Quickest Detection of Unknown Power Quality Events for Smart Grids

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Abstract—In this work, we study a change-point approach to provide the quickest detection of power quality (PQ) event occurrence for smart grids. Despite that both the occurrence time and the PQ event type are unknown beforehand, knowledge of the statistics of post-PQ event signals is required to implement the change-point approach. To circumvent this obstacle, we propose to model the unknown PQ events using different statistical distributions, namely the Gaussian, Gamma and inverse Gamma distributions. It is shown by computer simulation that all distributions under consideration can provide accurate PQ event detection. In particular, the inverse Gamma distribution

Index Terms—Power quality (PQ), Change-point detection theory, cumulative sum (CUSUM) algorithm.

I. INTRODUCTION

The emerging smart grid has presented an unprecedented opportunity to catapult the aging electric power infrastructure into the digital grid by leveraging state-of-the-art information technology. Equipped with two-way information and power exchange between utilities and customers, the smart grid is envisaged to dramatically improve future power delivery with significantly improved efficiency and reliability. However, realizing the promise of the smart grid critically hinges upon adoption and integration of innovative smart grid technologies. In particular, power quality (PQ) monitoring technology is indispensable for highly reliable power delivery by providing reliable and real-time network surveillance. More specifically, the sinusoidal power waveform generated by electric utilities is often distorted over transmission lines. In general, distortions can be classified into two categories; namely, PQ variations and PQ events [1], [2]. In contrast to PQ variations characterized by small and gradual deviations from the sinusoidal voltage/current waveforms, PQ events incur large waveform deviations. Thus, PQ events are more detrimental to the power distribution network since they may potentially inflict more severe damages such as power outages. Therefore, the occurrence of PQ events has to be accurately and timely detected to facilitate appropriate amending actions. In practice, PQ event monitoring consists of two steps: 1) detection and 2) classification. In the first step, the occurrence of a PQ event is declared when the waveform change is detected to exceed a pre-defined threshold. In the second step, the

distorted waveforms are fed into a classifier to identify the cause of the PQ event before further analysis is performed. Figure 1 illustrates voltage transient and sag events generated with the IEEE 14-bus test setup shown in [3] at 0.06s. In this work, we focus on developing novel detection schemes in the first step. For readers interested in the classification step of the PQ event monitoring, we refer to [2] and references therein for a more comprehensive treatment. Furthermore, for presentational simplicity, we concentrate on the voltage-based PQ events in this work while its extension to the current-based PQ events can be done in a straightforward manner.



Fig. 1. Illustration of a voltage transient event.

Three PQ event detection methods have been proposed in the current literature. The first one keeps tracking the root mean squared (rms) value of the voltage waveform over a moving window. The likelihood of PQ event occurrence is evaluated based on the rms change across windows. Despite its simplicity, the rms-based method is effective in detecting amplitude-related distortions. The second one detects the distortion in the frequency domain by transforming the time waveform into the frequency waveform using either the wavelet or the short-time Fourier transform (STFT) [2]. The third one decomposes the waveform into a sum of damped sinusoids using super-resolution spectral analysis techniques such as signal estimation via a rotational invariance technique (e.g., ESPRIT) or multiple signal classification (e.g., MUSIC) [4]. The distorted waveform is detected by comparing the decomposed frequency-domain components of a monitored waveform with those of the normal one. Apparently, the latter two are more agile to frequency distortions. Note that a sliding window is also required in all these methods to segment the waveform into blocks before any computational operation is applied [2]. As a result, the time resolution of all three methods is restricted by the sliding window size. To cope with this problem, [5] has studied the PO monitoring problem in a change-point detection theoretic framework. In [5], a cumulative sum (CUSUM)-based sequential detection scheme [6] has been proposed by exploiting the difference of statistical distributions of power waveforms before and after the PQ event occurrence. Since the CUSUM scheme proposed in [5] performs sample-by-sample evaluation, it can achieve the quickest detection with the finest time resolution.

