# Energy Efficient Cooperation for Two-Hop Relay Networks

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***Abstract*—We analyze the impact of cooperation on the energy efficiency of two-hop relay transmissions. While cooperation has been demonstrated to improve spectral efficiency, the benefits in terms of energy consumption are not well characterized. We show that cooperation is not always beneficial in this context, since the energy required to facilitate the cooperation can sometimes outweigh its benefit. In this work, we first characterize the optimal energy efficient strategy for a single source-destination transmission aided by multiple relay nodes as a function of network parameters and the transmission rate. We then extend this result to a relay-assisted cellular broadcast network and determine optimal solution which provides guidelines on the cooperative strategy that improves energy efficiency in such networks.**

## I. Introduction

Large network coverage and high spectral efficiency are two central features of wireless networks, and the deployment of relay nodes [1] together with the concept of cooperation [2] have been identified as the principal mean to achieve both these goals. Relay-assisted networks have also been considered in the standardization process for the next-generation mobile broadband communication systems [3]. Recently, driven by the explosive growth of wireless data traffic and the ever increasing economic and environmental costs associated with network operation, energy efficiency has become an important wireless network design consideration. In sensor networks, relaying and cooperation are known to improve the energy efficiency of the network. For example, diversity based node cooperation is analyzed in [4] and shown to provide a significant reduction in total energy consumption. Energy aware routing and cooperation is shown to prolong the lifetime of sensor networks in [5].

In cellular networks, transmission is generally performed over long distances, hence the circuit power consumption is negligible compared to the transmit power. In addition, the communication channel (for downlink direction) is modeled as a point-to-multipoint broadcast channel, where a single base station is serving multiple users at any one time. Several methods have been proposed to design an energy efficient cellular network, such as introducing a sleep mode capability to the base station together with better interference management and resource allocation [6], as well as optimizing the network architecture in terms of base station antenna placement and macro-cell/micro-cell selection [7]. While it is well understood that cooperative relaying enlarges the achievable rate region of a relay-assisted broadcast channel [8]-[9], it is not guaranteed to be the most energy efficient strategy. On the one hand, allowing all the relay nodes to fully cooperate during the second phase of transmission will reduce the relay transmit energy. On the other hand, more energy is required at the base station during the first phase to facilitate full relay cooperation, as all relay nodes must have a complete knowledge of all users' messages. The motivation of this work is to analyze the impact of cooperation on the energy efficiency of a network. In particular, we study the optimal level of cooperation for maximum energy efficiency along with its relationship to network parameters such as transmission rate and relay node position.

Previous work on energy efficiency of cooperative relaying [10]-[11] focused on an opportunistic scheme that used channel state information (CSI) to decide with which subset of the relay nodes to cooperate. The scheme considered in this work does not require CSI tracking, and the level of cooperation is determined by the amount of common information available at the relay nodes. Moreover, instead of considering just one specific scheme such as distributed space time coding or beamforming, we consider a general multi-terminal transmission scheme, and use the corresponding achievable rate region for the analysis.

The remainder of this paper is organized as follows. Section II describes the system and channel model under consideration. Focusing on a canonical system with two relays and one destination node, Section III studies the optimal level of cooperation under different network parameters. Section IV applies the result to a relay-assisted broadcast transmission, and the corresponding numerical analysis is presented in Section V. Section VI summarizes the paper with concluding remarks.

## II. System and Channel Model

A downlink cellular broadcast system is considered in which one base station transmits to $N_D$ destination nodes with the

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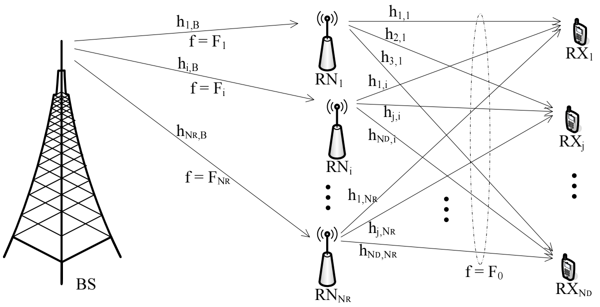



Fig. 1. Relay-assisted cellular broadcast system.

aid of $N_R$ relay nodes. Communications from the base station to the relay nodes is done over orthogonal channels, which occupy different frequency bands from the channel between relay nodes and destination nodes. This model is in accordance with the out-of-band Type-1 Relay deployment scenario in the LTE-Advance specification [12], in which the backhaul link and access link are allocated different frequency bands. Communications from the relay nodes to the destinations, however, is done over a common channel, which opens up the opportunity for cooperative transmission. An illustration of this communication scenario is provided in Fig. 1.

Let $W_j \in \mathcal{W}_j = \{1 \cdots 2^{nR_j}\}$ denote the message to be transmitted from the base station to receiver $j$, where $n$ is the block length and $R_j$ is the rate of this message. In the first communication phase, the base station distributes the message to the relay nodes. The same message can be sent to multiple relays and a message is sent to at least one relay. More formally, let $\mathcal{D} = \{1 \cdots N_D\}$ and define $\mathcal{A}_i \subseteq \mathcal{D}$ to be the set of messages $W_j$ decoded at relay node $i$ with $\bigcup_{i=1}^{N_R} \mathcal{A}_i = \mathcal{D}$. The encoding function performs a mapping

$$\boldsymbol{u}_i : \prod_{j \in \mathcal{A}_i} \mathcal{W}_j \to \mathcal{U}_i^n$$

to generate the codeword $\boldsymbol{u}_i(W_{\mathcal{A}_i})$ drawn from the input alphabet $\mathcal{U}_i$, which is then transmitted to relay $i$ over frequency $F_i$ with an average transmit power $P_B^{(i)} = E\left[\frac{1}{n}\|\boldsymbol{u}_i\|^2\right]$. The message $W_{\mathcal{A}_i} \in \{1 \cdots 2^{n\sum_{j\in\mathcal{A}_i} R_j}\}$ is related to the individual message intended to destination $j \in \mathcal{A}_i$ through[1]

$$W_{\mathcal{A}_i} = \sum_{k=1}^{|\mathcal{A}_i|} W_{\mathcal{A}_i[k]} \left( \prod_{l=1}^{k-1} 2^{nR_{\mathcal{A}_i[l]}} \right),$$

where $|\mathcal{A}_i|$ and $\mathcal{A}_i[k]$ are, respectively, the cardinality and the $k^{th}$ element of the set $\mathcal{A}_i$. The received signal at relay node $i$ is expressed as

$$\boldsymbol{v}_i[t] = h_{i,B}\, \boldsymbol{u}_i[t] + \boldsymbol{z}_{r,i}[t], \tag{1}$$

where we have used an abuse of notation $[t]$ to denote an element of a vector at position $t$, $h_{i,B}$ is the channel gain between the base station and relay node $i$, and $\boldsymbol{z}_{r,i}$ is a white Gaussian noise vector with variance $N_{r,i}$. The decoding function at relay $i$ produces an estimate of the transmitted message

$$\widehat{W}_{\mathcal{A}_i} = f_i(\boldsymbol{v}_i).$$

In order to guarantee $\lim_{n\to\infty} \Pr\left[\widehat{W}_{\mathcal{A}_i} \neq W_{\mathcal{A}_i}\right] < \epsilon$ for an arbitrarily small $\epsilon$, the required transmit power is obtained by inverting Shannon's capacity formula as

$$P_B^{(i)} \geq \left(2^{\sum_{j\in\mathcal{A}_i} 2R_j} - 1\right) N_{r,i}\, \Gamma\, d_{i,B}^{\lambda} \qquad \forall i, \tag{2}$$

