Two-stage Compensation for Non-ideal Effects in MIMO-OFDM Systems

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Abstract—In this paper, we propose a two-stage estimation and compensation of the carrier frequency offset (CFO) and the IQ imbalance in MIMO-OFDM systems. In the first stage, the receiver IQ imbalance is compensated along with CFO estimation. In the second stage, the transmitter IQ imbalance is compensated by taking advantage of structure of Alamouti spacetime block codes. Robust least square estimations are applied for all compensation processes. Compared with the conventional system that only compensates receiver IQ imbalance, simulation results show that BER performance of the proposed system is significantly improved.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been developed and widely used for high-speed wireless communication for years. Because of its simple equalization for frequency selective fading and excellent spectral efficiency, it is adopted for several international standards, *e.g.* IEEE 802.11a/g Local Area Network (WLAN) standard, and the European digital video broadcasting system(DVB-T). Similarly, multiple-input multiple-output (MIMO) technology has attracted much attention in emerging high-speed wireless communications. It provides higher performance and data throughput rates without additional transmit power and bandwidth. Therefore, the MIMO-OFDM system is the strongest candidate for emerging communication standards, such as IEEE 802.11n WLAN standard [1], and the 4G wireless systems like IEEE 802.16e(WiMAX) and 3GPP Long-term evolution (LTE).

For hardware implementation, zero-IF direct conversion becomes popular because of its lower cost and easier integration comparing with the super-heterodyne architecture. However, the zero-IF transceiver introduces IQ imbalance due to inevitable impairment in the analog components, leading to a mismatch between In-phase and Quadrature-phase branch which has no exactly 90° phase difference and non-equal amplitudes. The orthogonality of space-time block codes (STBC) in MIMO systems may be compromised because the circuits may suffer from different IQ imbalance in different antennas. Another non-ideal effect is CFO, which is an offset of carrier frequency between transmitter and receiver. CFO destroys the orthogonality between sub-carriers in OFDM systems. As a result, MIMO-OFDM systems is sensitive to both effects and the system performance may degrade severely.

Many researchers have presented different algorithms to estimate and compensate the IQ imbalance and the CFO in digital baseband domain since handling non-idealities in digital domain would be easier than designing an analog frontend circuit without imperfection. Horlin *et al.* [2] propose a joint estimation of CFO and receiver IQ imbalance. Liang *et al.* [3] modify the algorithm in [2] by a simple least square (LS) estimation. However, these literature focus on CFO and receiver IQ imbalance without the transmitter IQ imbalance. Tarighat *et al.* [4], [5] well compensate the effects of IQ imbalance in MIMO-OFDM systems, but the effects of CFO are not considered. Tandur *et al.* [6] propose an adaptive algorithm to compensate CFO and both transmitter and receiver IQ imbalance for SISO-OFDM system, but may suffers from slow convergence.

In this paper, we extend the algorithm in [3] to a twostage algorithm. The transmitter IQ imbalance, the receiver IQ imbalance and the CFO are estimated by using two known repetitive identical training sequence. Furthermore, we propose a compensation scheme for the transmitter IQ imbalance in Alamouti-STBC[7] based OFDM systems.

This paper is organized as follows. The system model for MIMO-OFDM with CFO and Tx/Rx IQ imbalance is introduced in Section II. In Section III, a two-stage joint estimation algorithm is proposed. Simulation results are shown in Section IV. Section V concludes this paper.

In this paper, the operator $(.)^*$, $(.)^T$, and $(.)^H$ denote the complex conjugate, transpose, and hermitian, respectively. The operator \otimes denotes the linear convolution between two signals. Boldface uppercase and lowercase letters are used for matrices and vectors, respectively.

II. SYSTEM MODEL

In a multiple-antenna based OFDM system, different antennas experience different IQ imbalances. However, we assume that different antennas are affected by the same CFO since they share the same local oscillators.

A. Transmitter IQ Imbalance Model

For the direct-conversion architecture, the transmitter antenna IQ imbalance model is illustrated in Fig. 1. The IQ imbalance is modeled by the amplitude mismatch ε and phase mismatch $\Delta \phi$. The baseband signal which goes through the IQ imbalance model in Fig. 1 is derived as:

$$\gamma(t) = \gamma_I(t) + j\gamma_Q(t)$$

= $\alpha_{TX}s(t) + \beta_{TX}s^*(t),$ (1)



Fig. 1. IQ imbalance model in Transmitter antenna.

where the signal gain $\alpha_{\scriptscriptstyle TX}$ and mirror gain $\beta_{\scriptscriptstyle TX}$ are difined as:

$$\alpha_{TX} = \cos\left(\Delta\phi\right) - j\varepsilon\sin\left(\Delta\phi\right) \tag{2}$$

$$\beta_{TX} = \varepsilon \cos\left(\Delta\phi\right) - j\sin\left(\Delta\phi\right). \tag{3}$$

If no IQ imbalance appears ($\varepsilon = \Delta \phi = 0$), then $\alpha_{TX} = 1$ and $\beta_{TX} = 0$, and (1) is reduced to $\gamma(t) = s(t)$. In the next subsection, this single-antenna transmitter IQ imbalance model is extended to fit a MIMO system.