However, the CUSUM scheme requires knowledge about the statistics of signals before and after PQ events. While the statistics of the sinusoidal signal before PQ events can be well characterized, the post-change signal statistics are usually unknown, depending on the nature of the underlying PQ event(s). To circumvent this uncertainty, [5] proposed to model the post-event signal using the Gaussian distribution by invoking the central limit theorem (CLT). Furthermore, assuming that the Gaussian noise is zero-mean, [5] then modeled the variance of the post-event signal as a Gaussian random variable. Finally, capitalizing on the changepoint detection theory with unknown parameters, the weighted CUSUM algorithm is employed to compute the *expected* loglikelihood ratio (LLR) over the variance. Simulation results in [5] showed that this approach could successfully capture the characteristics of uncertain PQ events. While modeling the variance as a Gaussian-distributed random variable is reasonable, other distributions have also recently been proposed for uncertainty modeling in the literature. For instance, [7] adopted the inverse gamma distribution to model uncertainty in channel estimation.

In this work, we examine the performance obtained with different variance models. More specifically, we model the unknown post-event signal variance using three distributions, namely the Gaussian distribution, the Gamma distribution and the inverse Gamma distribution. Through simulation, we compare the resulting performance of the change-point approach in PQ monitoring using the change-point detection technique developed in [5].

II. PROBLEM FORMULATION

In this section, we will introduce the signal model for PQ monitoring. Denote by t_e the PQ event occurrence time, the goal is to detect the PQ event with the minimum delay and the highest detection accuracy.

A. Pre-event PDF

The continuous-time signal *before* the PQ event is measured and sampled with the k-th sample being modeled as

$$y[k] = s_{\boldsymbol{\theta}_0}[k] + n[k], \tag{1}$$

where n[k] is the additive white Gaussian noise (AWGN) with zero-mean and variance σ_n^2 , denoted by $\mathcal{N}(0, \sigma_n^2)$, and

$$s_{\theta_0}[k] = a_0 \cdot \sin(2\pi f_0 T_s k + \phi_0),$$
 (2)

is the undistorted power waveform with T_s being the sampling duration, $\theta_0 \stackrel{\text{def}}{=} [a_0, f_0, \phi_0]^T$, where $a_0 = 1$ is the signal amplitude gain, and f_0 and ϕ_0 are the fundamental frequency and the initial phase of the power waveform, respectively. Note that we have implicitly assumed the variance of n[k] is independent of k.

To facilitate our detection algorithm development, we first transform y[k] in (1) into z[k] as

$$z[k] \stackrel{\text{def}}{=} y[k] - s_{\boldsymbol{\theta}_0}[k] = n[k], \quad 0 \le t < t_e.$$
(3)

Since s_{θ_0} is deterministic, the probability density functions (PDF) of z is simply

$$p_{\boldsymbol{\theta}_0}(z) = \mathcal{N}\left(0, \sigma_n^2\right). \tag{4}$$

B. Post-event PDF

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Next, we model the power waveform after the PQ event as

$$y[k] = s_{\theta_1}[k] + n[k], \quad t \ge t_e, \tag{5}$$

where

$$\boldsymbol{\theta}_{\boldsymbol{\theta}_1}[k] = a_1 \cdot \sin\left(2\pi f_1 T_s k + \phi_1\right) + \xi_{\boldsymbol{\varphi}}[k], \tag{6}$$

with $\xi_{\varphi}[k]$ being the additive distortion parameterized by φ . Furthermore, $\boldsymbol{\theta}_1 \stackrel{\text{def}}{=} [a_1, f_1, \phi_1, \varphi^T]^T$ are the signal amplitude gain, the fundamental frequency, the initial phase of the post-event power waveform and additive distortion power, respectively.