where $\Gamma$ is the SNR gap parameter to capture the imperfection of practical modulation and coding relative to the capacity-achieving strategy [13]. The above calculation takes into consideration the distance-dependent path-loss introduced by the propagation channel $h_{i,B}$, which is proportional to the transmission distance $d_{i,B}$ as well as the path-loss exponent $\lambda$. The total transmit power at the base station is given by

$$P_B = \sum_{i=1}^{N_R} P_B^{(i)}. \tag{3}$$

During the second phase, all relay nodes forward to all destinations. The codeword generation at relay node $i$ is described as follows. Firstly, for each message $W_j$ with $j \in \mathcal{A}_i$ known at relay $i$, the encoding function performs a mapping

$$\boldsymbol{x}_j : \mathcal{W}_j \to \mathcal{X}_j^n$$

to produce an intermediate codeword $\boldsymbol{x}_j(W_j)$ drawn from the alphabet $\mathcal{X}_j$. For a given $j$, the same mapping above is used by all relay nodes to facilitate cooperation. The actual codeword transmitted by the relay node $i$ is a weighted sum of the intermediate codewords given by

$$\boldsymbol{r}_i = \sum_{j\in\mathcal{A}_i} \sqrt{\boldsymbol{\mu}_i^{(j)}}\boldsymbol{x}_j, \tag{4}$$

which is then transmitted to all destination nodes over frequency $F_0$ with an average transmit power

$$P_R^{(i)} = E\left[\frac{1}{n}\|\boldsymbol{r}_i\|^2\right] = \sum_{j\in\mathcal{A}_i} \boldsymbol{\alpha}_i^{(j)} P_R^{(i)}.$$

From the above expression, we have $\sum_{j\in\mathcal{A}_i} \boldsymbol{\alpha}_i^{(j)} = 1$, and

$$\boldsymbol{\alpha}_i^{(j)} P_R^{(i)} = \boldsymbol{\mu}_i^{(j)} E\left[\frac{1}{n}\|\boldsymbol{x}_j\|^2\right].$$

The received signal at destination node $j$ is then expressed as

$$\boldsymbol{y}_j[t] = \sum_{i=1}^{N_R} h_{j,i}\, \boldsymbol{r}_i[t] + \boldsymbol{z}_j[t], \tag{5}$$

where $h_{j,i}$ is the channel gain between relay node $i$ to destination node $j$, and $\boldsymbol{z}_j$ is a white Gaussian noise vector with variance $N_j$. The estimate of $W_j$ is generated using the decoding function

$$\widehat{W}_j = g_j(\boldsymbol{y}_j).$$

Since the channel between all the relay nodes to destination node $j$ forms an $N_R$-to-1 multiple access channel (MAC), to

[1]Note that there are many ways to establish the one-to-one correspondence between $W_{\mathcal{A}_i}$ and $\{W_j : j \in \mathcal{A}_i\}$. The one used in this work is chosen due to its simple interpretation as multilevel binning of the component messages.

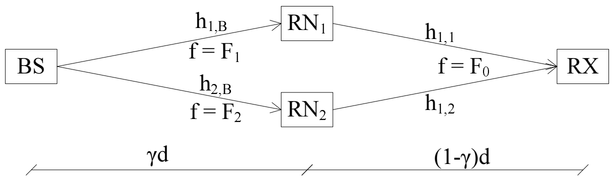



Fig. 2. Canonical relay system with 2 relay nodes

guarantee that $\lim_{n\to\infty} \Pr\left[\widehat{W}_j \neq W_j\right] < \epsilon$ for an arbitrarily small $\epsilon$, the rate vector $(R_1, \cdots, R_{N_D})$ has to be within the corresponding MAC capacity region as given by [14, Th. 14.3.5]. Considering the imperfection of the modulation and coding choice as well as the distance dependent path loss, this corresponds to the following constraints on the relay transmit powers:

$$\sum_{k\in\mathcal{S}}\left(\sum_{i=1}^{N_R} I_{\{k\in\mathcal{A}_i\}}\sqrt{\boldsymbol{\alpha}_i^{(k)}\frac{P_R^{(i)}}{d_{j,i}^{\lambda}}}\right)^2 \geq \left(2^{\sum_{k\in\mathcal{S}}2R_k}-1\right)\, N_j\,\Gamma \quad \forall\mathcal{S}\subseteq\mathcal{D} \text{ and } j\in\mathcal{D}, \quad (6)$$

where the indicator function $I_{\{e\}}$ is equal to 1 when the event $e$ is true, and is equal to 0 otherwise. The total relay transmit power is then given by

$$P_R = \sum_{i=1}^{N_R} P_R^{(i)}. \quad (7)$$

It is apparent from (6) that as more information is available at the relay nodes, a higher degree of cooperation can be achieved, hence enlarging the capacity region and correspondingly minimizing the required relay transmit power. However, in order to distribute more information to the relay nodes, a higher transmit power at the base station is required. The best level of cooperation in terms of energy efficiency is the one that minimizes total power

$$P_{\text{tot}} = P_B + P_R.$$

In the following section, we consider a canonical relay system with two relay nodes and a single destination to demonstrate the best level of cooperation for this model.

## III. Energy-Optimal Level of Cooperation

Following the discussion in the previous section, we define the level of cooperation $\delta$ as the amount of information that is transmitted through both of the relay nodes. Denoting the communication rate as $R$, this corresponds to transmitting common information at rate $R_c = \delta R$ through both relays, and private information at rate $R_p = (1-\delta)R \triangleq \overline{\delta}R$ through only one of the relays. The private information can optionally be further rate-split into $R_{p_1} = \eta\overline{\delta}R$ and $R_{p_2} = (1-\eta)\overline{\delta}R \triangleq \overline{\eta}\overline{\delta}R$.

For simplicity, both relay nodes are located at the same relative distance $\gamma$ with respect to the source-destination distance $d$ as shown in Fig. 2. Hence, $d_{1,B} = d_{2,B} = \gamma d$ and $d_{1,1} = d_{1,2} = (1-\gamma)d \triangleq \overline{\gamma}d$. The noise variance $N_{r,i}$ and $N_j$ at relay $i$ and destination $j$ are also assumed to be equal and denoted as $N$. We consider two scenarios depending on whether or not a further rate splitting is done on the private information.

### A. Case 1: Without Private Information Rate Splitting

We can equivalently consider this scenario as having $\mathcal{D} = \{c, p\}$, $\mathcal{A}_1 = \{c, p\}$, and $\mathcal{A}_2 = \{c\}$. The base station transmit power constraint is given by evaluating (2) and (3) as follows

$$P_B \geq \left(\left(2^{2\delta R}-1\right) + \left(2^{2R}-1\right)\right)\; N\,\Gamma\,(\gamma d)^{\lambda}. \quad (8)$$

Note that both elements in $\mathcal{D}$ refer to the same physical destination node, and the differentiation between the two rates is merely to facilitate rate splitting. Similarly, the relay transmit power constraint is given by evaluating (6) as follows

$$\left(\sqrt{P_R^{(2)}}+\sqrt{\boldsymbol{\alpha}_1^{(c)}P_R^{(1)}}\right)^2 \geq \left(2^{2\delta R}-1\right)N\Gamma(\overline{\gamma}d)^{\lambda} \quad (9a)$$

$$\left(1-\boldsymbol{\alpha}_1^{(c)}\right)P_R^{(1)} \geq \left(2^{2\overline{\delta}R}-1\right)N\Gamma(\overline{\gamma}d)^{\lambda} \quad (9b)$$

$$P_R^{(1)}+P_R^{(2)}+2\sqrt{\boldsymbol{\alpha}_1^{(c)}P_R^{(1)}P_R^{(2)}} \geq \left(2^{2R}-1\right)N\Gamma(\overline{\gamma}d)^{\lambda} \quad (9c)$$

Where we have used the substitution $\boldsymbol{\alpha}_1^{(p)} = 1-\boldsymbol{\alpha}_1^{(c)}$. Note that given (9b) and (9c), the constraint (9a) is redundant. The optimization problem can then be formulated as

$$\begin{array}{ll}\text{Minimize:} & P_B + P_R^{(1)} + P_R^{(2)}\\ \text{Subject to:} & (8)\ \&\ (9),\end{array}$$

which can be solved as follows. Given a fixed relay position $\gamma$ and level of cooperation $\delta$, the base station transmit power in (8) is fixed. The following proposition is useful to find the minimum relay transmit power required: $P_R^{(1)}$ and $P_R^{(2)}$.