B. MIMO Channel Model

The 2×2 MIMO channel model is illustrated in Fig.2. The received baseband signals μ 's are the linear convolution between the channel responses *h*'s and the transmitted baseband signal γ 's

$$\mu_{RX0} = h_0 \otimes \gamma_{TX0} + h_1 \otimes \gamma_{TX1} + n_0 \tag{4}$$

$$\mu_{RX1} = h_2 \otimes \gamma_{TX0} + h_3 \otimes \gamma_{TX1} + n_1, \tag{5}$$

where n_i denotes the channel noise, and γ_{TX0} and γ_{TX1} are the transmitted signals derived from (1)

$$\gamma_{TX0} = \alpha_{TX0} s_0 + \beta_{TX0} s_0^* \tag{6}$$

$$\gamma_{TX1} = \alpha_{TX1} s_1 + \beta_{TX1} s_1^*, \tag{7}$$

where s_0 and s_1 denote original signals transmitted by antenna TX0 and antenna TX1, respectively.

C. Receiver IQ Imbalance and CFO Model

In the following, we investigate the effect of receiver IQ imbalance and carrier frequency offset. The received signal can be modeled as [2], [3], [6]

$$y_{RX0} = \alpha_{RX0} e^{j(\Delta\omega t)} \mu_{RX0} + \beta_{RX0} e^{-j(\Delta\omega t)} \mu_{RX0}^* + v_0 \quad (8)$$

$$y_{RX1} = \alpha_{RX1} e^{j(\Delta\omega t)} \mu_{RX1} + \beta_{RX1} e^{-j(\Delta\omega t)} \mu_{RX1}^* + v_1, \quad (9)$$

where the parameter $\alpha_{\scriptscriptstyle RX}$ and $\beta_{\scriptscriptstyle RX}$ are receiver IQ imbalances defined as:

$$\alpha_{RX} = \cos\left(\Delta\phi\right) + j\varepsilon\sin\left(\Delta\phi\right) \tag{10}$$

$$\beta_{RX} = \varepsilon \cos\left(\Delta\phi\right) - j\sin\left(\Delta\phi\right),\tag{11}$$

and $\Delta \omega$ denotes CFO between the oscillator in transmitter and that in receiver. The signal v_i denotes the noise which goes through the analog front-end circuit in the receiver.

The whole system model of the 2×2 MIMO-OFDM with Alamouti-STBC is shown in Fig.3. We consider different IQ imbalances for different antennas, while the CFO effect is lumped together in the receiver.



Fig. 2. 2×2 MIMO channel model

III. ESTIMATION AND COMPENSATION OF CFO AND IQ IMBALANCE

In this section, we propose the compensation of CFO and IQ imbalance for the 2Tx2Rx WLAN communication system based on IEEE 802.11n standard. In [1], it specifies three different physical layer convergence procedure protocol data unit(PPDU) formats, including Non-High Throughput (Non-HT) format, HT-mixed format, and HT-greenfield format. We estimate CFO and IQ imbalance by using two repetitive long training sequences defined in these PPDU. Our estimation algorithm is composed of two stages. In the first stage, we estimate the CFO and the receiver antenna IQ imbalance by LS-method which has been proposed in [3]. We jointly estimate two parameters ϕ and $\overline{\beta}$ defined in [2],[3] at each receiver antenna. After the first-stage estimation, we compensate the long training sequence signal. As in [2], the signal y's in (8) and (9) can be approximately restored to the signal μ 's in (12) and (13). Based on equation (1)-(7), the compensated long training sequences become

$$\hat{\mu}_{RX0} = \alpha_{TX0} h_0 \otimes \mathbf{z}_{\mathbf{TX0}} + \beta_{TX0} h_0 \otimes \mathbf{z}^*_{\mathbf{TX0}} + \alpha_{TX1} h_1 \otimes \mathbf{z}_{\mathbf{TX1}} + \beta_{TX1} h_1 \otimes \mathbf{z}^*_{\mathbf{TX1}} + \mathbf{w}_0 \qquad (12)$$
$$\hat{\mu} = \alpha_{TX0} h_0 \otimes \mathbf{z}_{TX0} + \beta_{TX0} h_0 \otimes \mathbf{z}^*$$

$$+ \alpha_{TX1} h_3 \otimes \mathbf{z}_{\mathbf{TX1}} + \beta_{TX1} h_3 \otimes \mathbf{z}_{\mathbf{TX1}}^* + \mathbf{w}_1,$$
 (13)

where \mathbf{z}_{TX0} and \mathbf{z}_{TX1} are known transmitted long training sequences, and \mathbf{w}_i denotes the noise sequence.

