# Hardware-Efficient EVD Processor Architecture in FastICA for Epileptic Seizure Detection

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Abstract- Independent component analysis (ICA) is a key signal processing technique to improve the detection accuracy of epileptic seizures. It separate artifacts and epileptic signals, which facilitates the succeeding signal processing for seizure detection. FastICA is an efficient algorithm to compute ICA through proper pre-processing. In the preprocessing stage of the FastICA, eigenvalue decomposition (EVD) is applied to reduce the convergence time of iterative calculation of weights for demultiplexing received multi-channel signals. To calculate EVD efficiently, the Jacobi method is preferable since an array structure is proposed to decompose matrix efficiently by leveraging givens rotations. Multiple diagonal and off-diagonal processing elements run in parallel to calculate EVD. The microrotations can be realized efficiently by coordinate rotation digital computer (CORDIC), which calculates trigonometric functions using only addition, shift, and table lookup without dedicated multipliers. In this work, an approximate Jacobi method is adopted instead to reduce the number of iterations significantly. Optimized rotation angles can be calculated efficiently using shift-add operations for multiplications with coefficients of power of 2 in the diagonal processing elements. Normalization operation in the original mathematical formulation can be omitted due to signal re-scaling in both diagonal and offdiagonal processing elements. The number of processing cycles is reduced by 6 times for each sweep due to the reduced number of pipelining stages in the critical path. The approximate Jacobi method provides a 6x speedup (185-252 cycles instead of 1440 cycles) for a 6-channel EVD. An overall 77.2% area reduction is achieved due to arithmetic simplification and hardware reduction. The hardware architecture is verified by testing the human electroencephalogram (EEG) signals from the Freiburg Seizure Prediction EEG (FSPEEG) database.

# I. INTRODUCTION

Epilepsy, caused by abnormal discharges in the brain, is one of the most common neurological disorders. Around 1% of the people in the world are affected. Unfortunately, 25% of the epilepsy patients cannot be treated sufficiently by any available therapies [1]. Recently, alternative techniques, such as deep brain stimulation [1], [2] have been proposed. One essential technique required for the development of such a system is a robust on-line seizure detection method that can drive an antiepileptic device to suppress the seizure when a seizure occurs. A variety of seizure detection methods by electroencephalogram (EEG) signals have been proposed. Most of them classify the EEG segments from available database instead of continuous EEG recordings. The epilepticform discharges of a patient, however, may interfere with severe motion artifacts which affect the performance of detection. Artifacts, such as muscle noise and movement artifact, are serious problems for EEG analysis and affect the detection accuracy. It is therefore essential to separate the EEG signals and artifacts for practical applications.

Independent component analysis (ICA) has been used to separate the individual signals from the mixture signal resulted by different sources [3]. In recent years, several ICA methods have been promising technique for separating independent sources and applied to EEG for extracting artifacts and meaningful components [4]. Among all ICA algorithms, fast independent component analysis (FastICA) algorithm is the most hardware-efficient algorithm [5]. The complexity of the FastICA is significantly reduced since a fixed-point iteration scheme is leveraged as a measure of statistical independence to maximize non-Gaussianity.

To further reduce the number of iterations for FastICA computation, whitening the received signals has been proven useful in the preprocessing stage [5]. Whitening can be executed by eigenvalue decomposition (EVD) to uncorrelate received multi-channel signals. The Jacobi method is adopted for EVD realization since it exhibits a significantly higher degree of parallelism [6]. There are two Jacobi methods: exact and approximate. For the exact Jacobi method, the off-diagonal element is annihilated once [6]. In contrast, the approximate Jacobi scheme does not nullify the off-diagonal elements but demands only angle reduction from step to step [7]. Generally, the number of iterations of the approximate Jacobi algorithm increases but with a lower computational cost per iteration compared to the exact algorithm.