*Proposition 1:* For a two user MAC channel, given a target rate pair $(R_1, R_2)$, the minimum total transmit power $P$ is achieved when the point $(R_1, R_2)$ lays on the surface of the capacity region. Moreover, it can be shown that the point $(R_1, R_2)$ corresponds to a corner point of a MAC region with a certain power allocation $(P_1, P_2)$ having the same total power $P_1 + P_2 = P$. As such, there is no loss of optimality in choosing the corner point to be $(R_1, R_2)$ when calculating the minimum total power required by the MAC channel.

Applying the above proposition to (9), it is clear that we should maximize the sum-rate by setting $\boldsymbol{\alpha}_1^{(c)}$ to its maximum value allowable as follows

$$\boldsymbol{\alpha}_1^{(c)} = 1 - \left(\left(2^{2\overline{\delta}R}-1\right)N\Gamma(\overline{\gamma}d)^{\lambda}/P_R^{(1)}\right).$$

The above choice of $\boldsymbol{\alpha}_1^{(c)}$ corresponds to choosing a corner point of the MAC capacity region. Substituting into (9), the problem of minimizing the total relay transmit power becomes

$$\begin{array}{ll}\text{Minimize:} & P_R^{(1)} + P_R^{(2)}\\ \text{Subject to:} & \sqrt{P_R^{(2)}} + \sqrt{P_R^{(1)} - K_2} \geq \sqrt{K_1 - K_2},\end{array}$$

where:

$$K_1 = \left(2^{2R}-1\right)N\Gamma(\overline{\gamma}d)^{\lambda} \quad (10)$$

$$K_2 = \left(2^{2\overline{\delta}R}-1\right)N\Gamma(\overline{\gamma}d)^{\lambda}. \quad (11)$$

The above optimization problem can be solved directly, giving the optimal relay transmit power of

$$P_R^{(1)} = (K_1 + 3K_2)/4 \quad (12)$$
$$P_R^{(2)} = (K_1 - K_2)/4. \quad (13)$$

The total power consumption is therefore given by

$$P_{\text{tot}}^* = \left(\left(2^{2\delta R} - 1\right) + \left(2^{2R} - 1\right)\right) N\,\Gamma\,(\gamma d)^\lambda + \left(\left(2^{2R}-1\right) + \left(2^{2\bar{\delta}R}-1\right)\right) N\Gamma(\overline{\gamma}d)^\lambda/2. \quad (14)$$

The above expression holds for any value of $\delta$ and $\gamma$. The best relay location can be found by taking the first derivatives of the above equation with respect to $\gamma$ and equating it to zero, which results in the following choice

$$\gamma_{\text{opt}} = \left(1 + \left(\frac{2\left(\left(2^{2\delta R} - 1\right) + \left(2^{2R} - 1\right)\right)}{\left(2^{2R}-1\right) + \left(2^{2\bar{\delta}R}-1\right)}\right)^{\frac{1}{(\lambda-1)}}\right)^{-1}.$$

Substituting the optimal $\gamma_{\text{opt}}$ back into (14), we obtain the minimum transmit power requirement as a function of the level of cooperation $\delta$, and the most efficient level of cooperation is the one that minimizes the total power.

### *B. Case 2: With Private Information Rate Splitting*

In the previous subsection, we assume that one of the relay nodes has complete information about the message while the other relay node has only part of it depending on the level of cooperation. Here, we consider the case where none of the relay nodes has complete information, and the private information is split among the two relay nodes. Equivalently, we can describe this scenario as having $\mathcal{D} = \{c, p_1, p_2\}$, $\mathcal{A}_1 = \{c, p_1\}$, and $\mathcal{A}_2 = \{c, p_2\}$. The base station transmit power constraint is given by evaluating (2) and (3) as follows

$$P_B \geq \left(\left(2^{2(\delta+\eta\bar{\delta})R}-1\right) + \left(2^{2(\delta+\overline{\eta}\bar{\delta})R}-1\right)\right) N\,\Gamma\,(\gamma d)^\lambda. \quad (15)$$

Similarly, the relay transmit power constraint is given by evaluating (6) as follows

$$\left(1-\boldsymbol{\alpha}_1^{(c)}\right) P_R^{(1)} \geq \left(2^{2\eta\bar{\delta}R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16a)$$
$$\left(1-\boldsymbol{\alpha}_2^{(c)}\right) P_R^{(2)} \geq \left(2^{2\overline{\eta}\bar{\delta}R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16b)$$
$$\left(\sqrt{\boldsymbol{\alpha}_1^{(c)} P_R^{(1)}} + \sqrt{\boldsymbol{\alpha}_2^{(c)} P_R^{(2)}}\right)^2 \geq \left(2^{2\delta R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16c)$$
$$\left(1-\boldsymbol{\alpha}_1^{(c)}\right) P_R^{(1)} + \left(1-\boldsymbol{\alpha}_2^{(c)}\right) P_R^{(2)} \geq \left(2^{2\bar{\delta}R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16d)$$
$$P_R^{(1)} + \boldsymbol{\alpha}_2^{(c)} P_R^{(2)} + 2\sqrt{\boldsymbol{\alpha}_1^{(c)} P_R^{(1)} \boldsymbol{\alpha}_2^{(c)} P_R^{(2)}} \geq \left(2^{2(\delta+(\eta\bar{\delta}))R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16e)$$
$$\boldsymbol{\alpha}_1^{(c)} P_R^{(1)} + P_R^{(2)} + 2\sqrt{\boldsymbol{\alpha}_1^{(c)} P_R^{(1)} \boldsymbol{\alpha}_2^{(c)} P_R^{(2)}} \geq \left(2^{2(\delta+(\overline{\eta}\bar{\delta}))R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16f)$$
$$P_R^{(1)} + P_R^{(2)} + 2\sqrt{\boldsymbol{\alpha}_1^{(c)} P_R^{(1)} \boldsymbol{\alpha}_2^{(c)} P_R^{(2)}} \geq \left(2^{2R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (16g)$$

Again, we have used the substitution $\boldsymbol{\alpha}_1^{(p_1)} = 1 - \boldsymbol{\alpha}_1^{(c)}$ and $\boldsymbol{\alpha}_2^{(p_2)} = 1 - \boldsymbol{\alpha}_2^{(c)}$ in the above. It is also apparent that to satisfy the above constraints, we only need to consider (16a), (16b), (16d), and (16g), while (16c), (16e), and (16f) are redundant. The optimization problem can now be formulated as

$$\begin{aligned} \text{Minimize:} \quad & P_B + P_R^{(1)} + P_R^{(2)} \\ \text{Subject to:} \quad & (15)\ \&\ (16). \end{aligned}$$

Compared to the case without private information rate splitting, this problem is more complicated as the constraint (16) involves a 3-dimensional polyhedron capacity region.