Clearly, the PDF of y in Eq. (5) depends on the specific type of PQ events under consideration. As a result, it is generally difficult to fully characterize the PDF denoted by $p_{\theta_1}(y)$ before the occurrence of the PQ event. To cope with this uncertainty of the post-event PDF, we also transform Eq. (5) in a similar manner as Similar (3) :

 $z[k] = y[k] - s_{\theta_0}[k] = x[k] + w[k],$

where

$$x[k] = a_1 \cdot \sin(2\pi f_1 t + \phi_1),$$
 (8)

(7)

$$w[k] = \xi_{\varphi}[k] - s_{\theta_0}[k] + n[k].$$
(9)

Since θ_1 is unknown, rather than evaluating the LLR $\frac{p_{\theta_1}(z_i)}{p_{\theta_0}(z_i)}$ directly, we compute the logarithm of the weighted likelihood ratio with the weighted CUSUM method as

$$s_{i} = \ln\left[\int_{\Theta_{1}} \frac{p_{\theta_{1}}(z_{i})}{p_{\theta_{0}}(z_{i})} dF_{\Theta_{1}}\left(\theta_{1}\right)\right],$$
(10)

where $F_R(\mathbf{r})$ is the cumulative density function (CDF) of the enclosed random variable R.

By invoking the central limit theorem, we can approximate the PDF of w as $\mathcal{N}(0, \sigma_w^2)$, where $\sigma_w^2 = \sigma_\xi^2 + \sigma_n^2 + \frac{1}{2}a_0^2$. Furthermore, recall that x[k] is approximately uniformly distributed over $[-a_1, +a_1]$ with $a_1 > 0$. Thus, it is shown in the Appendix that

$$p_{\theta_1}(z) = \frac{1}{4a_1} \left[\operatorname{erf}\left(\frac{z+a_1}{\sqrt{2} \cdot \sigma_w}\right) - \operatorname{erf}\left(\frac{z-a_1}{\sqrt{2} \cdot \sigma_w}\right) \right]. \quad (11)$$

With the assumption that x[k] and w[k] are statistically independent, we can express $F(\theta_1)$ as

$$F_{\Theta_1}\left(\boldsymbol{\theta}_1\right) = F_{A_1}\left(a_1\right) \cdot F_{\Sigma_w}\left(\sigma_w\right). \tag{12}$$

As a result, Eq. (10) becomes

$$s_{i} = \ln\left[\int_{A_{1}}\int_{\Sigma_{w}}\frac{p_{\boldsymbol{\theta}_{1}}(z_{i})}{p_{\boldsymbol{\theta}_{0}}(z_{i})}dF_{\Sigma_{w}}\left(\sigma_{w}\right)\,dF_{A_{1}}\left(a_{1}\right)\right].$$
 (13)

In the following, we propose to model $F(\cdot)$ using different statistical distributions.

III. MODELING UNCERTAINTY

A. CDF of Uncertainty

The most commonly used distribution of $F(\cdot)$ includes the uniform and Gaussian distributions [6]. In [5], the performance using the Gaussian distribution has been examined. Alternatively, we can also model $F(\cdot)$ using the inverse Gamma and Gamma distributions. More specifically, we denote the shape and scale parameters by α and β , respectively. The $F(\cdot)$ using the inverse Gamma distribution can be expressed as:

$$F_{\mathbf{x}}(x) = Q\left(\frac{\beta}{x}\right),\tag{14}$$

where we set the shape parameter $\alpha = 1$ and the Q function takes the following form:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{u^2}{2}\right) \, du. \tag{15}$$

Finally, the $F(\cdot)$ using the Gamma distribution can be written as

$$F_{\mathbf{x}}(x) = \int_0^{\frac{x}{\beta}} e^{-t} dt, \qquad (16)$$

with the shape parameter is also set to $\alpha = 1$.

Note that the two CDF's in Eq. (13), namely F_{Σ_w} and dF_{A_1} , can be modeled using various combinations of different statistical distributions. For simplicity, we use the same distribution for both CDF's in the following simulation.

B. Summary of Weighted CUSUM-based Schemes

The proposed weighted CUSUM-based PQ-event detection scheme is summarized in Algorithm 1.

