 representative images in TID2008 database. The computational scores (with nPSNR) are compared with the ground truth values (mean opinion score—MOS for short) based on three commonly-used performance criteria, namely non-linear regression correlation coefficient, Spearman rank-order correlation coefficient (SRCC), and outlier ratio (OR).

Comparing with the typical full-reference IQA (image quality assessment) methods, we can see from Table 1 that the proposed model improves the correlation between video objective evaluation and subjective evaluation in terms of coefficient of nonlinear regression, Spearman correlation coefficients respectively. Moreover, outlier ratio is also reduced.

TABLE I
EXPERIMENTAL RESULTS OF VARIOUS ALGORITHMS ON THE TID2008

Model	Nonlinear regression correlation coefficient	Spearman rank correlation coefficient	Outline Ratio
Proposed	0.9661	0.9544	0.3839
FSIM[13]	0.8906	0.8572	0.6900
FSIMC[13]	0.8919	0.8607	0.6741
MS-SSIM[14]	0.8607	0.8287	0.7159
VIF[15]	0.8391	0.7652	0.7835
SSIM[16]	0.8374	0.8031	0.7441
IFC[17]	0.7693	0.6559	0.8124
VSNR[18]	0.7222	0.6285	0.7888
NQM[19]	0.6868	0.5502	0.6868
PSNRHVS[6]	0.6471	0.5396	0.7771
PSNR	0.5428	0.5100	0.5428

V. CONCLUSIONS

In this paper, we have introduced a no-reference image quality evaluation model for DCT-based compressed images. The proposed model computes quality scores (as nPSNR: no-reference PSNR) based on the quantization parameters and DCT coefficients of compressed images. We test the proposed model on the TID 2008 database, and the initial experimental results show that the proposed model via nPSNR calculation correlates well with the human perception of quality. Compared with the relevant state-of-art full-reference image quality metrics, the proposed model is more consistent with the subjective evaluation.

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