Extending Proposition 1 to the three users case and applying it to the above problem, it is apparent that the parameters should be chosen to maximize the sum-rate. First, observe that the private information rate splitting parameter $\eta$ does not affect the sum-rate constraint since it does not appear anywhere in (16g). As such, it can be optimized against the base station power in (15) as follows

$$\eta_{\text{opt}} = \arg\min_{\eta} P_B = 0.5.$$

With the above choice of private rate splitting parameter, for a fixed level of cooperation $\delta$ and relay position $\gamma$, the base station transmit power is fixed. As stated earlier, we need to maximize the sum-rate to find the minimum relay transmit power required. There are two ways to achieve this corresponding to the following choice of $\boldsymbol{\alpha}_1^{(c)}$ and $\boldsymbol{\alpha}_2^{(c)}$ pair

$$\boldsymbol{\alpha}_1^{(c)} = 1 - K_3/P_R^{(1)} \quad \text{and} \quad \boldsymbol{\alpha}_2^{(c)} = 1 - (K_2 - K_3)/P_R^{(2)},$$

or

$$\boldsymbol{\alpha}_1^{(c)} = 1 - (K_2 - K_4)/P_R^{(1)} \quad \text{and} \quad \boldsymbol{\alpha}_2^{(c)} = 1 - K_4/P_R^{(2)},$$

where:

$$K_3 = \left(2^{2\eta_{\text{opt}}\bar{\delta}R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \quad (17)$$
$$K_4 = \left(2^{2\overline{\eta}_{\text{opt}}\bar{\delta}R}-1\right) N\Gamma(\overline{\gamma}d)^\lambda. \quad (18)$$

With $K_1$ and $K_2$ defined as (10) and (11), respectively, substituting the above into (16) results in the following problem formulation for minimizing the total relay transmit power

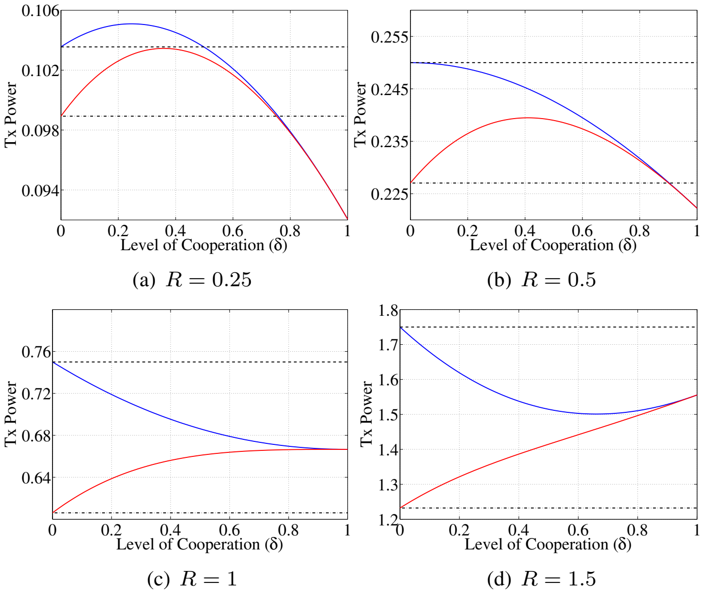
$$\begin{aligned} \text{Minimize:} \quad & P_R^{(1)} + P_R^{(2)} \\ \text{Subject to:} \quad & \sqrt{P_R^{(1)} - K_3} + \sqrt{P_R^{(2)} - (K_2 - K_3)} \geq \sqrt{K_1 - K_2} \\ & \qquad\text{or} \\ & \sqrt{P_R^{(1)} - (K_2 - K_4)} + \sqrt{P_R^{(2)} - K_4} \geq \sqrt{K_1 - K_2}, \end{aligned}$$

which can be solved directly, giving the optimal relay power

$$P_R^{(1)} = (K_1 - K_2)/4 + K_3 \quad (19)$$
$$P_R^{(2)} = (K_1 + 3K_2)/4 - K_3, \quad (20)$$

when the first constraint is used, or

$$P_R^{(1)} = (K_1 + 3K_2)/4 - K_4 \quad (21)$$
$$P_R^{(2)} = (K_1 - K_2)/4 + K_4, \quad (22)$$




(a) $R = 0.25$ (b) $R = 0.5$

(c) $R = 1$ (d) $R = 1.5$

Fig. 3. Total transmit power required at different levels of cooperation. Blue lines represent the case without private information rate splitting, red lines represent the case with private information rate splitting, dashed lines represent no cooperation utilizing only one relay node, and dot-dashed lines represent no cooperation with equal amount of information at both relays.

when the second constraint is used. Note that both choices result in the same total relay transmit power. Correspondingly, the overall transmit power can be calculated as

$$P_{\text{tot}}^{*} = \left(2\left(2^{2(\delta+\frac{\bar{\delta}}{2})R}-1\right)\right) N\,\Gamma\,(\gamma d)^{\lambda} + \left(\left(2^{2R}-1\right)+\left(2^{2\bar{\delta}R}-1\right)\right) N\Gamma(\bar{\gamma} d)^{\lambda}/2. \quad (23)$$

Using the same approach, the best relay location is given by

$$\gamma_{\text{opt}} = \left(1+\left(\frac{4\left(2^{2(\delta+\frac{\bar{\delta}}{2})R}-1\right)}{\left(2^{2R}-1\right)+\left(2^{2\bar{\delta}R}-1\right)}\right)^{\frac{1}{(\lambda-1)}}\right)^{-1}.$$

### C. *Performance Comparison*

In this subsection, the total transmit power calculated earlier as equation (14) and (23) are evaluated using the optimal relay location $\gamma_{\text{opt}}$. For comparison purposes, we also show the total power consumption of two non-cooperative schemes. The first one uses only one of the relay nodes to convey all the messages, while the second one distributes half of the messages to each relay node without any overlap.

From equation (14) and (23), it is observed that many of the parameters including noise variance $N$, SNR gap $\Gamma$, source-destination distance $d$, as well as the path loss exponent $\lambda$ only affect the scaling of the total power required. As such, they do not alter the trend on how the total transmit power behaves as the level of cooperation changes. Fig. 3 shows the total power of the different schemes at different target rates $R$. The accompanying optimal relative relay position $\gamma_{\text{opt}}$ for different levels of cooperation is given in Fig. 4.

Several observations can be made from the figure. Firstly, the performances of the two non-cooperative schemes are constant, and the one that uses both relays by distributing half of the disjoint messages to each of them is superior. Note that the two cooperative schemes collapse to the non-cooperative scheme when $\delta = 0$, therefore they coincide at the initial point. Secondly, the cooperative scheme with private information rate splitting always performs better than the one without rate splitting, but the performances of the two coincide when full cooperation is adopted (at $\delta = 1$). The optimal relay positions are also the same for both schemes at this cooperation level, whereby the relay nodes should be placed closer to the base station at one-third of the distance. Lastly, which is also the most interesting, is that a higher level of cooperation does not always result in lower transmit power. When the target rate is low, it is best to fully cooperate, and the relay nodes should be placed closer to the base station. As the target rate is increased, there is a point whereby cooperation is no longer preferable for any rate above it. In that regime, it is more efficient not to cooperate, and as the target rate is increased, the relay nodes should be placed closer to the destination. The reason for this behavior is that as the target rate becomes high, the cost of distributing a larger amount of messages to the relay nodes to facilitate cooperation outweighs the power saving advantage of cooperation.