In the second stage, the Least-Square estimation of the transmitter IQ imbalance is given by

$$\begin{bmatrix} \hat{\alpha}_{TX0} \\ \hat{\beta}_{TX0} \\ \hat{\alpha}_{TX1} \\ \hat{\beta}_{TX1} \end{bmatrix} = (\mathbf{P}^H \mathbf{P})^{-1} \mathbf{P}^H \begin{bmatrix} \hat{\mu}_{RX0} \\ \hat{\mu}_{RX1} \end{bmatrix}, \qquad (14)$$

where \mathbf{P} is defined as

$$\mathbf{P} = \begin{bmatrix} \hat{h}_0 \otimes \mathbf{z}_{\mathsf{TX0}} & \hat{h}_0 \otimes \mathbf{z}_{\mathsf{TX0}}^* & \hat{h}_1 \otimes \mathbf{z}_{\mathsf{TX1}} & \hat{h}_1 \otimes \mathbf{z}_{\mathsf{TX1}}^* \\ \hat{h}_2 \otimes \mathbf{z}_{\mathsf{TX0}} & \hat{h}_2 \otimes \mathbf{z}_{\mathsf{TX0}}^* & \hat{h}_3 \otimes \mathbf{z}_{\mathsf{TX1}} & \hat{h}_3 \otimes \mathbf{z}_{\mathsf{TX1}}^* \end{bmatrix},$$
(15)

where h_i is the estimated channel response. The algorithm of estimation of transmitter IQ imbalance can be applied to either single or multiple receiver antennas. For the single antenna



Fig. 3. The 2×2 MIMO-OFDM system with Alamouti-STBC scheme

$$\Lambda = \begin{bmatrix} \alpha_{TX0} H_0[n] & \alpha_{TX1} H_1[n] & \beta_{TX0} H_0[n] & \beta_{TX1} H_1[n] \\ \alpha_{TX1}^* H_1^*[n] & -\alpha_{TX0}^* H_0^*[n] & \beta_{TX1}^* H_1^*[n] & -\beta_{TX0}^* H_0^*[n] \\ \beta_{TX0}^* H_0^*[((-n))_N] & \beta_{TX1}^* H_1^*[((-n))_N] & \alpha_{TX0}^* H_0^*[((-n))_N] & \alpha_{TX1}^* H_1^*[((-n))_N] \\ \beta_{TX1} H_1[((-n))_N] & -\beta_{TX0} H_0[((-n))_N] & \alpha_{TX1} H_1[((-n))_N] & -\alpha_{TX0} H_0[((-n))_N] \end{bmatrix}$$
(20)

system, the equation (14) is reduced to

$$\begin{bmatrix} \hat{\alpha}_{TX0} \\ \hat{\beta}_{TX0} \\ \hat{\alpha}_{TX1} \\ \hat{\beta}_{TX1} \end{bmatrix} = (\mathbf{P}^H \mathbf{P})^{-1} \mathbf{P}^H \hat{\mu}_{RX0}, \qquad (16)$$

where **P** in (16) is defined as $[\hat{h}_0 \otimes \mathbf{z}_{\mathbf{TX0}}, \hat{h}_0 \otimes \mathbf{z}_{\mathbf{TX0}}^*, \hat{h}_1 \otimes \mathbf{z}_{\mathbf{TX1}}^*, \hat{h}_1 \otimes \mathbf{z}_{\mathbf{TX1}}^*].$

As shown in Fig. 3, the information is encoded by Alamouti STBC. The compensated signal in Rx antenna 0 in the first stage can be shown as

$$\hat{r}_{0}[n] = \alpha_{_{TX0}} H_{0}[n] s_{0}[n] + \beta_{_{TX0}} H_{0}[n] s_{0}^{*}[((-n))_{N}] + \alpha_{_{TX1}} H_{1}[n] s_{1}[n] + \beta_{_{TX1}} H_{1}[n] s_{1}^{*}[((-n))_{N}] + w_{0}[n]$$
(17)

$$\hat{r}_{1}[n] = -\alpha_{TX0}H_{0}[n]s_{1}^{*}[n] - \beta_{TX0}H_{0}[n]s_{1}[((-n))_{N}] + \alpha_{TX1}H_{1}[n]s_{0}^{*}[n] + \beta_{TX1}H_{1}[n]s_{0}[((-n))_{N}] + w_{1}[n],$$
(18)