The approximate rotation scheme can be combined efficiently with a binary data representation in the coordinate rotation digital computer (CORDIC) scheme [8]. A CORDIC processor calculates the rotation angles by a fixed length sequence of shift-add operations and a few comparisons without dedicated multiplications. Hardware implementation of the approximate Jacobi algorithm was demonstrated on FPGA in [9]. In this paper, we propose a modified approximate Jacobi method to reduce the number of iterations. Rotation angle is optimized using shift and add operations for multiplications with coefficients of power of 2. Normalization of observed signals is omitted to further reduce hardware resources.

This paper is organized as follows. Section II reviews the Jacobi algorithm and approximate rotations. The proposed

architecture is described and numerical examples are shown to demonstrate the efficiency. The performance of the proposed scheme is tested and the results are shown in Section III. Finally, Section IV concludes the paper.

#### II. JACOBI-BASED METHODS

For a covariance matrix  $C_X$  of the received signals **x**, its eigenvalue decomposition is described as follows.

$$\mathbf{C}_{X} = \begin{bmatrix} c_{11} & \cdots & c_{1p} & \cdots & c_{1q} & \cdots & c_{1n} \\ c_{p1} & \cdots & c_{pp} & \cdots & c_{pq} & \cdots & c_{pn} \\ c_{q1} & \cdots & c_{qp} & \cdots & c_{qq} & \cdots & c_{qn} \\ c_{n1} & \cdots & c_{np} & \cdots & c_{nq} & \cdots & c_{nn} \end{bmatrix}_{n \times n} = \mathbf{E} \mathbf{D} \mathbf{E}^{T}$$
(1)

where  $C_X$  is an  $n \times n$  symmetric matrix, **D** is a diagonal matrix formed by *n* eigenvalues, and **E** is an orthogonal matrix formed by *n* eigenvectors. To derive them, a sequence of Jacobi rotations  $J_{pq}$  is applied (see [7] for details).

$$\mathbf{E} = \mathbf{J}_{12}\mathbf{J}_{13}\cdots\mathbf{J}_{18}\mathbf{J}_{23}\cdots\mathbf{J}_{28}\mathbf{J}_{34}\cdots\mathbf{J}_{78}$$
(2)  
$$\mathbf{D} = \mathbf{F}^T \mathbf{C} \cdot \mathbf{E}$$
(3)

$$\mathbf{J}_{pq} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & \cos\theta & \cdots & \sin\theta & \cdots & 0 \\ 0 & \cdots & -\sin\theta & \cdots & \cos\theta & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}_{n \times n}$$
(4)

The rotation angle  $\theta$  is used to nullify the off-diagonal elements. If each off-diagonal element is nullified once, a so-called sweep is completed.

# A. CORDIC Arithmetic

The CORDIC algorithm implements a rotation operation by taking advange of several mico-rotations. To simplify the hardware complexity, the micro-rotation angle  $\alpha_j = \tan^{-1}(2^{-j})$  is chosen such that only shift and add operations are required. The rotation direction  $\mu_j$  is  $\pm 1$  according to the input. The number of rotations is decided by the wordlength *b* of the input. The total rotation angle  $\theta$  is represented by

$$\theta = \sum_{j=0}^{b} \mu_j \alpha_j.$$
<sup>(5)</sup>

There are two major operation modes for CORDIC: rotation and vectoring modes. In the rotation mode, a vector is rotated by a given angle, which equivalently calculates sine and cosine functions. In the vectoring mode, the vector is rotated to lie on the x-axis. The original angle and magnitude are obtained after the computation.