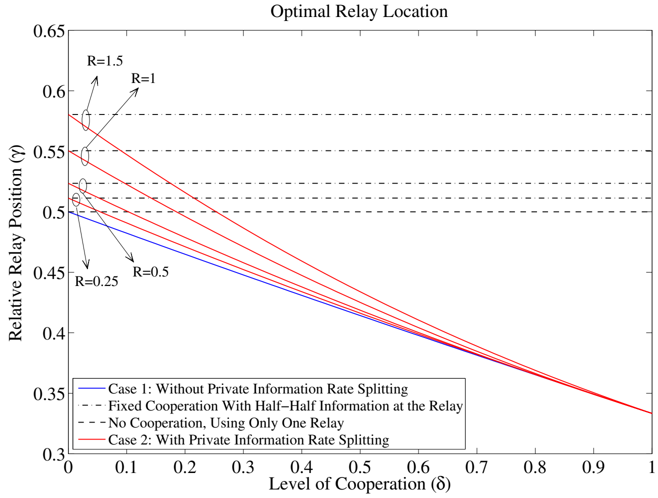



Fig. 4. Optimal relay position at different levels of cooperation.

## IV. Cooperative Strategy in Relay-Assisted Broadcasting

In the previous section, we have demonstrated that cooperation is not always a preferable choice in terms of energy efficiency. Here, we consider a relay-assisted cellular broadcasting system and utilize the result obtained earlier to study the most energy efficient cooperation strategy.

Each user in the network establishes a communication link of a certain rate $R_j$ with the base station. Without loss of generality, we assume $R_j \leq R_k$ if $j < k$. No rate splitting is considered as it is not usually adopted in practice, and a maximum transmit power limit of $P_B^{\max}$ at the base station and $P_R^{\max}$ at the relay nodes are considered. As such, there is a finite number of cooperation level possible between the relay nodes. We limit the analysis to the case of two relay and three destination nodes in the system, and we maintain the notation for the relay node placements $\gamma$ as the relative distance with respect to the source-destination distance $d$, which is assumed to be the same for all nodes. This equidistant assumption greatly simplifies the analysis, as all destinations will have the same signal quality as long as the noise variance $N$ at all nodes is equal. The following subsections discuss different cooperation levels and the corresponding total transmit power.

### A. Case 1: Both relay nodes decode all the messages

This corresponds to the full cooperation scenario, with $\mathcal{A}_1 = \mathcal{A}_1 = \{1,2,3\}$. The base station transmit power in this case can be readily calculated using (3) as follows

$$P_B = 2\left(2^{2(R_1+R_2+R_3)} - 1\right) N\,\Gamma\,(\gamma d)^{\lambda}. \tag{24}$$

When the above value is larger than $P_B^{\max}$, this level of cooperation is not supported.

The transmit power required at the relay nodes can be calculated using (7). Instead of dealing with the constraints given in (6), an easier approach to find the relay transmit power is by considering all the messages as one single equivalent message with rate $R_1+R_2+R_3$. Since both relay nodes have complete information about the message, the relay transmit power constraint is given by

$$\left(\sqrt{P_R^{(1)}} + \sqrt{P_R^{(2)}}\right)^2 \geq \left(2^{2(R_1+R_2+R_3)} - 1\right) N\,\Gamma\,(\overline{\gamma} d))^{\lambda}.$$

The minimum relay transmit power can then be calculated as

$$P_R^{(1)} = P_R^{(2)} = \frac{\left(2^{2(R_1+R_2+R_3)} - 1\right) N\,\Gamma\,(\overline{\gamma} d)^{\lambda}}{4}.$$

Similarly, when the above quantity is larger than $P_R^{\max}$, this level of cooperation is not supported. Combining with the base station transmit power, the total transmit power is given by

$$P_{\text{tot}} = \left(2^{2(R_1+R_2+R_3)} - 1\right) N\,\Gamma\,d^{\lambda}\left(2\gamma^{\lambda} + 0.5\overline{\gamma}^{\lambda}\right). \tag{25}$$

### B. Case 2: Only two messages are decoded at both relays

This is the scenario where one of the relay nodes receives all the messages while the other relay node receives only two out of the three messages from the base station. Consider the choice of $\mathcal{A}_1 = \{1,2,3\}$ and $\mathcal{A}_2 = \{2,3\}$, while the other choices can be similarly analyzed. The base station transmit power is given by

$$P_B = \left(\left(2^{2(R_1+R_2+R_3)} - 1\right) + \left(2^{2(R_2+R_3)} - 1\right)\right) N\Gamma(\gamma d)^{\lambda}. \tag{26}$$

Note that the above quantity is always smaller than (24), therefore it is more likely to satisfy the constraint $P_B \leq P_B^{\max}$.

The required transmit power at the relay nodes can be calculated by applying Proposition 1 and following the same approach as in Section IIIA. It can be shown that the relay transmit power minimization problem can be formulated as follows

$$\begin{aligned} \text{Minimize:} \quad & P_R^{(1)} + P_R^{(2)} \\ \text{Subject to:} \quad & \sqrt{P_R^{(2)}} + \sqrt{P_R^{(1)} - K_6} \geq \sqrt{K_5 - K_6}, \end{aligned}$$

where:

$$K_5 = \left(2^{2(R_1+R_2+R_3)} - 1\right) N\Gamma(\overline{\gamma} d)^{\lambda} \tag{27}$$

$$K_6 = \left(2^{2R_1} - 1\right) N\Gamma(\overline{\gamma} d)^{\lambda}. \tag{28}$$

The above optimization problem can be solved directly, giving the optimal relay transmit power of

$$P_R^{(1)} = (K_5 + 3K_6)/4 \tag{29}$$

$$P_R^{(2)} = (K_5 - K_6)/4. \tag{30}$$

It is apparent that the power assigned to the first relay node (who has the complete messages) is always larger than that assigned to the second relay node. When the maximum relay transmit power constraint is not satisfied, the best choice is to set

$$P_R^{(1)} = P_R^{\max} \tag{31}$$

$$P_R^{(2)} = \left(\sqrt{K_5 - K_6} - \sqrt{P_R^{\max} - K_6}\right)^2, \tag{32}$$

as long as both $K_6$ and the quantity $P_R^{(2)}$ given above do not exceed $P_R^{\max}$. Combining with the base station transmit power, the total transmit power is given by

$$P_{\text{tot}} = \begin{cases} \left(2^{2(R_1+R_2+R_3)} - 1\right) N\Gamma d^{\lambda}(\gamma^{\lambda} + 0.5\overline{\gamma}^{\lambda}) + \\ \quad \left(2^{2(R_2+R_3)} - 1\right) N\Gamma(d\gamma)^{\lambda} - 0.5\left(2^{2R_1} - 1\right) N\Gamma(\overline{\gamma} d)^{\lambda} \\ \qquad\qquad \text{if } (K_5 + 3K_6)/4 \leq P_R^{\max} \\ \left(\left(2^{2(R_1+R_2+R_3)} - 1\right) + \left(2^{2(R_2+R_3)} - 1\right)\right) N\,\Gamma\,(\gamma d)^{\lambda} + \\ \quad P_R^{\max} + \left(\sqrt{2^{2R_1}\left(2^{2(R_2+R_3)} - 1\right) N\Gamma(\overline{\gamma} d)^{\lambda}} - \right. \\ \qquad \left. \sqrt{P_R^{\max} - \left(2^{2R_1} - 1\right) N\Gamma(\overline{\gamma} d)^{\lambda}}\right)^2 \\ \qquad \text{if } \max\left(K_6, \left(\sqrt{K_5 - K_6} - \sqrt{P_R^{\max} - K_6}\right)^2\right) \leq P_R^{\max} \end{cases} \tag{33}$$

### C. Case 3: Only one of the messages is decoded at both relays

In this case, only one of the relay nodes receives all the messages, while the other relay node only receives one of the messages. Consider the choice of $\mathcal{A}_1 = \{1,2,3\}$ and $\mathcal{A}_2 = \{3\}$, while other choices can be similarly analyzed. The base station transmit power is given by

$$P_B = \left(\left(2^{2(R_1+R_2+R_3)} - 1\right) + \left(2^{2R_3} - 1\right)\right) N\,\Gamma\,(\gamma d)^{\lambda}. \tag{34}$$

Again, the above quantity is always smaller than both (24) and (26), therefore it is more likely to satisfy $P_B \leq P_B^{\max}$.