Algorithm 1 Weighted CUSUM-based PQ-event detection

Inputs: samples $\{y_k\}$ and a preset threshold *h* **States:** Initialize $t_e = 0$ **Procedure:**

for
$$k = 1, 2, \dots, \infty$$
 do
 $z_k = y_k - s_{\theta_0}(t_k);$
 $s_k = \ln \left[\int_{A_1} \int_{\Sigma_w} \frac{p_{\theta_1}(z_k)}{p_{\theta_0}(z_k)} dF_{\Sigma_w}(\sigma_w) dF_{A_1}(a_1) \right];$
 $S_k = \sum_{i=1}^k s_i;$
 $m_k = \min_{\substack{1 \le j \le k}} S_j;$
 $g_k = S_k - m_k;$
if $g_k \ge h$ then
 $\hat{t}_e = t_k;$
break;
end if
end for
Declare the detection of a PQ event at time \hat{t}_e if $\hat{t}_e \ne 0.$

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, simulation results are provided to compare the performance of the proposed CUSUM using different statistical distributions to model the uncertainty. We use ATP to simulate voltage transients with the IEEE 14-bus test setup specified in [3]. The PQ event is set to take place at $t_e = 0.06$ s in the simulation. Furthermore, we set the sampling rate at $T_s = 0.1$ ms and define the signal-to-noise ratio (SNR) as $\frac{1}{\sigma_n^2}$ while fixing $a_0 = 1$.

We compare three different distributions, namely the Gaussian, Gamma and inverse Gamma distributions. For the Gaussian distribution, we assume that both mean and variance are unity. For the Gamma and inverse Gamma distributions, we set the scale parameter β to 2 and 0.5, respectively. The simulation result is shown in Fig. 2 where the cumulated s_i value is depicted as a function of time.



Fig. 2. PQ monitoring performance with different uncertainty models.

First, Fig. 2 shows that all three distributions could accurately detect the occurrence of the PQ event at $t_e = 0.06s$. Furthermore, we argue that the slope of the performance curve at $t_e = 0.06s$ is a good metric to evaluate how well the PQ event can be detected. More specifically, steeper is the rising edge, more robust is the detection. Inspection of Fig. 2 reveals that the inverse Gamma modeling has slight edge over the other two distributions, though the advantage does not appear very significant.

V. CONCLUSION AND FUTURE WORK

Intelligent PQ event monitoring technology is indispensable for realizing the promise of the smart grid. In this study, we have extended the change-point detection scheme proposed in our previous work by modeling the uncertainty caused by PQ events using various statistical distributions. Simulation results have shown that all distributions investigated provided accurate PQ event detection while the inverse Gamma distribution demonstrated the best performance.

We may consider several extensions of our current study. For example, instead of using the same distribution for both F_{Σ_w} and dF_{A_1} , different distribution combinations can be employed to model these two CDF's. Furthermore, it is worthwhile to study the impact of fine-tuning distribution parameters such as the scale parameter β in the inverse Gamma distribution on the performance of the proposed approach.

APPENDIX

In this appendix, we will detail the derivation steps of Eq. (11). Recalling Eq. (7), we know that Z is the sum of a uniform-distributed random variable $X \sim U[-a_1, a_1]$ and a Gaussian-distributed random variable $W \sim N(0, \sigma^2)$. Exploiting the fact that X and W are statistically independent, the PDF of Z is simply given by the convolution of the PDF's of X and W:

$$p_Z(z) = \int_{-a_1}^{+a_1} \frac{1}{2a_1} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-x)^2}{2\sigma^2}} dx.$$
 (17)

Let $y = \frac{(z-x)}{\sqrt{2}\sigma}$. After replacing Y with Z in Eq. (17), we have

$$p_Z(z) = \int_{\frac{z-a_1}{\sqrt{2}\pi}}^{\frac{z+a_1}{\sqrt{2}\pi}} \frac{1}{2a_1} \frac{1}{\sqrt{\pi}} e^{-y^2} dx$$
(18)

$$= \frac{1}{4a_1} \left[\operatorname{erf}(\frac{z+a_1}{\sqrt{2}\sigma}) - \operatorname{erf}(\frac{z-a_1}{\sqrt{2}\sigma}) \right] \quad (19)$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$
 (20)

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