### B. Exact Jacobi Method

Exact Jacobi method is named due to the fact that  $c_{pq}$  is exactly annihilated ( $c_{pq}' = 0$ ) by computing the coefficients c (cos $\theta$ ) and s (sin $\theta$ ).

$$\tau = \frac{c_{qq} - c_{pp}}{2c_{pq}} \tag{6}$$

$$t = \tan(\theta) = \frac{sign(\tau)}{|\tau| + \sqrt{1 + \tau^2}}$$
(7)

$$c = \frac{1}{\sqrt{1+t^2}}, s = c \cdot t \tag{8}$$

It is achieved using CORDIC which restricts rotation angles to  $\theta = \tan^{-1}(2^{-l})$  with  $l \in \{0, 1, 2, ..., b-1\}$  where *b* is the word length of the hardware system for the ease of hardware implementation.

$$\widetilde{\mathbf{J}}_{pq} = \cos\theta \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$$
$$= \frac{1}{\sqrt{1+2^{-2l}}} \begin{bmatrix} 1 & sign(\tau) \cdot 2^{-l} \\ -sign(\tau) \cdot 2^{-l} & 1 \end{bmatrix}$$
(9)

Since the angles are fixed, only the rotation direction  $sign(\tau)$  should be decided. Vectoring mode is used first to obtain the direction of rotation angles and rotation mode is used in the rest of the rotation process.

## C. Approximate Jacobi Method

Approximate Jacobi method takes only one of the elementary angles that is closest to the desired angle  $\theta$  with the tradeoff of  $c_{pq}' \neq 0$ . We define  $c_{pq}' = dc_{pq}$ , where

$$d = c^{2} - s^{2} - 2\pi s$$
  
=  $\frac{1 - 2\pi t - t^{2}}{1 + t^{2}} = \frac{1 - 2|\tau| 2^{-l} - 2^{-2l}}{1 + 2^{-2l}}$  (10)

 $0 \le |d| < 1$  since  $c_{pq}$ ' is expected to converge to zero. Fig. 1 shows the cases for l = 0 to 6. In order to find the specific lthat maps the minimal  $d(\tau, l)$  and to avoid computationallyintensive comparisons over all possibilities (l = 0, 1, 2, ..., b), efficient estimation of l is required. The corresponding error  $d(\tau, l)$  of  $\tau$  is minimized by choosing the feasible solutions in the shadowed region. This also reduces the search space of *l*. From the plot, we observe that both  $d(\tau, l)$  and *l* are reduced for arbitrary  $\tau$  by setting  $|d| \le 1/3$ .



Fig. 1. Reduction factor *d* versus  $|\tau|$ , *l* for approximate rotation.

Specifically, substituting d in the above equation with 1/3 yields the boundaries as follows.

$$|\tau_i| \le (2^i - 2^{-i+2})/3 \tag{11}$$

For a given value of  $\tau$ , l can be decided through piece-wise segmentation.

$$|\tau| \in [2^{k-2}, 2^{k}], \ k = \exp(c_{qq} - c_{pp}) - \exp(c_{pq})$$
(12)  
if  $k \le -2$ , then  $l = 0$   
if  $-2 < k \le 0$ , then  $l \in \{0, 1\}$ (13)  
if  $k > 0$ , then  $l \in \{k - 1, k, k + 1\}$ 

## D. Modified Approximate Jacobi Method

For the sake of finding the unknown k in (12), two 16-to-4 priority encoders are needed for a 16-bit system. The hardware resources can be reduced by leveraging arithmetic techniques. A 3-stage approach with 8 segmentations is used to replace the direct-mapped implementation with 4 stages and 16 segmentations, as shown in Fig. 2. This modified approach reduces the critical path and the decision boundaries can also be realized efficiently using shift operation.



Fig. 2. Reduction factor *d* versus  $|\tau|$ , *l* for modified rotation.

# E. Hardware Complexity Reduction

The hardware complexity of the proposed modified approximate Jacobi method can be further reduced. Two *b*-to-1 priority encoders were replaced by seven shifters, one 2-to-1 multiplexer, 4-to-1 multiplexer, and XOR-gate in the angle decision part in a diagonal processor. Normalization operation in the original mathematical formulation can be omitted due to signal re-scaling in both diagonal and off-diagonal processing elements. Overall, *b*/4 *b*-bit adders, shifters, eight multipliers, *b* registers, and a *b*-to-1 multiplexer are saved. To evaluate the hardware resources, the proposed architecture is mapped onto FPGA. Xilinx XC2V6000-6 Virtex-II FPGA is chosen and the clock frequency is set to 73.1MHz for a fair comparison with [10]. An overall 77.2% area reduction (4540 [10] vs. 1034 slices) is achieved, as shown in Fig. 3.