Using the same technique as in the previous case, the optimal relay transmit power is given by the same formula as (29)-(30), with the term $K_6$ replaced by $K_7$ defined as

$$K_7 = \left(2^{2(R_1+R_2)} - 1\right) N\Gamma(\overline{\gamma} d)^{\lambda}. \tag{35}$$

When the maximum relay transmit power constraint is not satisfied, the optimal relay transmit power is given by (31) and (32) instead (with the same substitution of $K_6$ with $K_7$). Correspondingly, the total transmit power is given by

$$P_{\text{tot}} = \begin{cases} \left(2^{2(R_1+R_2+R_3)}{-}1\right) N\Gamma d^\lambda (\gamma^\lambda + 0.5\overline{\gamma}^\lambda) + \\ \quad \left(2^{2R_3}{-}1\right) N\Gamma(d\gamma)^\lambda - 0.5\left(2^{2(R_1+R_2)}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda \\ \qquad \text{if } (K_5+3K_7)/4 \le P_R^{\max} \\ \left(\left(2^{2(R_1+R_2+R_3)}-1\right)+\left(2^{2R_3}-1\right)\right) N\,\Gamma\,(\gamma d)^\lambda + \\ \quad P_R^{\max} + \Big(\sqrt{2^{2(R_1+R_2)}\left(2^{2R_3}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda} - \\ \quad \sqrt{P_R^{\max} - \left(2^{2(R_1+R_2)}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda}\Big)^2 \\ \qquad \text{if } \max\left(K_7, \left(\sqrt{K_5-K_7} - \sqrt{P_R^{\max}-K_7}\right)^2\right) \le P_R^{\max} \end{cases} \tag{36}$$

### D. Case 4: No relay cooperation using both of the relay nodes

Here, there is no common message sent to both of the relay nodes, hence the level of cooperation is zero. Consider the choice of $\mathcal{A}_1 = \{1,2\}$ and $\mathcal{A}_2 = \{3\}$, while the other choices can be similarly analyzed. The base station transmit power is then given by

$$P_B = \left(\left(2^{2(R_1+R_2)}-1\right)+\left(2^{2R_3}-1\right)\right) N\,\Gamma\,(\gamma d)^\lambda. \tag{37}$$

Since there is no cooperation involved, no optimization is required. The optimal relay transmit power is given by either

$$P_R^{(1)} = \left(2^{2(R_1+R_2)}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda \tag{38}$$

$$P_R^{(2)} = 2^{2(R_1+R_2)}\left(2^{2R_3}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda, \tag{39}$$

or

$$P_R^{(1)} = 2^{2(R_1+R_2)}\left(2^{2R_3}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda \tag{40}$$

$$P_R^{(2)} = \left(2^{2R_3}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda, \tag{41}$$

whichever pair meets the maximum relay transmit power constraint. When none of them meets the constraint, the best relay transmit power is to set one of the relay powers to $P_R^{\max}$ (provided it is greater than $\left(2^{2\max(R_1+R_2,R_3)}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda$), and the other relay power to

$$\left(2^{2(R_1+R_2+R_3)}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda - P_R^{\max}, \tag{42}$$

as long as it is less than $P_R^{\max}$. When no such solution exist, this level of cooperation is not supported.

Combining with the base station transmit power, the total transmit power is then given by

$$P_{\text{tot}} = \left(\left(2^{2(R_1+R_2)}-1\right)+\left(2^{2R_3}-1\right)\right) N\,\Gamma\,(\gamma d)^\lambda + \left(2^{2(R_1+R_2+R_3)}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda. \tag{43}$$

### E. Case 5: No cooperation using only a single relay node

In this case, the base station transmits all the messages to a single relay node during the first phase. Consequently, during the second phase, only one relay node will broadcast the messages to all destination nodes. In other words, it is either $\mathcal{A}_1 = \{1,2,3\}$ and $\mathcal{A}_2 = \emptyset$, or $\mathcal{A}_1 = \emptyset$ and $\mathcal{A}_2 = \{1,2,3\}$. The transmit power at the base station and the relay nodes are

$$P_B = (2^{2(R_1+R_2+R_3)} - 1) N\Gamma(\gamma d)^\lambda \tag{44}$$

$$P_R = (2^{2(R_1+R_2+R_3)} - 1) N\Gamma(\overline{\gamma}d)^\lambda. \tag{45}$$

As long as both of the above satisfy the maximum transmit power constraint, the total transmit power is

$$P_{\text{tot}} = \left(2^{2(R_1+R_2+R_3)} - 1\right) N\Gamma d^\lambda(\gamma^\lambda + \overline{\gamma}^\lambda). \tag{46}$$

It is worth noting that under the equidistant assumption, the total relay transmit power in this case is identical to the previous case where the messages are forwarded by both relays without cooperation. However, the total base station transmit power in (44) is larger compared to (37). As such, using only a single relay node is always inferior to using both relay nodes when no relay cooperation is involved.

### F. Case 6: Only one message is decoded at both relays, but none of the relays has all the messages

This scenario is similar to that described in Case 3, where only one of the messages is sent to both relay nodes. However, in this case, none of the relay nodes receives all the messages. Consider the choice of $\mathcal{A}_1 = \{1,3\}$ and $\mathcal{A}_2 = \{2,3\}$, while other choices can be similarly analyzed. The base station transmit power is then given by

$$P_B = \left(\left(2^{2(R_1+R_3)}-1\right)+\left(2^{2(R_2+R_3)}-1\right)\right) N\,\Gamma\,(\gamma d)^\lambda. \tag{47}$$

As far as the relay transmit power is concerned, it is observed that the same approach described in Section IIIB can be applied, resulting in the following two choices of power allocation parameters $\boldsymbol{\alpha}_1^{(3)}$ and $\boldsymbol{\alpha}_2^{(3)}$:

$$\boldsymbol{\alpha}_1^{(3)} = 1 - K_6/P_R^{(1)} \quad \text{and} \quad \boldsymbol{\alpha}_2^{(3)} = 1 - (K_7-K_6)/P_R^{(2)},$$

or

$$\boldsymbol{\alpha}_1^{(3)} = 1 - (K_7-K_8)/P_R^{(1)} \quad \text{and} \quad \boldsymbol{\alpha}_2^{(3)} = 1 - K_8/P_R^{(2)},$$

where

$$K_8 = \left(2^{2R_2}{-}1\right) N\Gamma(\overline{\gamma}d)^\lambda. \tag{48}$$