where $H_i[n]$ is the channel response of the *n*th sub-carrier, the notation $((n))_N$ denotes $(n \mod n n)$ which N is the length of FFT/IFFT, and $w_i[n]$ denotes the noise. Using the property of index reversal, the above equations can be written in a matrix form

$$\hat{\mathbf{r}} = \Lambda \mathbf{s} + \mathbf{w},\tag{19}$$

where $\hat{\mathbf{r}} = [\hat{r}_0[n], \hat{r}_1^*[n], \hat{r}_0^*[((-n))_N], \hat{r}_1[((-n))_N]]^T$, $\mathbf{s} = [s_0[n], s_1[n], s_0^*[((-n))_N], s_1^*[((-n))_N]]^T$, and Λ is shown in (20) at the top of this page. The transmitter IQ imbalance can be compensated by decoding the Alamouti coded signal as

$$\hat{\mathbf{s}} = (\Lambda^H \Lambda)^{-1} \Lambda^H \hat{\mathbf{r}},\tag{21}$$

where Λ is composed as (20) with estimated channel response $H_i[n]$ and estimated transmitter IQ imbalances α 's and β 's from (14). For the system with two receive antennas, the signals \hat{s}_0 and \hat{s}_1 are estimated in each receive antenna, and the output is averaged

$$\hat{s}_{0} = \frac{\hat{s}_{0}(\text{estimated in } \text{Rx}0) + \hat{s}_{0}(\text{estimated in } \text{Rx}1)}{2} \quad (22)$$
$$\hat{s}_{1} = \frac{\hat{s}_{1}(\text{estimated in } \text{Rx}0) + \hat{s}_{1}(\text{estimated in } \text{Rx}1)}{2}. \quad (23)$$

The above process can be easily generalized to the system with more receiver antennas.

IV. SIMULATION RESULTS

A typical 2×2 MIMO-OFDM system based on IEEE 802.11n standard is simulated to evaluate the standard deviation of the estimated transmitter IQ imbalance and the BER performance of the proposed scheme. The system parameters used in our simulation are : OFDM symbol length N = 64with a 20MHz bandwidth, cyclic prefix of CP = 16, HIPER-LAN/2 type A channel model with AWGN. The antenna correlation parameter is set to be 0.6 both in transmitter and receiver. The CFO is set to be 50kHz as in [2] and [3]. The IQ amplitude imbalance of $\varepsilon = 2-5\%$, and the IQ phase imbalance of $\Delta \phi = 2$ -5° in each antenna are considered. Fig. 4 shows the standard deviation of the estimated Tx IQ imbalance error. The mirror gain β estimation accuracy achieves the level of 10^{-2} at SNR of 21dB, but the estimation of the signal gain α is not accurate. Fig. 5-7 show the bit error rate (BER) curves for uncoded 64-QAM MIMO-OFDM system. The IQ imbalance is set with the amplitude mismatch $\varepsilon \approx 2\%$ and phase mismatch $\Delta \phi \approx 2^{\circ}$ for Fig. 5-6 and $\varepsilon \approx 5\%$, $\Delta \phi \approx 5^{\circ}$ for Fig. 7. Fig. It is evident from these simulations verify that the proposed scheme improves the BER performance compared with the



Fig. 4. Standard deviation of the Tx IQ imbalance estimation error as a function of SNR when the CFO = 50kHz, the IQ imbalance of $\varepsilon = 3\%$ and $\Delta \phi = 3^{\circ}$.



Fig. 5. BER versus SNR for a 2 \times 1 MISO-OFDM system. The CFO = 50kHz, IQ imbalance of $\varepsilon\approx 2\%$ and $\Delta\phi\approx 2^\circ$ at each antenna.

system proposed in [3], in which the transmitter IQ imbalance is not considered. Furthermore, the results show that the BER performance of the proposed system is close to that of the ideal system.

V. CONCLUSIONS

This paper addresses the joint effects of CFO and IQ imbalance in a MIMO-OFDM system with Alamouti-STBC scheme. A two-stage algorithm has been developed to compensate for such effect in the digital domain. In the first stage, our scheme reduces the effect of CFO and receiver IQ imbalance. The transmitter IQ imbalance is further compensated in the second stage. Simulation results show a significantly improvement via the second stage IQ imbalance compensation.

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Fig. 6. BER versus SNR for a 2×2 MIMO-OFDM system. The CFO = 50kHz, IQ imbalance of $\varepsilon \approx 2\%$ and $\Delta \phi \approx 2^{\circ}$ at each antenna.



Fig. 7. BER versus SNR for a 2×2 MIMO-OFDM system. The CFO = 50kHz, IQ imbalance of $\varepsilon \approx 5\%$ and $\Delta \phi \approx 5^{\circ}$ at each antenna.

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