Fig. 3. Area comparison between approximate Jacobi and the modified one.

#### III. PERFORMANCE EVALUATION

## A. Database

Invasive EEG (IEEG) data is obtained from the Freiburg Seizure Prediction EEG (FSPEEG) database with authorization [11]. Three of six contacts were chosen from the seizure onset zone (i.e. from areas involved early in ictal activity, in-focus electrodes) of all 11 patients with totally 42 seizures. The remaining three electrode contacts were selected as not involved or involved latest during seizure spread (outfocus electrodes).

#### B. Simulation Results

For prevention of infinite loop due to fixed-point rounding error, setting maximal number of rotation is essential. Considering 2-3 rotations on average within each step, we set 10 as the maximal number of rotations. In addition, l will be enlarged by 2 if the value keeps the same for three times. The algorithm converges when the calculated off-diagonal elements are 10,000 times smaller than the original ones.

To evaluate the performance loss, correlation coefficients (r) of both Jacobi methods after ICA are listed in Table. I. The number of processing cycles is reduced from 18 (as published in [10]) to 3 for each step due to the reduced number of pipelining stages in the critical path. The approximate Jacobi method provides a 6x speedup over the exact Jacobi method

(185-252 cycles instead of 1440 cycles) for a 6-channel EVD because only 2-3 rotations are in need instead of 16.

 TABLE
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 PERFORMANCE LOSS OF APPROXIMATE JACOBI METHOD

	Approximate Jacobi method		Modified Approximate Jacobi method	
EEG1	cycle	207	cycle	252
	r	0.9828 0.9902 0.9846 0.9961 0.9835 0.9960	r	0.9851 0.9558 0.9428 0.8719 0.9603 0.8930
	mean	0.98887	mean	0.93482
EEG2	cycle	180	cycle	218
	r	0.9985 0.9985 0.9935 0.9504 0.8652 0.8315	r	0.9829 0.9960 0.9966 0.9561 0.8436 0.8241
	mean	0.9396	mean	0.93322
EEG3	cycle	166	cycle	185
	r	0.9926 0.9937 0.9863 0.9920 0.7569 0.7348	r	0.9867 0.9200 0.9880 0.9935 0.8249 0.7348
	mean	0.90938	mean	0.90799

Fig. 4 is an example to show the original EEG signals and results after ICA for 3-channel (part of a set of 6 channels) EEG signals. In the original data, artifacts disturb EEG in all channels, as in Fig. 4(a). After ICA processing, artifacts are gathered in one channel, as in Fig. 4(b). After being identified, the artifacts can be removed to improve the overall detection accuracy. The correlation coefficients of the ICA result are between 0.7348 to 0.9966.



Fig. 4. Representative EEG signals vs. FastICA result

## IV. CONCLUSIONS

A low-complexity approximate Jacobi method and its highly-parallel hardware architecture for EVD used in the FastICA have been proposed. The algorithm combines implementation properties of CORDIC processors for vector rotation with approximate rotation schemes. The binary data representation for CORDIC rotation is obtained easily and the closest angle is applied. The hardware-efficient implementation is achieved through hardware sharing and logic simplification. The higher performance of the approximate algorithm is due to the fact that the savings in the rotation computations are significantly higher than the cost of the additional sweeps. With the modified Jacobi algorithm, hardware requirement is further reduced without redundant normalizations. The modified approximate Jacobi method provides a 6x speedup and an overall 77.2% area reduction compared to the conventional method. FastICA equipped with EVD is able to achieve good performance in improving detection time, accuracy, and false detection.

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