The terms $K_6$ and $K_7$ are defined in (28) and (35), respectively. Using the same value of $K_5$ as in (27), the problem of minimizing the total relay transmit power can be formulated as follows

$$\begin{aligned} &\text{Minimize:} && P_R^{(1)} + P_R^{(2)} \\ &\text{Subject to:} && \sqrt{P_R^{(1)}{-}K_6} + \sqrt{P_R^{(2)}{-}(K_7{-}K_6)} \ge \sqrt{K_5{-}K_7} \\ & && \text{or} \\ & && \sqrt{P_R^{(1)}{-}(K_7{-}K_8)} + \sqrt{P_R^{(2)}{-}K_8} \ge \sqrt{K_5{-}K_7}, \end{aligned}$$

which can be solved directly, giving the optimal relay power

$$P_R^{(1)} = (K_5 - K_7)/4 \;+\; K_6 \tag{49}$$

$$P_R^{(2)} = (K_5 + 3K_7)/4 \;-\; K_6, \tag{50}$$

when the first constraint is used, or

$$P_R^{(1)} = (K_5 + 3K_7)/4 \; - \; K_8 \tag{51}$$
$$P_R^{(2)} = (K_5 - K_7)/4 \; + \; K_8, \tag{52}$$

when the second constraint is used. Note that both choices result in the same total relay transmit power as long as they satisfy the maximum transmit power constraint. When none of the solutions satisfy the constraint, the best strategy is to set $P_R^{(2)} = P_R^{\max}$ (provided it is greater than $K_8$), and set

$$P_R^{(1)} = \left(\sqrt{K_5 - K_7} - \sqrt{P_R^{\max} - K_8}\right)^2 + (K_7 - K_8).$$

Correspondingly, the total transmit power is given by

$$P_{\text{tot}} = \begin{cases} 0.5\left(\left(2^{2(R_1+R_2+R_3)}-1\right) + \left(2^{2(R_1+R_2)}-1\right)\right) N\Gamma(\overline{\gamma}d)^\lambda + \\ \quad \left(\left(2^{2(R_1+R_3)}-1\right) + \left(2^{2(R_2+R_3)}-1\right)\right) \; N\,\Gamma\,(\gamma d)^\lambda \\ \qquad \text{if } \min(\max((49),(50)), \max((51),(52))) \leq P_R^{\max} \\ \left(\left(2^{2(R_1+R_3)}-1\right) + \left(2^{2(R_2+R_3)}-1\right)\right) \; N\,\Gamma\,(\gamma d)^\lambda + \\ \quad P_R^{\max} + \left(\sqrt{2^{2(R_1+R_2)}\left(2^{2R_3}-1\right) N\Gamma(\overline{\gamma}d)^\lambda} - \right. \\ \quad \left.\sqrt{P_R^{\max} - \left(2^{2R_2}-1\right) N\Gamma(\overline{\gamma}d)^\lambda}\right)^2 + \\ \qquad 2^{2R_2}\left(2^{2R_1}-1\right) N\Gamma(\overline{\gamma}d)^\lambda \\ \text{if } \max\left(K_8, \left(\sqrt{K_5-K_7}-\sqrt{P_R^{\max}-K_8}\right)^2+(K_7-K_8)\right) \leq P_R^{\max} \end{cases} \tag{53}$$

## V. Numerical Studies

In this section, we evaluate the total transmit power as derived in the previous section. A noise spectral density of $N_0 = -123.5$ dBm/Hz for a transmission over 10 MHz is considered. The path loss exponent $\lambda$ and source-destination distance $d$ are set to 3 and 5 Km respectively. The modulation and coding used are assumed to exhibit a $\Gamma = 2.5$ dB gap from Shannon's limit (which is achievable using either Turbo codes or LDPC codes [15]). The maximum transmit powers at the base station and relay nodes are set to 44.77 dBm (30 Watts) and 41.76 dBm (15 Watts), respectively. Two scenarios are considered, namely a uniform rate scenario where the requested rate from all users is identical, and a non-uniform rate scenario, where the requested rates are different.

Fig. 5 shows the total transmit power requirement for a uniform requested rate $(R_1, R_2, R_3) = (0.5, 0.5, 0.5)$ b/s/Hz employing six different cooperation strategies as described in the previous section. The solid lines represent the scheme where relay nodes cooperate, while the dashed lines represent the scheme where the relay nodes forward different subsets of messages without any cooperation.

It can be seen from the figure that the most energy efficient strategy changes with the relay nodes' position. When the relay nodes are close to the base station, the most efficient strategy is the one given in Case 1, whereby the base station transmits all the users' messages to all relay nodes during the first slot, and lets the relay nodes fully cooperate during the second slot. As the relay nodes are placed further away from the base station, at approximately one third of the source-destination distance, the most energy efficient strategy changes

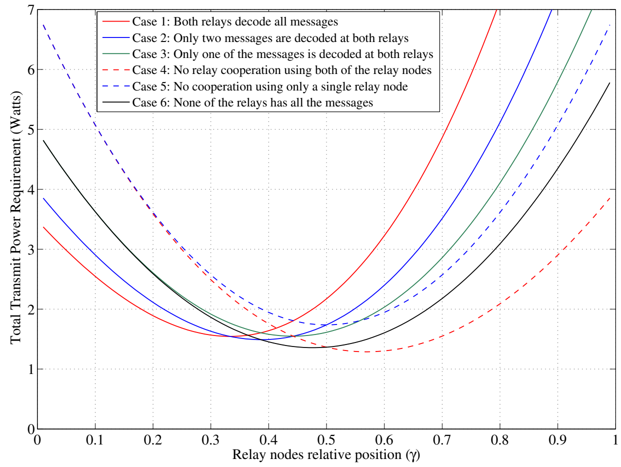



Fig. 5. Total transmit power required for $(R_1, R_2, R_3) = (0.5, 0.5, 0.5)$.

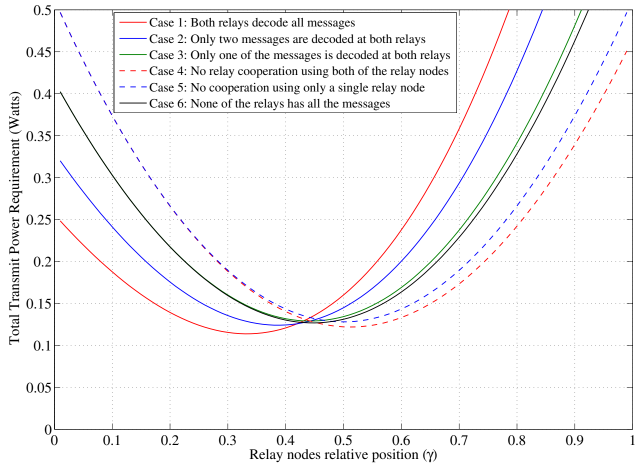



Fig. 6. Total transmit power required for $(R_1, R_2, R_3) = (0.1, 0.1, 0.1)$.

to the one given in Case 2. Here, only two out of the three users' messages are transmitted to both relay nodes, while the other user's message is transmitted to only one of the relay nodes. This indicates that the level of cooperation should be reduced as the relay nodes are further away from the base station. This trend continues to hold as observed in the figure, whereby the most efficient strategy changes to that given in Case 6 (cooperate on only one of the three users' messages) until the relay nodes are approximately at the middle point, and finally changes again to that given in Case 4 (no relay cooperation is used) for any relay position beyond that point. This behavior is reasonable, as the price of transmit power at the base station to facilitate cooperation is higher as the path loss (and correspondingly the distance) between the base station and the relay nodes are larger. Another interesting observation in Fig. 5 is that the global minimum of the total transmit power requirement is achieved using the non-cooperative strategy (Case 4) with the relay position around the middle point (slightly closer to the destination). When the requested rate is very low, in accordance with the discussion in Section III, the global minimum of the total transmit power is achieved using full cooperation as shown in Fig. 6 for

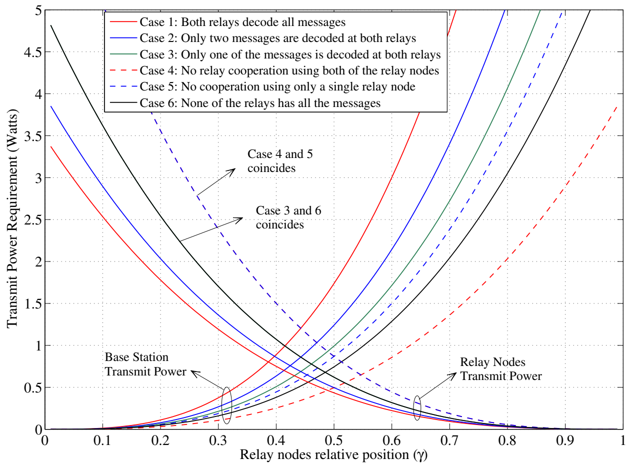



Fig. 7. Transmit power requirement at base station and relay nodes for $(R_1, R_2, R_3) = (0.5, 0.5, 0.5)$.

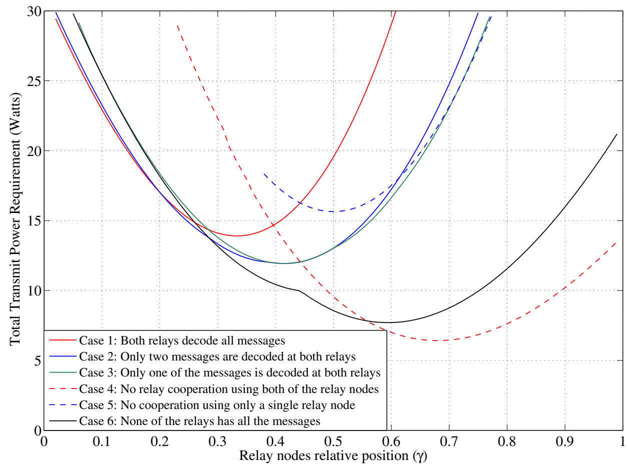



Fig. 8. Total transmit power required for $(R_1, R_2, R_3) = (0.5, 1, 1.5)$.

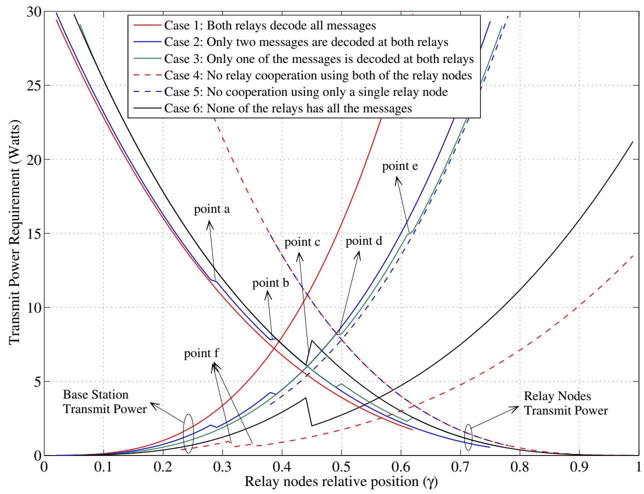



Fig. 9. Transmit power requirement at base station and relay nodes for $(R_1, R_2, R_3) = (0.5, 1, 1.5)$.

$(R_1, R_2, R_3) = (0.1, 0.1, 0.1)$ b/s/Hz.

We also plot the transmit power requirement at the base station and the relay nodes separately in Fig. 7 for the rate $(R_1, R_2, R_3) = (0.5, 0.5, 0.5)$ b/s/Hz. As observed in the figure, the savings on the base station transmit power when the non-cooperative scheme (Case 4) is used at the regime where the relay nodes are closer to the destinations are the most prominent. It is also observed that the transmit power required is relatively small, and the maximum power constraint is never exceeded. Fig. 7 also shows that the scheme in Case 4 and Case 6 are always better than Case 5 and Case 3, respectively. This is because, while the relay transmit power of the pair are equal, the base station transmit power is higher in Case 3 and Case 5. This is also the reason why the two schemes never produce the most energy efficient strategy in Fig. 5.

Fig. 8 shows the total transmit power requirement for a non-uniform requested rate $(R_1, R_2, R_3) = (0.5, 1, 1.5)$ b/s/Hz. The same observation is made as in the equal-rate case, whereby the most energy efficient level of cooperation depends on the relay nodes' positions. A high level of cooperation is the most efficient when the relay nodes are closer to the base station, while a lower level of cooperation is more efficient as the relay nodes are further away from the base station. As opposed to the previous scenario, the maximum transmit power constraint is exceeded in some cases, which is reflected by the absence of some curves at a certain regime of the relative relay position.

The transmit power at the base station and relay nodes are plotted separately in Fig. 9. Instead of smooth curves as obtained in the uniform rate case, there are some discontinuities on some of the curves. We have marked the discontinuity points in Fig. 9, and give the explanation in the following.

At point a, the base station reduces the level of cooperation by changing the set of messages to be sent to both relay nodes. Prior to this point, the messages of user 2 and user 3 were sent to both relay nodes for cooperation, hence the cooperation is used for transmission at rate $R_2 + R_3 = 2.5$. After this point, the messages of user 1 and user 3 are used for cooperation instead. This signifies that at that relay position, the base station transmit power saved by reducing the level of cooperation is larger than the extra power needed to relay the messages with less cooperation. A similar argument can be applied at point b, where the message of user 1 and user 2 are sent to both relay nodes for cooperation instead of the messages of user 1 and user 3.

At point c, the level of cooperation is reduced by choosing the message of user 1 to be transmitted cooperatively instead of user 3. Prior to this point, the base station transmits the messages for user 1 and user 3 to the first relay node, and the message for user 2 and user 3 to the second relay node. This again signifies that at this point the base station transmit power saved is more prominent than the extra relay transmit power required from reducing the level of cooperation. A similar argument applies to both point d and e, whereby the base station changes the cooperation level by altering the message to be sent cooperatively from user 3 to user 2, and from user 2 to user 1, respectively.

Finally, the discontinuity at point f is the most interesting, as it does not involve any change to the level of cooperation. At that point, the base station changes the set of users' messages to the two relay nodes. Initially, the message of user 1 is sent to the first relay node while the messages of user 2 and user 3 are sent to the second relay node. Although this is the least preferable as it requires the most transmit power at the base station, this enables the two relay nodes to satisfy their maximum transmit power constraint. As relay nodes are placed closer to the destination, the effect of the maximum relay transmit power constraint becomes less stringent, and therefore the base station can choose a better strategy by transmitting messages of user 2 to the first relay node and the messages of user 1 and user 3 to the second relay. This trend continues, and finally the base station chooses to send the message of user 3 to the first relay node and messages of user 1 and 2 to the second relay node. Note that this behavior will not occur if the maximum relay transmit power constraint is high or not imposed.

## VI. Conclusions

A study on the relationship between cooperation and the energy efficiency of two-hop relay systems has been presented. It was shown that a higher level of cooperation does not necessarily translate into a lower total power requirement. In some cases where the target rate is high, the most efficient strategy is not to cooperate but to simply relay disjoint sets of messages equally to all available relay nodes. This is because in those regimes, the transmission power cost at the base station to facilitate cooperation is dominant and outweighs the benefit of cooperation. This result is then applied to a relay-assisted broadcast system, in which the most energy-efficient strategy is proposed. It was shown that the scheme changes its level of cooperation as the transmission rate is varied, which is also dependent on the relay node position. A high level of cooperation is favored when the relay nodes are close to the base station, and decreases as the relay nodes move closer to the destination.

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